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Project VILAGIL-MaaS: literature review

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Institut de Recherche en Informatique de Toulouse
CNRS - Toulouse INP - UT3 - UT1 - UT2

Project VILAGIL-MaaS: literature review

Kamaldeep Singh Oberoi¹, Valérie Camps^{1,*}, Elsy Kaddoum², Stéphanie Combettes¹, Marie-Pierre Gleizes¹, Sébastien Parenty³, Isabelle Laplace³, Pratik Rai⁴, Guillaume Dufour⁴, Eugénie Avril⁵, Loïc Caroux⁵

¹ IRIT, Université de Toulouse, UT3 Paul Sabatier, Toulouse, France

² IRIT, Université de Toulouse, UT2J Jean Jaurès, Toulouse, France

³ ENAC, Université de Toulouse, Toulouse, France

⁴ ONERA, Centre de Toulouse, Toulouse, France

⁵ CLLE, Université de Toulouse, UT2J Jean Jaurès, Toulouse, France

Valerie.Camps@irit.fr

* contact author

May 17, 2022



Institut de Recherche en Informatique de Toulouse
CNRS - Toulouse INP - UT3 - UT1 - UT2

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Kamaldeep Singh Oberoi¹, Valérie Camps^{1,*}, Elsy Kaddoum², Stéphanie Combettes¹, Marie-Pierre Gleizes¹, Sébastien Parenty³, Isabelle Laplace³, Pratik Rai⁴, Guillaume Dufour⁴, Eugénie Avril⁵, Loïc Caroux⁵

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³ ENAC, Université de Toulouse, Toulouse, France

⁴ ONERA, Centre de Toulouse, Toulouse, France

⁵ CLLE, Université de Toulouse, UT2J Jean Jaurès, Toulouse, France

Valerie.Camps@irit.fr

* contact author

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Abstract. This document concerns the MaaS (Mobility As A Service) action of the VILAGIL project and contains the literature review carried out by each partner (CLLE, ENAC, ONERA and UPS with IRIT-SEPIA team and IRIT-SMAC team) involved in this action.

The objective of the MaaS research action is to develop innovative functionalities that can be integrated in a virtual mobility assistant in order to make it personalized. This assistant aims to proactively offer multimodal routes adapted to its user (according to his needs, mobility profile and mobility preferences) by informing him/her of the environmental impact of each proposed route.

Keywords: Mobility-as-a-Service, Personalisation, Gamification, Transport Mode Choice Modelling, Macroscopic Mathematical Traffic Modelling, Mobility Assistant

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Projet VILAGIL-MaaS

Revue de littérature

RÉSUMÉ

Ce rapport concerne l'action MaaS (Mobility As A Service) du projet VILAGIL et porte sur la revue de littérature réalisée par chaque partenaire (CLLE, ENAC, ONERA et UPS avec l'équipe IRIT-SEPIA et l'équipe IRIT-SMAC) impliqué dans cette action.

L'objectif de l'action de recherche MaaS est de développer des fonctionnalités innovantes pouvant être intégrées dans un assistant virtuel de mobilité afin de le rendre personnalisé. Cet assistant vise à proposer de manière proactive des itinéraires multimodaux adaptés à son utilisateur (en fonction de ses besoins, de son profil de mobilité et de ses préférences de mobilité) en l'informant de l'impact environnemental de chaque itinéraire proposé.

CONTRIBUTEURS

Kamaldeep Singh Oberoi¹, Valérie Camps¹, Elsy Kaddoum¹, Stéphanie Combettes¹, Marie-Pierre Gleizes¹, Sébastien Parenty³, Isabelle Laplace³, Pratik Rai⁴, Guillaume Dufour⁴, Eugénie Avril⁵, Loïc Caroux⁵.

1 - IRIT, Université de Toulouse, UT3 Paul Sabatier, Toulouse, France

2 - IRIT, Université de Toulouse, UT2J Jean Jaurès, Toulouse, France

3 - ENAC, Université de Toulouse, Toulouse, France

4 - ONERA, Centre de Toulouse, Toulouse, France

5 - CLLÉ, Université de Toulouse, UT2J Jean Jaurès, Toulouse, France

Contact : Valerie.Camps@irit.fr

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Chapter 1

Introduction and Context

1.1 Introduction

The ambition of the VILAGIL (2020-2030) project is to address the problem of congestion in the Toulouse area as well as its negative economic, environmental, social and public health externalities, thanks in particular to the deployment of new integrated mobility services. Concretely, the VILAGIL project aims to encourage the inhabitants of the Toulouse metropolis to leave their personal cars to adopt more virtuous modes of displacement.

VILAGIL's ambition is divided into 4 objectives:

1. to decarbonize the territory by promoting electromobility and the use of active mobility;
2. to decongest the territory by developing a better link between the mobility of goods and people and regional development;
3. to make Toulouse a territory of excellence, a showcase for new forms of mobility by stimulating the development of the mobility of the future, in particular, urban air mobility;
4. to promote mobility for all by using new technologies in order to develop and integrate new mobility solutions.

VILAGIL is made up of 15 key actions (including 9 investment actions and 6 subsidy actions) that act on both the supply and demand of future mobility, notably by seeking to transforming demand by changing usage behavior and by seeking to enriching the supply of alternative transport to the private car.

Paul Sabatier University contributes to this project by being responsible for 4 of the 6 subsidy actions :

1. Urban planning action: This action concerns the use of digital simulations at all stages of development of a planning project, by all these actors to optimize both the use of space and the means available;
2. AutOCampus action: This action aims at setting up an experimental field (a first 1800 meters course, with a 600m extension) of connected and/or autonomous vehicles on the site of the University of Toulouse Sabatier;
3. Data and mobility action: This action aims to define mechanisms for integration, storage and automatic access to masses of heterogeneous data, but also to identify and deal with the legal risks associated with the use of these masses of heterogeneous data;
4. MaaS services action: The objective of this action is to develop innovative functionalities that can be integrated into a virtual mobility assistant in order to make it personalized so that it can offer the most suitable multimodal transport choice to its user.

This research report concerns the last action, namely the MaaS action. It presents a synthesis of the positioning work carried out by the different partners involved.

1.2 Context

The global objective of the MaaS action is fully in line with the global objective of the VILAGIL project, i.e. to encourage the inhabitants of the Toulouse

metropolitan area to give up their private cars and adopt more virtuous modes of transport. The idea in this action is to increase the use of new forms of mobility such as carpooling and multi-modality in transport.

More concretely, the objective of the MaaS research action is to develop innovative functionalities that can be integrated into a virtual mobility assistant in order to make it personalized so that it can **proactively** offer the most suitable multimodal transport choice to its user.

Four partners are involved in this action: CLLE, ENAC, ONERA and UPS with IRIT-SEPIA team and IRIT-SMAC team. This MaaS research action is organized into four sub-actions, including the mobility assistant personalization action, which brings together all the sub-actions to provide a service adapted to the user. These sub-actions include:

1. Blocks for customizing the mobility assistant. These blocks will be constructed by observing the user in his or her daily activities, in order to extract and learn from these observations his or her mobility profile and propose multimodal transport solutions that best suit him or her at a given moment. The aim will be to integrate carpooling into these proposals, in the same way as existing public transport (UPS IRIT-SMAC team);
2. Economic modeling of user behavior to identify the factors that determine multimodal mobility choices for each user and measure their relative importance in order to propose the best travel solution at a given time (ENAC);
3. Optimized mathematical traffic models to predict the evolution and state of traffic in the short and medium-term in order to better estimate travel times and optimize travel (reasoned proposals for travel modes in response to a request) (ONERA and UPS IRIT-SEPIA team);
4. Gamified interfaces design, centered on the needs, expectations and constraints of the final users (to be adapted to a large population such as elderly people who may have a handicap due to their age), which meet the need while creating an emulation in its use, thus arousing a motivation for its users (CLLE).

These sub-actions as well as their links are illustrated in 2.4.

The rest of this research report aims at gathering the different contributions of the different partners involved in the MaaS action, in terms of literature review.

Chapter 2

Customization of a mobility assistant (IRIT-SMAC)

2.1 Introduction

In World Economic Forum's (WEF) annual meeting held in Davos in 2021, the founder of WEF, Klaus Schwab, launched his book entitled "Stakeholder Capitalism: A Global Economy that Works for Progress, People and Planet" which is co-authored with Peter Vanham. Klaus has been an advocate of *stakeholder capitalism*¹ as opposed to *shareholder capitalism* which has largely been accepted in most countries of the world. In a nutshell, the idea behind stakeholder model is for a company to take into account the long-term effects of its businesses on all the stakeholders which are directly or indirectly affected by its decisions and functioning. These stakeholders can be the employees of the company, its customers, the government as well as the planet and the environment. Whereas shareholder model focuses only on the short-term profit of the shareholders (sometimes at the expense of other stakeholders), in stakeholder model, people and the planet are at the center. Although the concept of stakeholder capitalism has existed since 1970s, with the discussion at WEF's annual event in 2021, one can see the willingness of the policy makers from around the world to gradually move towards more inclusive socio-economic models which should result in the sustainable growth in different sectors of the global economy while keeping the environment preservation at the forefront.

In the mobility and transportation sector, several public and private actors have started collaborating for the development of environment-friendly, inclusive and affordable solutions. For example, European Union (EU) funded Personal TRansport Advisor (PETRA)² project in 2014 in order to bring together different mobility service providers and end-users via an integrated platform. This platform takes into account the data about various modes of transport as well as the uncertainty in the city transit network to provide up-to-date information about multi-modal transportation services to the end-users. The uncertainty of transportation services is modeled in terms of delay in arriving time, difficulty in boarding a crowded bus, weather conditions etc. [1]. This project led to the public-private-academic collaboration in two European cities (Rome and Venice in Italy) and one Asian city (Haifa, Israel). Whereas in Rome and in Haifa the objective was to provide environment-friendly on-demand journey planning abilities to the commuters, in Venice the platform was used to enable tourists in finding interesting places to visit while maintaining crowd distribution at famous tourist spots [1].

The Seamless Integrated Mobility System (SIMSystem) project, launched by WEF in partnership with Deloitte Consulting LLP in 2017, has the aim of developing a mobility system which, while integrating different modes of transport, improves the accessibility of marginalized communities to various mobility services [2]. The region of Detroit (U.S.) was chosen as the testing ground for the project due to its multi-stakeholder mobility setup and the socioeconomic disparities between different sections of its population [3]. With successful public-private cooperation, SIMSystem project's pilot initiative showed the potential of data-driven technologies for improving the quality of life of the economically weaker sections. The findings from the region of Detroit were also extrapolated and applied to other cities around the world, in U.S.: Los Angeles, San Diego and Ann Arbor, in Lisbon (Portugal), in London (U.K.), in Singapore, in Tokyo (Japan), in Tel Aviv (Israel) and in Windsor (Canada) [4].

Toulouse, the fourth largest city in France, is home to Airbus and European aerospace industry employing around 70,000 people. Since Toulouse has be-

¹www.weforum.org/agenda/2021/01/klaus-schwab-on-what-is-stakeholder-capitalism-history-relevance

²cordis.europa.eu/project/id/609042

come the industrial and research hub in the region, the population of the city has seen constant growth over the years. In order to understand the environmental impact of home-work trips of the employees in the industrial zone close to the airport Toulouse-Blagnac, the city of Toulouse launched EU-funded project called COMMUTE (Collaborative Mobility Management for Urban Traffic and Emissions reduction)³ in 2017. The project saw very fruitful public-private collaboration between the city of Toulouse and major industrial partners such as Airbus, Sopra-Steria, Safran among others. In fact, this collaborative governance model was presented as an innovative feature of the project [5]. One of the main objectives of the project, besides public-private collaboration, was to incentivize the change in travel behaviour of the employees of the industrial partners and encourage them to use sustainable means of transport (cycling and car-pooling to and from work were given importance). In addition, new ways of working such as work-from-home and modulated work schedules were tested which in turn reduced the need for travel and eased the regular traffic congestion.

From the projects discussed above, one can notice that for successful development and deployment of urban mobility services different public, private and academic stakeholders come together, define common goals and keep user needs and environment conservation at the center of their policies. These practices highlight the guidelines which should be followed for an equitable development of mobility services. WEF consolidated eight such guidelines [6] which are summarized below:

- **Data sharing:** Different stakeholders should share anonymous and useful data among themselves and, if possible, with the general public.
- **Public space usage and infrastructure impact:** Mobility partners should gradually shift from car- and truck-centric urban road network and redesign public space to encourage the use of more environment-friendly modes of transport.
- **Safety:** In addition to fulfilling in-vehicle safety requirements, safety of passengers in public transports and that of vulnerable road users should also be considered.
- **Inclusion and equity:** Mobility services should be provided to all sections of the society as well as physically-challenged users should be kept in mind while developing such services.
- **Fair work:** Working conditions of the employees of different mobility partners should be adequate and a comprehensive training program should be put in place in order to ensure the safety of all the workers.
- **Shared mobility and pooling:** Use of shared modes of transport should be encouraged and pooling services should be integrated within the existing public transit networks.
- **Clean transition:** Users should be incentivized for opting environment-friendly mode of transports and adequate infrastructure, like charging stations, should be set-up in order to ensure the continuity of sustainable mobility services.
- **Multi-modal integration:** Mobility coverage should be maximized by providing multi-modal transportation services to isolated communities using Mobility-as-a-Service (MaaS) platforms.

³www.toulouse-metropole.fr/projets/commute

Over the past few years, Mobility-as-a-Service (MaaS) has become a hypernym for mobility partners proposing multi-modal integration and payment facilities. Due to a variety of existing services, it is not easy to define, in concrete terms, what constitutes MaaS. From the plethora of definitions discussed in [7], one can synthesize that MaaS platform provides a *digital interface* for *user-centric, flexible, multi-modal and sustainable* mobility services which result from the *cooperation between public and private stakeholders* and encourage users to *reduce private car dependency* (move from asset ownership to access on-demand). Furthermore, users can *plan their journeys* and *buy mobility services provided by same or different operators* using *personalized payment and/or subscription options for various mobility packages*.

From this brief description about MaaS, one can notice that the major objectives of MaaS (or MaaS-like products) align with (some of) the guidelines of city mobility proposed by WEF. Indeed, the idea of multi-modal integration, development of sustainable mobility services and access to those services, by various sections of the society, are at the center of different MaaS trials. Since one of the goals of MaaS is to promote the use of bike-sharing services, for its successful deployment, the existing urban infrastructure will have to be re-purposed to support such modes of travel. Finally, MaaS platforms and services can not be implemented without fruitful collaborations between public and private organizations and transport operators [8]. And one (perhaps, the most important) of the talking points between different partners is the sharing of different kinds of data. In the report published by the House of Commons of the Parliament of U.K. ([9], Page 7), it is noted that *We recognise that if data sharing is not encouraged there is a serious risk schemes will never be developed at a scale that moves beyond piloting activity and remains sustainable. Data must be shared in an anonymised and aggregated manner to protect passengers' personal information and the commercial interests of transport operators and MaaS platform providers*.

2.2 Mobility-as-a-service (MaaS)

2.2.1 What is MaaS?

The concept of MaaS was introduced by Sampo Hietanen as *a mobility distribution model in which a customer's major transportation needs are met over one interface and are offered by a service provider* [10]. He further mentioned the idea of developing transportation system *with, to and by the users* as an *inter-connected ecosystem consisting of transport infrastructure, transportation services, transport information and payment services*. As this concept gained traction, more definitions were proposed by the research community. For example, [11] defines MaaS as *The provision of transport as a flexible, personalised on-demand service that integrates all types of mobility opportunities and presents them to the user in a completely integrated manner to enable them to get from A to B as easily as possible*. [12] defines MaaS as *user-centric, intelligent mobility management and distribution system, in which an integrator brings together offerings of multiple mobility service providers, and provides end-users access to them through a digital interface, allowing them to seamlessly plan and pay for mobility*. The MaaS operator acts as an intermediary between the end-users (demand side) and the mobility service providers (supply side) and is responsible for integrating multiple modes of transports along with payment methods and subscriptions into a single digital platform (Figure 2.1). In addition to the core mobility service providers, mobility supporters (such as entertainment service providers, parking operators etc.) can also be integrated into the platform and

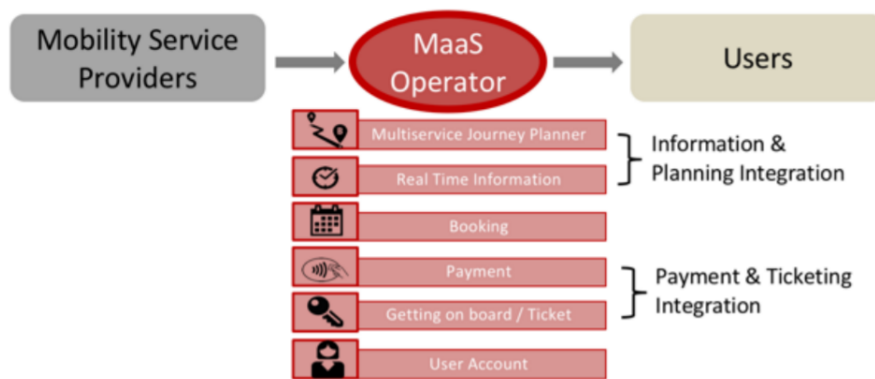


Figure 2.1: MaaS Concept as defined in [12]

the user subscription plans. The reader is referred to [7] for a (non-exhaustive) list of definitions.

2.2.2 Integration of services in MaaS

From the above discussion, it is clear that MaaS as a concept is not very well defined and different stakeholders and organisations interested in the development of MaaS propose definitions and general vocabulary according to their understanding of this concept. In other words, it can be said that even though everyone shares a common vision about MaaS, there is a need to concretize this vision. Towards this direction, in the literature, several authors have defined a *topology* of MaaS which demonstrates the integration of various services that MaaS should or could offer in the short/long term.

One of the widely agreed upon topology of MaaS has been developed in [7]. The authors propose 5 (0-4) levels of integration (Figure 2.2), with higher levels integrating more number of services, although the authors have precised that it should not be considered that higher levels are better than the lower levels since they are independent and vary with customer needs. Different levels of integration proposed are as follows. Since Level 0 represents no integration, it is not discussed in detail.

- **Level 1:** It represents the *integration of information*. The services at this level incorporate the multi-modal travel planner and price information for individual trips. Google Maps is an example where users can search for the best trip along with the associated travel cost.
- **Level 2:** This level represents the *integration of booking and payment*. Here, the users can find, book and pay for their individual trips. This level is useful in combination with level 1 in order to help the users identify and choose between travel options.
- **Level 3:** It represents the *integration of service offers*. It gives the opportunity to the users to purchase subscriptions plans and/or mobility bundles for individuals as well as family members and in exchange use multiple modes of transport for all travel needs from single digital platform. The applications which provide level 3 services can be considered as MaaS applications.
- **Level 4:** It represents the *integration of societal goals* in the long run. The services at this level incorporate the services at lower levels as well as



Figure 2.2: MaaS topology with different levels of integration as defined in [7]

Level	Description	Explanation
0	Base level, relative to today	There are account-based systems in place, individual modes of transportation already have a digitized interface and the traveler has information available online for each
1	One-to-one integration between some private services	Services start to develop joint offerings. For example, tolling and car park; private car and ferry; and park-and-ride bus services
2	Integrated payment and ticketing across modes of limited public and private modes of transportation services	Greater integration of services occurs, but this time between privately operated and public transportation. Integration shows promise but other public transit modes are skeptical and continue to stay separate
3	Unified interface for single account used in multiple modes of transport services	Instead of having multiple channels, an interface is unified across modes, providers and services, where the traveler can plan their journeys
4	All modes are integrated, private and public, including routing, ticketing and payment	Open data and standards are defined and commonly used
5	Active artificial intelligence choices are taken based on travel preferences and near real-time data for ad-hoc changes	Based on traveler-specific behavior and profiling, minimal intervention is needed by the traveler for an end-to-end journey
6	MaaS connects beyond mobility, interfacing with the IoT, smart buildings and smart cities	As MaaS evolved, so did other systems that were involved in the traveler's day. For example, food, groceries, entertainment and sport, to provide seamless interfaces that encompass the traveler's ecosystem

Figure 2.3: MaaS topology as defined in [13]

government policies and regulations proposed in order to incentivize the change in travel preferences to more sustainable modes.

In 2018, an article published in Traffic Technology International [13] presented a technology profile in which 7 (0-6) levels of integration were proposed. Since this publication is geared towards industry, the article did not go into the technical details of different integration levels. However, at higher integration levels, it goes beyond mobility and discusses the possibilities to integrate various digital services like smart home and IoT as well within MaaS platforms. The description of different integration levels discussed in this article is given in Figure 2.3.

Another important contribution related to the integration of services in MaaS is put forth in [14]. The authors propose 6 (0-5) levels of integration by taking into account the user perspective and the cognitive user effort required to use the mobility services. It is argued that since MaaS wants to replace the use of private car, choosing which gives the flexibility to the users and is the easiest choice to make (given one owns a private car) in terms of the required cognitive effort, the services proposed within MaaS should be as easy, convenient and flex-

ible to use in order to make them scalable. According to the proposed taxonomy, lower levels of integration require higher cognitive effort whereas higher level requires lower cognitive effort. The levels proposed are as follows:

- **Level 0** represents *no integration* of any sort. The services offered are unimodal, however, there is possibility to book and pay for those services. Highest cognitive effort is required in order to make multi-modal door-to-door journeys.
- **Level 1** represents *basic integration* of information for some travel modes. A multi-modal journey planner is incorporated into the services, however, the payment for using different modes is kept separate.
- **Level 2** represents *limited integration* and introduces the possibility of coordination between different modes of travel. To some extent, the payment and booking of multi-modal journeys is also possible.
- **Level 3** represents *partial integration* and allows for the planning, booking and payment of multi-modal journeys through a single platform with the possibility of making door-to-door journeys under limited circumstances.
- **Level 4** represents *full integration under certain circumstances* and allows the informational and transactional integration for multi-modal journeys, although not in every geographical area.
- **Level 5** represents *full integration under all conditions* and supports complete integration required to undertake multi-modal door-to-door journeys in every geographical area with minimum cognitive effort required from the user.

2.2.3 Some existing MaaS trials

Ever since its inception in 2014 [10], MaaS has seen an increasing interest, both from private and public entities, for its successful development, deployment and acceptance on large scale. This growing interest has lead to a lot of MaaS pilot projects, some of which are listed below:

- **Whim:** The most well documented and well studied pilot project which has lead to the launch of MaaS Global⁴ as one of the pioneers in the industry. This project has been operational in Helsinki, Finland since 2016 and integrates public transport with rental car services, taxi services, bike and car sharing service. It offers different payment methods to the users, including both Pay-As-You-Go (PAYG) schemes as well as subscription plans [8, 15].
- **UbiGo:** The pilot version of UbiGo commenced in Gothenburg, Sweden in 2013-14 and proposed the concept of “transportation smorgasbord”, i.e., the assortment of multiple mobility services tailored to the users [8, 16, 17]. The project received positive feedback from the users which lead to the launch of the second version UbiGo 2.0 in 2019 for the city of Stockholm integrating public transport, taxi, rental car, and car sharing options.
- **Tripi:** The trial⁵ started in Sydney, Australia in 2019 with the aim of replicating the results of European MaaS trials and addressing the existing knowledge gaps. Sydney, having an open-data platform and multiple public and

⁴whimapp.com/about-us

⁵imoveaustralia.com/project/maas-trial-sydney

private mobility service providers, has the perfect setting for implementing MaaS integrating public transport (including train, tram, ferry, and bus) and car-based transport services (traditional taxi, ridesourcing, and car sharing) [8, 18].

- **EC2B:** Another successful trial⁶ which was implemented in Gothenburg, Sweden in 2019. The objective of the trial was to test the deployment of a mobility platform in an apartment complex targeting its tenants. Due to the absence of a dedicated car parking facility within the residential area, people were nudged towards making sustainable mobility choices. This project integrated bike- and car-sharing travel modes with the traditional public transport. Recently, the project has been developed further (project MoJo⁷) to include the employees at Chalmers University of Technology and other companies in the area.
- **NaviGoGo:** Scotland's first MaaS pilot⁸ operational in 2017-18. Specifically designed for young people (16-25 years old), it provided a personalized journey planner responsive to the user's profile, taxi-splitter tool to split the taxi fare between friends, a 'Discover a Destination' database containing local transport information etc. [19]. Interestingly, the trial participants were actively involved in the decision-making process and provided recommendations to the project team in order to develop the complete system. Hence, the system is designed 'by the young people, for the young people' [19].
- **Real Estate × MaaS:** MaaS pilot⁹ initiated in September 2020 in Tokyo, Japan with the aim of improving existing real estate using the MaaS concept. Tested by a major real estate developer, this project aims to provide mobility services based on the specific needs of (residential or commercial) properties and create new lifestyles for the people living or working within such properties. They have partnered with MaaS Global Ltd., the creators of Whim, to provide mobility services to hotels, retail facilities, office buildings etc., ultimately increasing the real estate value.

There are many more MaaS pilot projects which have been or are being launched. The reader is referred to [8] (Chapter 3, Page 42) for the list of existing MaaS schemes and their current status.

2.2.4 Some concerns about MaaS

Although MaaS has seen a growing interest, there are still some apprehensions related to its acceptance and scalability. These have been observed by many researchers which may make its acceptance unlikely in the short term and even over long term. Some of the points raised in different scientific publications are discussed below.

- The report published by KiM Netherlands Institute for Transport Policy Analysis [20], while discussing the changes in travel preferences and travel behaviour, raised some important points to be kept in mind while designing MaaS (or MaaS-like) platforms.

⁶ec2b.se

⁷<https://www.chalmers.se/en/news/Pages/Mobility-as-a-service-tested-on-Campus-Johanneberg.aspx>

⁸www.the-espgroup.com/project/navigogo

⁹www.mitsufudosan.co.jp/english/corporate/news/2020/1215_02/

- One of the goals of MaaS is to create circumstances which lead to the change in travel preferences and travel behaviour of the users in the long term. However, in this report the authors mention, while presenting the results of various MaaS trials, that **it is not an easy task to change travel behaviour of the people**, especially when there is no trigger to do so. This puts into question one of the major objectives of MaaS. It is observed by the authors that people have *travel behaviour inertia* as well as safety related concerns when asked to change their travel preferences. Recall that one of the guidelines proposed by WEF is related to safety. Furthermore, it is observed that people are more likely to change travel behaviour for exceptional trips rather than daily work-related trips.
 - Another objective of MaaS is to **replace private cars** with public and shared transport modes. The authors observe that this **is another challenge** related to the acceptance of MaaS schemes since for many people, especially belonging to weaker sections of the society, owning a private car represents an important milestone in their lives and it provides them the necessary freedom and social status. The authors recommend that for the general acceptance of MaaS, it should not require a lot of compromise on the autonomy, flexibility and reliability that people enjoy with private cars. In addition, since a mobile application is the main interface for people to use MaaS, it should have a simple and intuitive design and should include behaviour change support systems [21] (like persuasion strategies and gamification-based systems) so as to motivate and encourage the use of sustainable means of traveling.
 - Finally, **personalisation to the users' preferences is a key requirement** for MaaS. Users' preferences regarding the payment methods and bundles as well as their mobility preferences should be taken into consideration while designing MaaS platforms.
- The report published by the House of Commons Transport Committee of the U.K. Parliament [9] has also made some recommendations to the (U.K.) government for creating a supportive ecosystem for the development of MaaS.
 - The report notices that **MaaS may contribute to the worsening of digital and social exclusion** since it is mainly catered towards younger people who are tech-savvy, live in urban areas, have a bank account and the income to pay for mobility services and have good internet connection. It is recommended that the government pushes the academic research and entrepreneurial ventures to develop MaaS services for all the sections of society.
 - Another important recommendation is to avoid **boundary limitations** i.e., MaaS solutions being available in some places and not others. There is a need to have a national level plan for the development and deployment of MaaS.
 - Government should also propose regulations so that small businesses are able to compete with well established companies in the industry and market monopolization can be avoided.
 - There is also an immediate need of **regulations in order to protect customers' rights** especially in the absence of services initially promised by the MaaS operators.

- As mentioned before, **data sharing between different stakeholders and organisations is crucial** and the government must ensure that it is undertaken by these organisations.
- In a recently published book [8], the Australian authors have discussed five MaaS trials and presented the shortcomings of the existing MaaS schemes.
 - For example, the authors note that the emergence of **multiple MaaS schemes in one country is counter-intuitive to the larger goal of integration** which MaaS schemes want to implement in the transportation sector. This point is synonymous with the point raised in the report presented in the U.K. parliament about the need to have (at least) a national vision for MaaS.
 - Another point raised in the book is about the **absence of standards for collaborating and sharing data** between different public and private partners involved in the development of MaaS. Here, as mentioned before, the government has an important role to play.
 - The book also discusses the **social exclusion as well as the boundary limitations** that MaaS can cause which again is not in the same direction as the goals originally set for MaaS schemes.
 - The question about the **willingness of the people to use MaaS platform** on a daily basis is also raised as well their willingness to pay for such services. Here, the personalisation of payment bundles and subscriptions comes into picture. Authors discuss four categories of people, *Car shedders* who want to try to live without owning a car; *Car accessors* who want to gain access to a car without purchasing one; *Simplifiers* who want to use an easier way to manage their multi-modal mobility needs; and *Economizers* who want to have a cheaper access to public transport, originally presented in [22], who are willing to try and pay for the mobility services provided by MaaS platform.
 - Finally, the authors note that since in the current market scenario, different services are proposed by different public and/or private organisations, integrating them onto a single platform is a behemoth task which presents an **institutional barrier** for the large scale deployment of MaaS.

As can be seen from the points noted above, the organisations, researchers and governing bodies from different countries share (more or less) the same concerns regarding the large scale and everyday use of MaaS. Finding solutions to such concerns should be the center of attention during the future MaaS trials.

2.3 Project Vilagil

2.3.1 Introduction

Toulouse is the capital of Haute-Garonne department and is located in Occitanie region in the south-west. Being home to Airbus and European aerospace industry, Toulouse has seen a constant growth in the number of inhabitants since 1968¹⁰. According to the National Institute of Statistics and Economic Studies (INSEE), the population of the metropolis of Toulouse in 2018¹¹ was around

¹⁰www.insee.fr/fr/statistiques/2011101?geo=EPCI-243100518#consulter-sommaire

¹¹www.insee.fr/fr/statistiques/1405599?geo=EPCI-243100518

783K, out of which 26.1% was in the age group of 15-29 years, followed by 20.9% in the age group of 30-44 years. Total area of the metropolis is around 458.2 km², hence the population density (number of people living in an area of 1 km²) is around 1710. Toulouse is also considered as a cultural hub in the region which attracts a large amount of tourists every year.

Due to the constant increase in population, especially younger people having active lifestyle, and the presence of aviation and aerospace industry, Toulouse suffers from traffic congestion and low air quality index. Hence, the metropolis of Toulouse has decided to invest in projects which would help in promoting sustainable means of transport and encourage people to chose public and/or shared transport services instead of private cars for travel. With these objectives in mind, and building on the success of the recently terminated project COMMUTE [5], another project called Vilagil¹² was launched in 2020 by the metropolis of Toulouse in partnership with the local state-owned mobility operator Tisséo Collectivités. Main objectives of this project can be summarized as follows:

- Data-driven development of smart and sustainable modes of transport
- Promote user-centric change in everyday travel behaviour
- Ecological transformation of the existing infrastructure and better management of home-work trips
- Promote “mobility for all” by involving citizens in the governance of innovative solutions

By the year 2030, through this project, the metropolis aims to reduce the number of vehicles by 5%, develop electro-mobility and make Toulouse the hub for smart mobility. Furthermore, this project promotes public-private-academic partnerships by bringing together public authorities such as SICOVAL, PETR Portes de Gascogne, research universities and laboratories such as Université Toulouse Paul Sabatier, ENAC, CNRS and Onera, and industrial partners such as Airbus, Continental, Kaufman & Broad, Bouygues Energie & services, Geopost and Aura Aero. It is noteworthy that the objectives set for this project and the fact that it brings together public, private and research partners, are in accordance with the guidelines for smart mobility ([6]) defined by WEF (Section [Section 2.1](#)).

2.3.2 Vilagil-MaaS

One of the tasks within Vilagil is to develop an intuitive and personalised MaaS platform which would combine different modes of transport and payment methods to offer inclusive mobility services. Taking into account the factors affecting the users’ choice of travel mode depending on their age, gender, level of education etc. and their daily movement patterns captured using GPS data from their smartphones, this platform will construct the users’ mobility profile. This mobility profile will be dynamic i.e., it will get updated as the users’ preferences change. Using the knowledge about the users’ mobility patterns, their preferred mode of transport as well as the real-time traffic data, the MaaS platform will proactively propose optimal travel itineraries between origin and destination locations at a given time instant. The user feedback acquired through the platform interface will be used to improve the recommendations over time. Speaking of the interface, gamification-based strategies will be applied in order to understand understand the users’ current travel behaviour and persuade

¹²www.toulouse-metropole.fr/-/toulouse-metropole-laureate-de-l-appel-a-projet-territoires-d-innovation-avec-vilagil

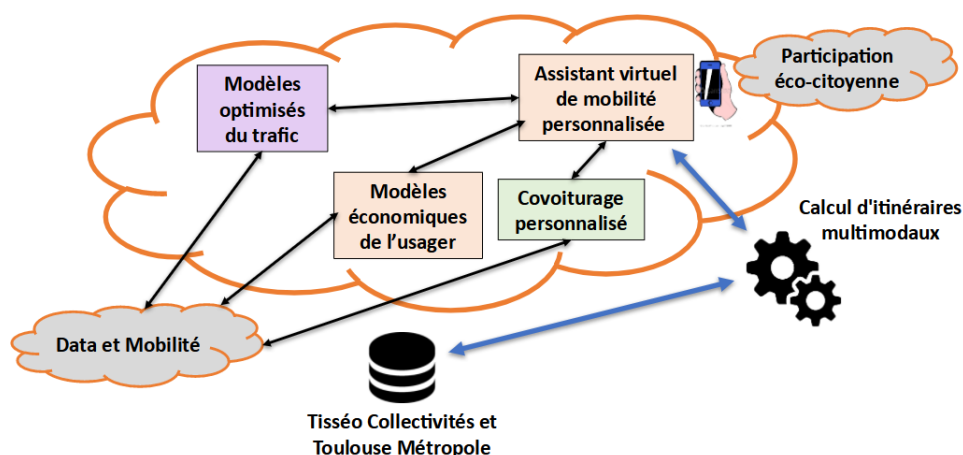


Figure 2.4: Different modules involved in the development of Vilagil-MaaS

them to opt for more sustainable transport services. The modes of transport considered within this platform include public transport as well as bike-sharing and car-pooling services. The global structure combining various modules involved in the development of this MaaS platform is shown in Figure 2.4. The development of this platform brings together different research partners: IRIT¹³, CLLE¹⁴, ENAC¹⁵ and ONERA¹⁶, each working on a specific module of this platform, along with the support from the metropolis of Toulouse and Tisséo Collectivités as data providers.

2.3.3 Proposed system architecture

In this subsection we will take a look at the system architecture proposed for the development of MaaS platform within the framework of Vilagil. The complete system architecture can be decoupled into four distinct modules.

2.3.3.1 User interface

This module developed by CLLE has the following objectives:

- Understand the current mobility behavior of the users and identify their needs and constraints
- Use gamification-based design strategies focused on the needs, expectations and constraints of the end-users
- Testing the effect of these strategies on the objective mobility performance and the subjective human evaluation
- Study the evolution of the user experience, acceptability and appropriation of the services made available to users over time

In addition to these objectives, acquiring user feedback about proposed trips will be an integral part of the user interface since it will be used in other modules of the system to improve the recommendations over time.

¹³www.irit.fr/en/home/

¹⁴clle.univ-tlse2.fr/

¹⁵www.enac.fr/en

¹⁶www.onera.fr/en

2.3.3.2 Transport mode choice model

This module will be developed by ENAC. Its main objectives are summarized as:

- Provide relevant modeling of multi-modal travel demand
- Highlight the factors that lead to multi-modal travel choices
- Calculate the probability that a given user will choose a particular mode of transport depending on factors like age, gender, level of education etc.
- Measure their relative importance to propose the best travel solution at a given time

2.3.3.3 Mathematical traffic model

ONERA is working on this part of the MaaS platform. The main objectives are:

- Modeling traffic and its dynamics
- Numerical simulation of the short and medium term evolution of traffic conditions, interactions with other transportation networks
- Propositions for travel arrangements in response to an individual request
- Allow a better real time estimation of trip travel time
- Evaluation of multiple aspects of the cost of a trip: duration, finances, environment, etc.

2.3.3.4 Personalised mobility profile

This module is being developed by IRIT. It will combine the information from different above mentioned modules to generate the mobility profile of a given user which will be used to recommend suitable trips according to the user's needs. The objectives of this module can be summarized as:

- Personalized and continuous learning of mobility preferences and habits
- Spontaneous propositions of multi-modal trips according to user's habits and preferences
- Integration of carpooling in multi-modal route proposals
- Evaluation of multiple aspects of the overall cost of a trip
- Respecting user's privacy

2.4 Conclusion

Mobility-as-a-Service (MaaS) aims to promote sustainable modes of transportation while integrating the possibility of booking and paying for multi-modal mobility services onto a single digital platform. It provides flexible and on-demand mobility options which are personalised according to the user's needs and can be customised according to the user's habits. The general goals which MaaS aims to achieve are in-line with some issues raised by the World Economic Forum (WEF) about the existing transportation and mobility sector.

At the moment, private enterprises have shown interest in developing MaaS platforms and services, which is evident from most of the trials conducted till date. However, to go from pilots to large-scale deployment, governments and other public institutions need to step in and work with private entities so as to provide a level playing field to every stakeholder and protect larger public interests. The existing research has raised some concerns about MaaS and has called into question its long-term goals, and it has been noted that without successful public-private partnership, MaaS cannot be deployed for the use of general public.

Over the past years, governments in different countries have started taking interest in understanding the need for MaaS and are working to propose regulations to protect user rights in case of absence of the promised services. In addition, the need for regularizing technical aspects of MaaS has been put forth which would allow various entities to share data and collaborate successfully during the development phase of MaaS services.

In Toulouse (France), the project VILAGIL, and particularly the action MaaS, aims to bring together public stakeholders (Toulouse Métropole and Tisséo) and research institutions (IRIT-UT3, ENAC, CLLE-UT2J, ONERA) for developing an integrated and personalised MaaS platform to provide multi-modal transport services to the users. Taking into account the user's past movement behaviour and his or her preferences as well as the current traffic conditions, this platform will be able to proactively recommend multi-modal routes to the user using public and shared modes of transport. In addition, this platform will employ gamification-based strategies to motivate users to change their travel behaviour and opt for sustainable modes of travel. Since public entities are involved from the beginning of this project and follow keenly the development of the MaaS platform, they are better equipped to later propose regulations for the benefit of the general public. With this collaborative approach, some concerns about MaaS which have been raised in the literature will be avoided.

Chapter 3

Economic modeling of user behavior (ENAC)

3.1 Introduction

3.1.1 The transport market and the demand for mobility

With 37 municipalities and 762,956 inhabitants, the Toulouse metropolis was the sixth largest in France in 2016¹, and the first in terms of demographic growth. Indeed, over the 2011-2016 period alone, the metropolis gained 48,624 inhabitants - an increase of 6.8 % - and this trend does not seem to be slowing down since 2020. In this context, understanding users' transportation choices in order to anticipate mobility demand is becoming a crucial issue for the city of tomorrow.

Every day, the Toulouse metropolitan area sees an average of 100,000 vehicles on its ring road, while the Tisseo metro network absorbs 150,000 passengers on its two lines A and B and the equivalent on its bus network. Several hundred thousand trips are therefore made every day, and this number has been rising steadily since the 1980s. For the city, the challenge is to understand why users use one mode of transport rather than another, for both economic reasons (to improve traffic flow, increase metro/bus trips, etc.) and environmental reasons (to reduce road traffic). It is in this context that the *Vilagil*² project is placed and its "Mobility As A Service" (MAAS) component, of which we represent here the econometric part.

Like any market, the transportation market in a city such as Toulouse has its own particularities that will interest the economist. On the one hand, users - who represent the demand - make a choice of transport mode for a given trip: train, metro, streetcar, bus, car, carpooling, cab, VTC, bicycle, walking and soft modes. This context places us in transport economics in the context of modal choice, as defined by [23]: "*a decision-making process that leads to a choice between several transport alternatives and that is determined by a combination of individual, socio-demographic and spatial factors, and influenced by socio-psychological factors*". These combinations of factors form the inputs of the user's utility function: his choice is then determined by the maximization of this utility under constraints.

On the other hand, the supply is made up of several actors: a public transport monopoly (Tisseo) for the metro, streetcar, and bus supply; private actors competing for cabs and VTCs; and "exit options" involving different costs for cars, bicycles, and soft modes. This supply structure, combining public and private actors offering the same service - a mode of transport - places us within the framework of the theory of **mixed oligopoly** [24, 25], which have, by definition, at least one private and one public firm offering substitutable goods or services. However, we will not study the structure of supply here, but only its attributes, and will remain focused on demand throughout the rest of this article. Our objective is thus to present several econometric models that attempt to explain users' modal choice.

Nevertheless, supply structure is highly dependent on the geographical scale of studies. Indeed, apart from the trade-off between travel time and cost, which is common to all scales of analysis, the question of modal choice is posed in different terms depending on the geographical scale at which it is studied. The regional scale has received relatively little attention in the literature [26, 27, 28] and involves, although they are not entirely exclusive to this scale, a number

¹Data available in www.toulouse-metropole.fr

²VILAGIL is a long-term project (horizon 2030) aiming to improve the travel conditions of the populations throughout the Toulouse urban area. The objective is to encourage the emergence of new forms of mobility and sustainable economic development in order to improve the living conditions of the inhabitants.

of original features because of the modes of transport modes involved: train, bus and car. As the scale increases and the study extends to greater distances, most papers focus on the study of competition between trains, cars and planes. Most of the time, the articles focus on a particular route, i.e., an origin-destination, where these three modes compete. For example [29] has a look on the Cologne-Berlin line for a train-plane competition model (the car is taken as an outside option), or [30] studies inter- and intramodal competition in the London-Paris passenger market between plane and Eurostar. The latter shows that in this market, demand is elastic, passengers are heterogeneous in their valuation of fares and accessibility and Aviation and high-speed rail are homogenous in unobserved effects. The large market share of the Eurostar and the withdrawal of aviation alternatives indicate that competition will decline in the long-run. In any case, this article focuses on urban mobility, i.e. on the scale of a city or agglomeration, where by definition there are neither high-speed trains nor airplanes. The recent arrival of new modes of transport sharing such as dockless scooter-share and e-bikes has been the subject of particular study by economists and public policy makers (see [31] for a survey on this topic). For instance, work by [32] found that an increase in ride-hailing trips had a negligible impact on public transit, negatively impacted taxi ridership, and contributed positively to active modes of travel such as biking.

Thus, understanding tomorrow's urban mobility requires taking into account all the factors likely to influence users' choices, whether through innovative means of sharing (carpooling, bicycle rental, etc.) or through changes in awareness, particularly environmental, or the emergence of new habits. This is probably the reason why most of the literature in transport economics focuses on the user and the determinants of his transport choice through an estimation tool : the discrete choice models.

3.1.2 Determinants of modal choice

Discrete choice models have long been used to explain why a proportion of consumers choose one good over another. In economics literature [33, 34, 35], these are models where consumers have a random component in their preferences, are statistically identical and their choices are often described by logit-multinomial models [36]. The utility of the consumer is determined by his own characteristics as well as those of the products, and the demand for each good is obtained by aggregating the demand of all consumers who prefer this product to all others.

Discrete choice models are widely used in transportation economics [37, 38, 39]. It is a matter of the user making a choice between different modes of travel from one point to another, depending on various factors. In most of the cases, the study is made on a single trip in a specific Origin-Destination line [29] with a competition between two or three different mode, typically train-car-plane or train-car for regional trips. [40] gives us a simple example, applied to passengers coming from all over Spain and going to Madrid Bajas airport to take a flight abroad. For m a mode of transport among train (express and regional), airplane and car, the utility U_m of the user is expressed by :

$$U_m = k_m + \alpha C_m + \beta T_m + \beta' W_m$$

Where the parameter k_m represents the basic utility of each mode of transport, C_m , T_m and W_m being respectively the price, the time and the waiting time of the trip, and α , β and β' their elasticities. Once this utility has been estimated, the proportion of users adopting the m mode is given by the *standard logit* model

[41]. Initiated by [42], this technique is based on the fact that the probability for each individual to choose option m is the probability that this option maximizes his utility:

$$P(m) = \frac{\exp U_m}{\sum_j \exp U_j}$$

$P(m)$ being then the proportion of users having chosen the mode of transport m among the totality of the known journeys. As expected, the estimated elasticities are negative, each explanatory variable inducing a disutility for the user: $k_m = 0$; $\alpha = -0.007$; $\beta = -0.36$; $\beta' = -0.104$; the time of the trip being thus the factor influencing the choice of the users the most, before the waiting time and the price.

Another example of these simple discrete choice models is given by [43]. The authors use this same model with the addition of variables explaining a choice between rail (train and metro) and bus using the attributes of the offer: reliability, information, comfort, safety and accessibility. They show that once travel time and cost are controlled for, preferences for rail are explained by better service quality.

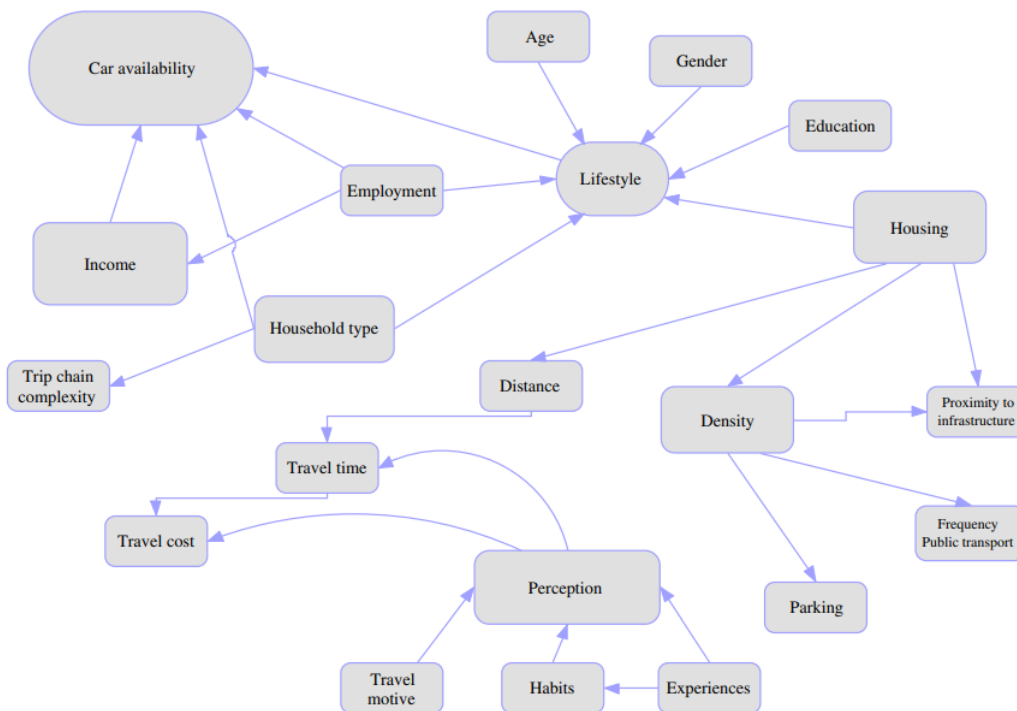
However, these specifications only take into account certain attributes of the offer and multiple other factors are likely to explain users' choice. In their review of the literature on the link between modal choice and mobility, [23] proposes a comprehensive set of explanatory variables, classified into three groups:

- **Supply attributes** : network density, diversity, parking, frequency, proximity, time, cost
- **Individual uses and characteristics**: habits, age, gender, income, household, education, motorization, trip purpose, weather, loops
- **Preferences**: experiences, familiarity, lifestyle, perception

Among these determinants, the authors show that the most studied in the literature are travel time and cost, age, gender, network density and motorization³. In many cases, these determinants influence each other, which explains the many links between them. For example, there is a positive relationship between income and car use and an inverse relationship with public transport use. [44].

Similarly, the reason for travel is closely related to the need for travel, which drives travel demand and is therefore an important determinant of modal choice. The literature examines three main trip motives, namely commuting, work trips and leisure trips. [45] In particular, they indicate that for home-to-work trips, the share of public transport is systematically higher than for other trip purposes. [23] summarizes these interdependencies of the determinants of modal choice in the following figure:

³The authors analyze 36 articles and these variables are present in the models studied in more than 45% of the cases.



In practical terms, this means that each variable determining modal choice is nested within another. For example, lifestyle will be determined by age, gender, education and employment level as well as by the composition of the household of the person concerned. And the combination of these different factors will likely explain whether or not a person owns a vehicle and thus increase his or her ability to choose. Similarly, the cost of the trip is the result of a chain of different factors: the place of residence will determine the distance, which in turn will determine the travel time which will have an important influence on the final cost of the trip. Past experiences, habits and travel motive will form the travel perception which also will impact the importance of travel cost and time. For the econometrician, this implies that the endogenous variables are correlated with each other and this must be controlled during the estimation of the model.

3.1.3 Multi-modal mobility demand

Some of the determinants of modal choice are observable, such as supply-side attributes or certain individual characteristics, such as age or gender, and others are unobservable, such as perception or environmental awareness. As transport mode choice is a multi-parametric decision process, it is determined by the characteristics of the transport alternatives and a combination of individual socio-economic, psychological and spatial factors. Psychological factors are unobservable, latent variables and remain among the least studied factors in discrete choice models. These latent variables are de facto excluded from many studies and limit the scope of analysis. Indeed, for [46], economic models that assume rational, context-invariant preferences that depend only on observable characteristics are not sufficient to explain why a mode is chosen.

Some articles try to answer this problem by using ICLV (Integrated Choice

and Latent Variable) models. These models are a combination of the structural equation model and the discrete choice model, allowing the integration of latent variables, typically of the psychological type (perception). In the general formulation of [47], two components can be distinguished: a multinomial discrete choice model and a latent variable model. Each of these sub-models is composed of a structural component and a measurement component. Based on a choice experiment conducted in the Rhône-Alpes region, [48] estimates an ICLV model to assess how sensitivity to public transport travel time, i.e., the willingness to pay to save an hour of travel time, is distributed as a function of comfort level according to objective and latent measures of public transport comfort. Comfort is modeled as a function of objective attributes and individual perceptions of comfort: the guarantee of a seat, the quality of the travel experience (feelings), the (perceived) use of travel time during the trip, and the overall ease of using public transport (perceived behavioral control). [48] The utility formulation of the ICLV models is then used:

$$U_{i,m} = V_{i,m}(Y_{i,m}, X_i, \xi_i) + \epsilon_{i,m}$$

Where $V_{i,m}$ is the deterministic part of the utility of user i using mode m ; $Y_{i,m}$ is a vector of the level of attributes of mode m ; X_i is a vector of observable individual characteristics; ξ_i is a vector containing the latent variables and $\epsilon_{i,m}$ is the random error term. The main results are that objective comfort, i.e., having a guaranteed seat on trains, and individual feelings when traveling by public transport (bus and train) robustly explain variations in the value of time. Their model estimation shows that guaranteed seating decreases the value of time, as does having positive feelings about public transport and perceiving train and coach travel time as useful time.

[49] extend this model with the incorporation of a latent variable of *environmental awareness*. Using ICLV models applied to transport users' choices in the Rhône-Alpes region (with stated preference surveys of 1900 individuals), they test whether people with high environmental awareness have a better perception of the usefulness of public transport and thus a higher propensity to use them. Environmental awareness is measured by three dimensions that are derived from the *Value-Belief-Norm* [50] theory: a cognitive dimension that measures environmental beliefs through the "New Environmental Paradigm" scale, an affective dimension that measures the feeling of responsibility towards the environmental problem, and a cognitive dimension that measures the awareness of the environmental problem. The authors estimate different models to measure whether the mediation relationship is complete, partial or non-existent. The results show that a favorable perception of public transport partially mediates the relationship between environmental awareness and public transport use patterns. Affective and symbolic motivations for car use are well explained by environmental awareness. In contrast, the relationship between affective and symbolic motivations and car use patterns is not robust.

Thus, a major challenge in adding variables such as environmental awareness or affective relationship to a particular mode of transport is that they are typically unobservable and must be inferred from other observations. This is what we propose to do in the next section.

3.2 Data

3.2.1 The household survey of Toulouse Metropole

The Tisséo network has seen a 75% increase between 2004 and 2013. This exceptional growth is the result of a global policy that has enabled the implementation of two metro lines, a tramway line, a hierarchical bus network, a proactive ticketing and fare policy, 7,000 of park-and-ride facilities, transit hubs... In just a few years, the Tisséo network has become the second most frequented network in the French's province (excluding Paris). The proportion of residents who subscribe to the network is more and more important, 8 students out of 10 have a Tisséo subscription. In addition, the growth in ridership has also regional (TER) and departmental (Arc en Ciel) networks, particularly in terms of the Tisséo network. Intermodal travel is growing very strongly, boosted by the various exchange systems put in place : parking lots, intermodal ticketing and the Multimodal Information Center. That's why Tisséo decided in 2012 to make a survey in order to estimate the future demand for multi-mobility in the city, and that's the Data we will use in the rest of this paper.

The household survey of Toulouse Metropole⁴ took place from October 16, 2012 to April 13, 2013. The survey records all trips made by residents on weekdays (excluding weekends, vacations and public holidays) and provides a snapshot of the travel patterns of an urban population on an average weekday. A total of 6,666 households were surveyed, with 13,680 people interviewed and a total of 48,357 trips recorded. In total, the inhabitants of the greater Toulouse area make 3.8 million trips during the course of a day, whether on foot, by car, by public transport, by bicycle or with another mode of transport.

31 different travel modes were surveyed. In order to simplify reading, they are usually aggregated into 7 categories: private vehicle (PV), urban public transport (UPT), other public transport (OPT), walking, cycling, motorized two-wheeler, other modes. It should be noted that only trips made entirely on foot are counted in the "walking" mode. When several modes are used during the same trip, only one of the modes is taken into account in the presentation of the results: this is the "main mode", which is derived from a hierarchy of modes defined at the national level (see order of priority).

A "standard" questionnaire forms the basis of the survey. It is structured in four parts:

- Household: the number of persons, information on the housing, on the equipment in car, in motorized two-wheelers, in bicycle...
- Person: age, main occupation, possession of a driver's license, public transport pass, place of study or work, travel habits...
- Trips: all trips made the day before the survey day (excluding weekends and school vacations) are recorded and described in great detail (origin and destination, reason⁵, used modes, hours, time...)
- Opinion: A person 16 years of age or older is randomly selected and asked to answer opinion questions about certain aspects of urban life and transportation.

An important point of the survey is that just one person in a household is asked about her opinion. A very strong assumption comes then to assume that

⁴https://www.tisseo.fr/sites/default/files/Enquete_menage_deplacement.pdf

⁵The motive is an activity carried out by the respondent: work, shopping, visiting friends, etc. There are 37 motives in the questionnaire, grouped into 6 categories

everybody in the same household has the same opinions whether in general politics or in various opinions.

This questionnaire was supplemented by "local" questions, in order to respond to the concerns of actors in the Toulouse area: residential mobility and criteria for choosing a place of residence, housing/transportation budget, parking attached to the home, bicycle parking, carpooling, use of the ring road and expressways, discomfort when walking. The different variables of the first three sheets are listed in the following table:

Trip	Person	Household
Motive	n° and person's name	Housing
Origin	Type (student, active, etc..)	Infrastructures Proximity
Departure Time	Gender	Motorization
Arrival Time	Age	Ressources
Number of mechanized mode	Licence	Fare
Waiting Time	School level	
Transportation mode	Employement	
	Place of work	
	Transport's uses	

In addition to these three cards, there is a fourth one, the "Opinion" card, where the respondent has to classify between: 1) Rather important, 2) Rather not important, 3) No opinion + 1 priority choice (choose the subject that the respondent sees as a priority before the others) among 3 themes:

- **City's Life** : safety, environment, recreation, education, transportation, employment, housing
- **Urban's moves** : accident risk, noise, pollution, fashion, parking, traffic
- **Some Opinions** : construction of parking, future of the bicycle, limitation of the car, etc..
- **Qualification of modes use** : The respondent is asked to choose 3 adjectives from a list of 41 (fast, convenient, cheap, ecological etc...) on the 4 modes of travel: car; public transport; bicycle; walking.

We present in page 4 the part "Some opinions" and "Qualification of the use of the modes"

According to this data provided by Toulouse Metropole, we define 4 travel alternatives, or modes $m \in \mathbf{M} = \{\text{Car}; \text{Public transport}; \text{bicycle}; \text{walking}\}$. The utility of a user i to use mode m is :

$$U_{i,m} = V_{i,m}(Y_{i,m}, X_i, \xi_i) + \epsilon_{i,m}$$

Where $V_{i,m}$ is the deterministic part of utility of user i taking the transportation mode m ; $Y_{i,m}$ is a vector of attributes level of mode m ; X_i a vector of observable individuals characteristics; ξ_i a vector with latent variables and $\epsilon_{i,m}$ random error term. Following [23], we can define :

- The vector of **Supply attributes**⁶, $Y_{i,m} = (y_1, y_2, y_3, y_4, y_5, y_6)_{i,m}^T$:

- y_1 : Network density = 1;2;3;4;5
- y_2 : Diversity = nb of transport modes
- y_3 : Parking = 1;2;3;4;5
- y_4 : Waiting time = nb of minute
- y_5 : Time = Arival time - Departure time
- y_6 : Costs

- The vector of **Individual Characteristics** $X_i = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)_i^T$:

- x_1 : Age = years
- x_2 : Gender = 1 if man, 0 if woman
- x_3 : Income = slices
- x_4 : Type = student, full time worker, part-time worker, unemployed, retired
- x_5 : Household = number of people living in the house
- x_6 : Education = number of years as a student
- x_7 : Motorisation = number of vehicles
- x_8 : Motive = house-work, house-school, house-grocery, others.

- The vector of **perceptions** $\xi_i = (\xi_1, \xi_2)_i^T$:

- ξ_1 : Car's vision = 1 or 0
- ξ_2 : Environmental awareness = 1 or 0

- The vector of **endogenous variable** $Y_i = (y_1, y_2)_i^T$:

- y_1 : Frequency of the four transport mode use : never (0), exceptional (1), monthly (2), weekly (3), daily (4)
- y_2 : The mode used on a particular journey : Car, Bike, Bus/Metro and walk.

Perceptions are dummy variables and are calculated from the answers to 2 questions asked to users. They are asked to choose the adjective they consider most important for the life of the city: safety, environment, employment, education, housing, leisure and traffic. Similarly, regarding transportation, they are asked to choose among: accident, noise, cyclists, pedestrians, pollution, parking, public transport and car. If the respondent has mentioned the environment and/or pollution in these two answers, he/she is given a score of 1 in environmental awareness. If the respondent answered traffic and/or car in the second question, he/she receives a score of 1 for his/her view of the car as a priority, and 0 in the other cases for both variables.

Define the alternative m :

$$c_{i,m} = \begin{cases} 1 & \text{iff } U_{i,m} \geq U_{i,m'} \text{ pour } m' \in M \\ 0 & \text{otherwise.} \end{cases}$$

⁶Here, i doesn't represent an individual but the supply geographical characteristics.

We've got hence :

$$Pr(c_{i,m} = 1|Y_{i,m}, X_i, \xi_i) = \frac{\exp V_{i,m}(Y_{i,m}, X_i, \xi_i)}{\sum_i \exp V_{i,m}(Y_{i,m}, X_i, \xi_i)}$$

This equation, mainly used in the articles quoted above, embodies the probability of having a higher utility to use one transport mode in particular. Hence, using this model calibration allows us to set our 2 econometrics equations : the ordinal logit model :

$$Frequency_m = \beta_0 + \beta_1 Male + \beta_2 Age + \beta_3 Income + \beta_4 Education + \beta_5 Education + \beta_6 Environment$$

and the multinomial logit model determining the probability of using a transport mode for a given trip. Next two tables presents the results. We can see that being a male will increase the use of car and bike when decrease this use of metro/bus. Identically to what we found in the literature, income has a negative impact for public transport use while a positive one for car and bike. One can remark that the typical profile of a high user of car and bike is the same, except for the environmental awareness grade.

Table 3.1

	<i>Dependent variable: Frequency by Mode</i>		
	Car (1)	TC (2)	Bike (3)
Male	0.403*** (0.052)	-0.339*** (0.047)	0.468*** (0.055)
Age	-0.008*** (0.003)	-0.023*** (0.002)	-0.017*** (0.003)
Income	0.147*** (0.016)	-0.091*** (0.014)	0.036** (0.016)
Education	0.052** (0.024)	0.136*** (0.021)	0.181*** (0.024)
Student	-1.351*** (0.153)	0.525*** (0.137)	-0.240 (0.158)
Unemployment	-0.299** (0.118)	-0.120 (0.110)	-0.286** (0.133)
Worker	0.935*** (0.098)	-1.027*** (0.088)	-0.198* (0.105)
Environment	-0.167** (0.067)	0.102* (0.060)	0.199*** (0.068)
Observations	6,252	6,252	6,252
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01		

Table 3.2: Results

	<i>Dependent variable:</i>	
	Mode_Car (1)	Mode_TC (2)
groupe_age25-44	0.951*** (0.048)	-0.933*** (0.048)
groupe_age45-59	0.791*** (0.050)	-0.824*** (0.051)
groupe_age60 et +	1.293*** (0.053)	-1.147*** (0.053)
Male	0.134*** (0.027)	-0.187*** (0.028)
Education	0.712*** (0.171)	-0.368** (0.187)
FqVeloHebdomadaire	-0.201*** (0.058)	0.066 (0.063)
FqVeloJamais	-0.053 (0.035)	0.077** (0.038)
FqVeloMensuel	-0.098 (0.061)	-0.095 (0.067)
FqVeloQuotidien	-1.366*** (0.060)	-0.258*** (0.070)
Revenu	0.275*** (0.010)	-0.354*** (0.012)
Distance	0.00004*** (0.00000)	-0.00004*** (0.00000)
Constant	-1.563*** (0.176)	1.235*** (0.190)
Observations	33,199	33,199
Log Likelihood	-18,146.070	-16,346.990
Akaike Inf. Crit.	36,324.150	32,725.990
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01	

3.3 Conclusion

Articles about economic models of multi modal travel demand share many common features and similar results. First, in the vast majority of cases the literature considers discrete choice models to determine demand functions, i.e. the ability of the user to maximize utility given a remaining number of possible choices.

Second, these choices are determined by three main types of variables: the attributes of the supply, the individual characteristics of users, and the so-called latent, unobservable variables. Among these variables, some are more present than others in the articles, in particular the cost of trips, distance and travel time for the supply attributes. For individual characteristics, we regularly find age, gender, income, socio-professional category and level of education. Finally, questions concerning latent variables, typically opinions on environmental issues, are more recent but are the subject of growing interest in the literature.

Once the variables are selected and the model is calibrated, several results are recurrent throughout the articles. For example, income has a positive impact on car use and a negative impact on the use of public transport and carpooling. This and other results are consistent with what we find in our own estimates based on our data, whether in the ordinal model - estimating frequency of use - or the multinomial model - the probability of using a mode of transport on a particular trip. These calculated probabilities and elasticities can then be used to better target the users likely to use such or such means of transport, and thus be used within a MAAS tool to, among other things, personalize the gamification of the tool for the user.

Chapter 4

Optimized mathematical traffic models to predict traffic state and evolution (ONERA)

4.1 Introduction

The mathematical modelling of traffic flow started gaining interest in the middle of the twentieth century, due to a sudden increase in the use of cars and buses for urban transportation, and to this day it has continued to be a very vibrant area of research. Part of the appeal towards modelling traffic flow is the sheer complexity of the problem where one may need to account for the complexity of road networks, the dynamics of traffic congestion, the heterogeneity of vehicle sizes and performances, drivers' behavior and the large number of vehicles. Additionally, in today's time, the modelling and accurate prediction of traffic has significant environmental impact as it helps in reducing congestion and vehicular pollution which are contributing to climate change in the long-term [51]. Furthermore, we are entering the era of autonomous vehicles which will revolutionize urban transportation in not too distant future. Contrary to classical drivers, these vehicles will be routed following mathematical algorithms, which will provide full optimal control of the trajectories with respect to given constraints. In this context, traffic could, in the future, be routed in order to minimize its adverse impact like emissions and congestion. Considering all these reasons, modelling of traffic flow and the development of advanced models has become increasingly crucial, especially in the context of urban planning.

Towards the goal of designing advanced traffic flow models, we have compiled this report where an overview has been provided of different macroscopic traffic flow models that are actively being used today, with a specific focus on urban applications. It must be mentioned, that the literature is vast with respect to new traffic flow models, however recent results have only been a variant of the models presented in this report, where modification has been applied to cater to a particular scenario. The models presented here rely on the theory of hyperbolic partial differential equations and applies the principles of fluid dynamics to averaged quantities such as vehicle density and mean traffic velocity. These models either comprise of a single scalar conservation law, such as the celebrated Lighthill-Whitham-Richards model [52, 53] or they can be defined using a coupled system of first order differential equations, such as the model by Payne [54, 55], Aw-Rascle-Zhang [56, 57] or the phase transition model [58]. Here we recall these models and summarize their mathematical properties.

4.2 The Lighthill-Whitham-Richards (LWR) model

A first attempt to model vehicular traffic flow based on the theory of hyperbolic partial differential equations (PDEs) can be found in the seminal works by Lighthill and Whitham from 1955 [52], and separately by Richards in 1956 [53]. Within these works, the authors replaced the distribution of vehicles over a stretch of road with a continuous density function $\rho := \{\rho(t, x) : [0, +\infty) \times \mathbb{R} \rightarrow \mathbb{R}, 0 \leq \rho \leq \rho_{\max}\}$, while defining the mean vehicular velocity as $v := \{v : \rho \mapsto v(\rho), 0 \leq v \leq v_{\max}\}$. Here the density and the velocity of the vehicles are nonnegative functions that are bounded with respect to their maximum admissible values ρ_{\max} and v_{\max} , respectively, where the maximum density essentially represents bumper to bumper traffic and the maximum velocity is traffic under free flow.

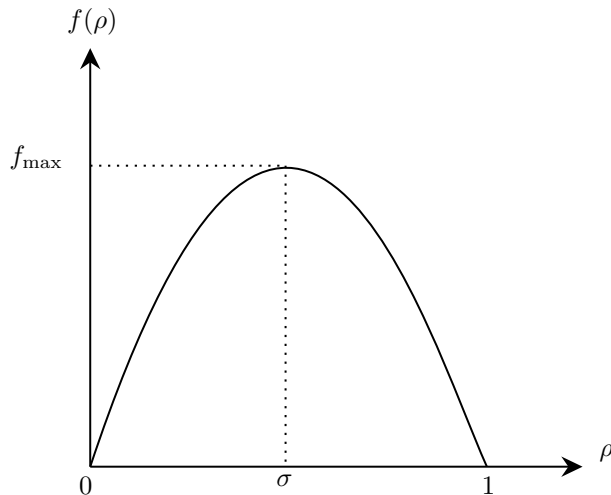


Figure 4.1: The fundamental diagram.

Some of the choices for defining the velocity function are as follows:

$$v = v_{\max} \left(1 - \frac{\rho}{\rho_{\max}}\right), \quad (4.1a)$$

$$v = v_{\max} \ln \left(\frac{\rho_{\max}}{\rho}\right), \quad (4.1b)$$

$$v = v_{\max} e^{(-\rho/\rho_{\max})}, \quad (4.1c)$$

$$v = v_{\max} \left(1 - \left(\frac{\rho}{\rho_{\max}}\right)^n\right), \quad (4.1d)$$

$$v = v_{\max} \left(\frac{1}{\rho} - \frac{1}{\rho_{\max}}\right). \quad (4.1e)$$

$$(4.1f)$$

In all our forthcoming discussions, the density and velocity will be considered as normalized functions, where the normalization is with respect to their maximum values respectively.

According to the Lighthill-Whitham-Richards (LWR) model, the evolution of traffic density in space-time is governed by the following scalar conservation law

$$\partial_t \rho + \partial_x f(\rho) = 0, \quad (4.2)$$

where $f := \{f : \rho \mapsto f(\rho) = \rho v, 0 \leq f(\rho) \leq 1\}$ is a concave function that represents the flux of the vehicles moving through a point on the road in a given time period, and it satisfies the following reasonable assumptions

$$f(0) = f(1) = 0, \quad \exists \sigma \in [0, 1] : f'(\sigma) = 0, \quad (\rho - \sigma)f'(\rho) < 0, \rho \neq \sigma, \quad (4.3)$$

where σ is the density at which $f(\rho)$ attains a unique maximum, and henceforth we will refer to it as the critical density, see Figure 4.1.

In the present work, we are concerned with the Cauchy problem for (4.2), which can be constructed by supplying (4.2) with the initial data $\rho(0, x) = \rho_0(x)$.

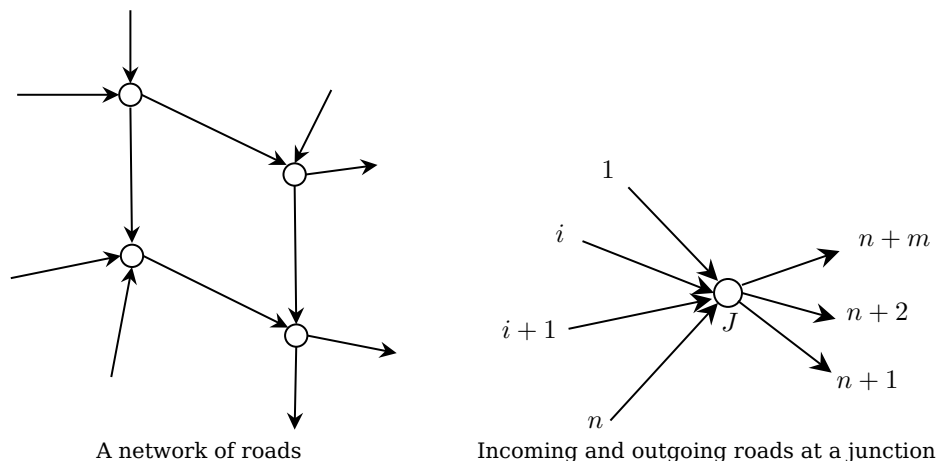


Figure 4.2: Left: Network of roads represented as a directed graph. Right: flow of traffic through a junction.

Hyperbolicity of (4.2) requires that $\partial_\rho f(\rho)$ be real-valued, and solutions to the Cauchy problem take the form of shocks or rarefaction waves. A physical realisation of these solutions can be seen under the following scenarios: suppose a free flowing traffic suddenly comes to a halt, perhaps as a result of a red light at a traffic junction, or due to an accident on the road, then, within a small neighborhood of the incident, the cars apply a sudden break and come to a stop. Any approaching traffic behind them are required to decelerate as well and finally come to a halt. This phenomenon is seen as a formation of shock which is travelling backwards for the oncoming traffic. Inversely, if a traffic jam is set free then the cars at the head of the traffic accelerate away from the cluster of cars, and we interpret this as the formation of rarefaction waves.

One of the most prominent uses of the LWR model have been the study of distribution of traffic over a network of roads, where the road network is considered as a finite, connected directed graph with the edges that model individual roads with unidirectional traffic flow and the vertices representing the junctions. A schematic representation of this representation can be seen in Figure 4.2. The first results towards flow on road networks was presented by Helge and Holden in [59] and by Garavello and Coclite in [60]. The essential idea in these works is to find a unique traffic density at the junctions such that the flow is maximized. The Cauchy problem with Riemann initial data can then be solved on individual roads with the density of vehicles either approaching or exiting the junction. Some examples of recent works towards the application of the LWR model to traffic flow over networks can be found in [61, 62, 63, 64, 65]. In this report, we present a prototype problem for the flow over a network of roads by recalling the ideas from original work by Helge and Holden [59].

Let there be a network of roads comprising of N separate roads and M junctions. The density of vehicles on each road is denoted by $\rho_i(t, x)$, $x \in [a_i, b_i]$, where $a_i = -\infty$ or $b_i = \infty$ if the ends of road are not connected to any junction. We restrict ourselves to unidirectional traffic and assume that the traffic flows from a_i to b_i . Furthermore, for the vector-valued function of the density around a junction, we will use the notation

$$\rho(t, x_1, \dots, x_N) = (\rho_1(t, x_1), \dots, \rho_N(t, x_N)), \quad t \geq 0, \quad x_i \in [a_i, b_i], \quad i = 1, \dots, N. \quad (4.4)$$

The Cauchy problem for the LWR model away from the junction is

$$\partial_t \rho_i + \partial_x f_i(\rho) = 0, \quad i = 1, \dots, N, \quad (4.5a)$$

$$\rho(0, x) = \rho_0(x), \quad x \in [a_i, b_i] \quad i = 1, \dots, N. \quad (4.5b)$$

The roads are coupled through boundary data obtained at the junctions.

For the sake of simplification, it is reasonable to restrict the present analysis to a single junction J due to the finite speed of propagation of the waves. At J , we consider that vehicles from n roads enter the junction and they have a choice of m roads for exiting. Thus, J connects $N = n + m$ roads, which are indexed as $i = 1, \dots, n$ for the incoming roads and $i = n + 1, \dots, n + m$ for the outgoing roads, see Figure 4.2. For roads that are connected through J , the ends of the road at the junction are labelled as

$$c_i = \begin{cases} b_i, & \text{for } i = 1, \dots, n, \\ a_i, & \text{for } i = n + 1, \dots, n + m. \end{cases}$$

Weak solution. Let $\phi = (\phi_1(t, x_1), \dots, \phi_N(t, x_N)) \in \mathbf{C}_0^1([0, +\infty[\times[a_i, b_i])^N$ be a set of smooth test functions that are \mathbf{C}^1 across junctions

$$\phi_i(t, c_i) = \phi_j(t, c_j), \quad \partial_x \phi_i(t, c_i) = \partial_x \phi_j(t, c_j). \quad (4.6)$$

The weak solutions of (4.5) are the set of functions $\rho_i(t, x_i)$ that satisfy

$$\sum_{i=1}^N \left(\int_0^{+\infty} \int_{a_i}^{b_i} (\rho_i \partial_t \phi_i + f(\rho_i) \partial_x \phi_i) dx dt + \int_{a_i}^{b_i} \rho_{i,0}(x) \phi_i(0, x) dx \right) = 0. \quad (4.7)$$

Note that the number of vehicles are always conserved at the junction, and the flux satisfies the Rankine-Hugoniot relation

$$\sum_{i=1}^n f(\rho_i(\cdot, b_i)) = \sum_{i=n+1}^N f(\rho_i(\cdot, a_i)), \quad \forall t > 0. \quad (4.8)$$

Note that irrespective of whether the initial data satisfies the RH condition, the solution must satisfy this relation at all times $t > 0$. However, satisfying the RH relation does not guarantee uniqueness as (4.8) is underdetermined for the N unknowns. Therefore we need to define entropy solutions, that ensure uniqueness.

Let us denote the solution at the junction as

$$\rho_J(t) = \rho(t, c_1, \dots, c_N) = (\bar{\rho}_1, \dots, \bar{\rho}_N). \quad (4.9)$$

The objective is to seek a unique ρ_J in a manner that maximizes the flow at the junction (by satisfying the entropy condition). Once the solution at the junction is obtained the Riemann problem can then be solved in the usual way on each road where $\bar{\rho}_i$ will be the right state for the incoming road $i \leq n$, and similarly it will be the left state for outgoing roads $i \geq n + 1$. The solutions emerging from the junctions will comprise of waves that will have negative speed for incoming roads (recall that shocks travels backwards in a queuing traffic jam) and positive speed for outgoing roads. We can now define the set of admissible solutions at the junction.

Admissible solution. For any $\rho \in [0, 1]$, $\rho \neq \sigma$, we define $\tau(\rho)$ as a unique number with $\tau(\rho) \neq \rho$ such that $f(\tau(\rho)) = f(\rho)$. Furthermore, if $\rho > \sigma$ then $\tau(\rho) < \rho$, else if $\rho < \sigma$ then $\tau(\rho) > \rho$ and $\tau(\sigma) = \sigma$. To be specific, the density function at the junction for a given road takes the following values:

- For incoming roads $i = 1, \dots, n$

$$\bar{\rho}_i \in \begin{cases} [\sigma, 1], & \text{for } \rho_{i,0} > \sigma, \\ \{\rho_{i,0}\} \cup [\tau(\rho_{i,0}), 1], & \text{for } \rho_{i,0} < \sigma, \end{cases} \quad (4.10)$$

- For outgoing roads $i = n+1, \dots, n+m$

$$\bar{\rho}_i \in \begin{cases} [0, \sigma], & \text{for } \rho_{i,0} < \sigma, \\ [0, \tau(\rho_{i,0})] \cup \{\rho_{i,0}\}, & \text{for } \rho_{i,0} > \sigma. \end{cases} \quad (4.11)$$

Therefore, we will consider ρ_J as an admissible solution if it belongs in

$$\Omega = \Omega_I \times \Omega_O \subseteq \tilde{\Omega} = \prod_{i=1}^n [\sigma, 1] \times \prod_{i=n+1}^{n+m} [0, \sigma] \subset \mathbb{R}^{n+m}, \quad (4.12)$$

where

$$\Omega_I = \prod_{i=1}^n [\tau(\rho_{i,0}), 1], \quad \Omega_O = \prod_{i=n+1}^{n+m} [0, \tau(\rho_{i,0})]. \quad (4.13)$$

Although, observe that it is quite possible that for an incoming road $\bar{\rho}_i = \rho_{i,0}$ for $\rho_{i,0} < \sigma$ and so $\rho \in \Omega$ is not guaranteed. In this case, we can modify the weak solution on a set of measure zero to obtain an admissible solution.

Entropy function. The aim of this condition is to find a unique ρ_J that satisfies the RH condition, while also maximizing the flow through the junction. More precisely, we maximize the flow through the junction which is measured by a concave function $g(\gamma_i)$, where $\gamma_i = f(\bar{\rho}_i)/f(\sigma)$ for $i = 1, \dots, N$. We define the entropy function at a junction J as

$$E_J = \sum_{i=1}^N g(\gamma_i). \quad (4.14)$$

Introduce the Hugoniot locus

$$H_J = \left\{ \rho_J \in \mathbb{R}^N \mid \sum_{i=1}^n f(\bar{\rho}_i) = \sum_{i=n+1}^N f(\bar{\rho}_i) \right\}, \quad (4.15)$$

the entropy solution $\rho_J = (\bar{\rho}_1, \dots, \bar{\rho}_N) \in H_J \cap \Omega$ which maximizes E_J .

From the definition of $g(\gamma_i)$, the entropy function E_J is a concave function that takes values from a convex set, thus it has a unique maximum. This maximum can be either found as an interior point where $\nabla E = 0$, or at the boundary of the domain. Using γ_i , the RH condition (4.8) can be written as

$$\sum_{i=1}^n \gamma_i = \sum_{i=n+1}^N \gamma_i. \quad (4.16)$$

The entropy function is rewritten as

$$E_J \stackrel{(4.16)}{=} \sum_{i=1}^{N-1} g(\gamma_i) + g\left(\sum_{i=1}^n \gamma_i - \sum_{i=n+1}^{N-1} \gamma_i\right). \quad (4.17)$$

Thus

$$\partial_{\gamma_i} E_J = g'(\gamma_i) + g'(\gamma_N) \partial_{\gamma_i} \gamma_N, \quad (4.18)$$

where we adopt the following convention

$$\partial_{\gamma_i} \gamma_N = \begin{cases} +1 & \text{for } i = 1, \dots, n, \\ -1 & \text{for } i = n+1, \dots, N-1. \end{cases}$$

Therefore, $\nabla E = 0$ implies that $g'(\gamma_i) = -g'(\gamma_N)$ for $i = 1, \dots, n$ and $g'(\gamma_i) = g'(\gamma_N)$ for $i = n+1, \dots, N-1$. Thus

$$\gamma_i = \begin{cases} \frac{m}{n} \gamma_N, & \text{for } i = 1, \dots, n, \\ \gamma_N, & \text{for } i = n+1, \dots, N-1, \end{cases} \quad (4.19)$$

where γ_N is the solution of the equation

$$g'\left(\frac{m}{n} \gamma_N\right) = -g'(\gamma_N), \quad \gamma_N \in [0, 1]. \quad (4.20)$$

For $\rho_J \in \Omega$, the corresponding fluxes will be $\gamma_J = (\gamma_1, \dots, \gamma_N) \in \prod_{i=1}^N [0, \kappa_i(\rho_{i,0})] \subseteq \prod_{i=1}^N [0, 1]$, where

$$\kappa_i(\rho) = \begin{cases} 1, & \text{if } i \leq n \text{ and } \rho \geq \sigma, \text{ or } i > n \text{ and } \rho \leq \sigma, \\ \gamma(\rho) & \text{otherwise.} \end{cases} \quad (4.21)$$

Once we have determined the unique value of the solution at the junction, we construct the solution on individual roads.

Solution to the Riemann problem. Let a single discontinuity separate a scalar function $u(t, x)$ into the left and right states u_l and u_r , where the jump of any variable across the discontinuity is denoted as

$$[[\cdot]] = (\cdot)_r - (\cdot)_l.$$

The Cauchy problem for the scalar conservation law for u with Riemann initial data is

$$\begin{aligned} \partial_t u + \partial_x f(u) &= 0, \\ u(0, x) &= \begin{cases} u_l, & \text{for } x \leq 0, \\ u_r, & \text{for } x \geq 0, \end{cases} \end{aligned} \quad (4.22)$$

where the flux function $f(x)$ is assumed to be concave.

If $u_l \leq u_r$, the solution to (4.22) is a shock discontinuity, which travels at speed $s = [[f(u)]]/[[u]]$ given by the RH condition. Therefore, the solution is simply

$$u(t, x) = \begin{cases} u_l & \text{for } x < st \\ u_r & \text{for } x > st. \end{cases} \quad (4.23)$$

On the contrary, if $u_l > u_r$, then we have rarefaction waves

$$u_l = u_0 < u_1 < u_2 = u_r, \quad (4.24)$$

with the corresponding flux function $f(u_i) \in \mathbb{R}$ for $i = 1, 2, 3$, such that f is linear on $[u_i, u_{i+1}]$,

$$u \in [u_i, u_{i+1}] \Rightarrow f(u) = \frac{f(u_{i+1}) - f(u_i)}{u_{i+1} - u_i} (u - u_i) + f(u_i), \quad i = 1, 2, 3. \quad (4.25)$$

Similarly, for the LWR model (4.2), which is endowed with a flux function $f(u)$ that satisfies (4.3), the Riemann problem at the junction J has a unique solution that satisfies the entropy condition. Let ρ_J denote the entropy solution at the junction. Then the solution to the Riemann problem on each road can be defined as follows:

- For incoming roads $i = 1, \dots, n$, if $\bar{\rho}_i > \rho_{i,0}$, we have

$$\rho_i(t, x) = \begin{cases} \rho_{i,0} & \text{for } x < st \\ \bar{\rho}_i & \text{for } x > st, \end{cases} \quad (4.26)$$

while if $\bar{\rho}_i < \rho_{i,0}$

$$\rho_i(t, x) = \begin{cases} \rho_{i,0} & \text{for } x < f'(\rho_{i,0})t, \\ (f')^{-1}\left(\frac{x}{t}\right) & \text{for } f'(\rho_{i,0})t < x < f'(\bar{\rho}_i)t, \\ \bar{\rho}_i & \text{for } x > f'(\bar{\rho}_i)t. \end{cases} \quad (4.27)$$

- For outgoing roads $i = n + 1, \dots, n + m$, if $\bar{\rho}_i < \rho_{i,0}$, we have

$$\rho_i(t, x) = \begin{cases} \rho_{i,0} & \text{for } x > st \\ \bar{\rho}_i & \text{for } x < st, \end{cases} \quad (4.28)$$

while if $\bar{\rho}_i > \rho_{i,0}$

$$\rho_i(t, x) = \begin{cases} \bar{\rho}_i & \text{for } x < f'(\rho_{i,0})t, \\ (f')^{-1}\left(\frac{x}{t}\right) & \text{for } f'(\bar{\rho}_i)t < x < f'(\rho_{i,0})t, \\ \rho_{i,0} & \text{for } x > f'(\bar{\rho}_i)t. \end{cases} \quad (4.29)$$

4.3 The LWR model with non-local flux

Although the LWR model, introduced in (4.2), is quite effective in predicting traffic, yet it carries a major drawback: the velocity of the vehicles can be discontinuous, which translates to infinite acceleration values. As a result, such a model cannot be combined with atmospheric and noise pollution models, which require an accurate estimation of the acceleration terms. To overcome this predicament, the authors in [66, 67] propose a variant of the LWR model with a non-local flux function which accounts for the adaptation of the velocity based on downstream traffic. Here the velocity function is assumed to depend on a weighted mean of the downstream density. Due to the action of a convolution product, the velocity becomes a Lipschitz function of the space and time variables, guaranteeing bounded acceleration. For a more comprehensive explanation on the effects of non-local flux in pedestrian dynamics and traffic flow, one can refer to [68, 69, 70, 71, 72]. Here we briefly introduce this model and recall some well-posedness results.

The mass conservation equation (4.2) now takes the following form with non-local mean velocity:

$$\partial_t \rho + \partial_x \rho v(\rho *_d w_\eta) = 0 \quad \text{for } t > 0, \quad x \in \mathbb{R}, \quad (4.30)$$

where the velocity $v(\cdot)$ is defined as a function of the downstream convolution product

$$\rho *_d w_\eta = \int_x^{x+\eta} \rho(t, y) w_\eta(y - x) dy, \quad \eta > 0. \quad (4.31)$$

In the above, the convolution kernel $w_\eta \in \mathbf{C}^1([0, \eta]; \mathbb{R}^+)$ is a non-decreasing function such that $\int_0^\eta w_\eta(x) dx = 1$. Possible choices for the kernel can be $w_\eta(x) \equiv 1/\eta$ or $w_\eta(x) \equiv 2((1 - x/\eta)/\eta)$. The choice of the velocity function in this model allows the drivers to adapt the speed of their vehicles based on downstream traffic (density of vehicles in front of them), see Figure 4.3.

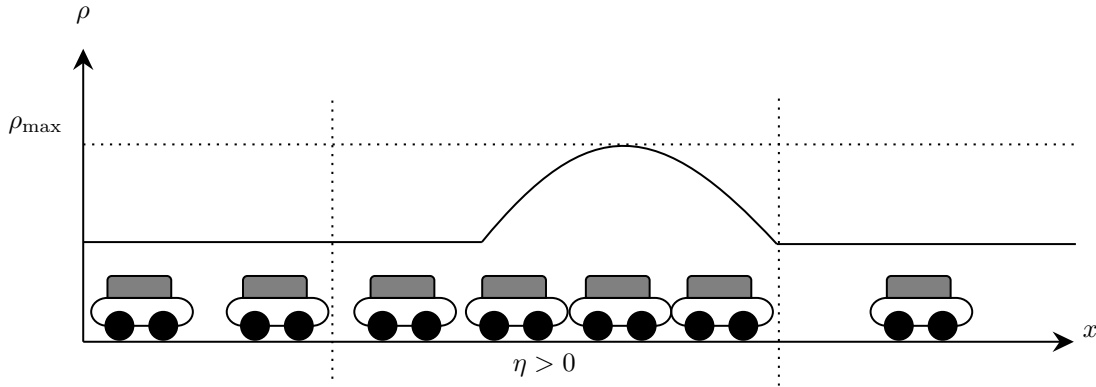


Figure 4.3: Illustration of a non-local traffic flow model

For the sake of simplifying the notation, let $\vartheta(t, x) = v(\rho *_{d} w_{\eta})(t, x)$. The equation (4.30) can be rewritten as

$$\partial_t \rho + \partial_x \rho \vartheta = 0, \quad (4.32)$$

which is coupled with the initial data $\rho(0, x) = \rho_0(x) \in \text{BV}(\mathbb{R}; [0, 1])$ to form the Cauchy problem.

A measurable function $\rho \in (\mathbf{L}^1 \cap \mathbf{L}^\infty \cap \text{BV})(\mathbb{R}^+ \times \mathbb{R}; \mathbb{R})$ is said to be weak solution of (4.32) if it satisfies

$$\int_0^{+\infty} \int_{-\infty}^{+\infty} (\rho \partial_t \phi + \rho \vartheta \partial_x \phi) dx dt + \int_{-\infty}^{+\infty} \rho_0(x) \phi(0, x) dx = 0, \quad (4.33)$$

where $\phi \in \mathbf{C}_c^1(\mathbb{R}^2; \mathbb{R})$.

Furthermore, $\rho \in (\mathbf{L}^1 \cap \mathbf{L}^\infty \cap \text{BV})(\mathbb{R}^+ \times \mathbb{R}; \mathbb{R})$ is an entropy weak solution if it satisfies the Kruřkov-like [73] condition

$$\begin{aligned} \int_0^{+\infty} \int_{-\infty}^{+\infty} (|\rho - \kappa| \partial_t \phi + |\rho - \kappa| \vartheta \partial_x \phi - \text{sgn}(\rho - \kappa) \kappa \phi \partial_x \vartheta)(t, x) dx dt \\ + \int_{-\infty}^{+\infty} |\rho_0(x) - \kappa| \phi(0, x) dx \geq 0 \quad \forall \phi \in \mathbf{C}_c^1(\mathbb{R}^2; \mathbb{R}), \quad \kappa \in \mathbb{R}. \end{aligned} \quad (4.34)$$

The existence and uniqueness of solution for (4.32) is proved in [67] by constructing a converging sequence of finite volume approximate solutions, defined using an adapted Lax-Friedrichs scheme. The results shown in [67] do not require a smoothness criteria for the kernel w_{η} , and provides \mathbf{L}^∞ bounds on the solution through the maximum principle. In [67], we also see a comparison of the solution under different choices of the kernel function, and the well-posedness of the entropic solutions using an anisotropic kernel was shown in [74]. It must also be mentioned that the choice of the convolution product (4.31) is not limited to the weighted mean of the downstream density, and the authors in [75] choose the velocity function in the convolution product while demonstrating well-posedness results by constructing a Godunov type scheme.

We end this section by recalling some important well-posedness results for (4.32), [67, Theorems 1 and 2]. The proofs can be found in [67] and we refer to [76, 77] for further details. The results shown here were obtained by using the doubling of variables technique introduced by Kruřkov [73] and exercising Gronwall's lemma.

Theorem 4.3.1. *Let $\rho_0 \in \text{BV}(\mathbb{R}; [0, 1])$ and $w_{\eta} \in \mathbf{C}^1([0, \eta]; \mathbb{R}^+)$ be a non-increasing*

function such that $\int_0^\eta w_\eta(x)dx = 1$. Then the Cauchy problem

$$\begin{aligned}\partial_t \rho + \partial_x \rho v &= 0, \quad t > 0, x \in \mathbb{R}, \\ \rho(0, x) &= \rho_0(x), \quad x \in \mathbb{R},\end{aligned}\tag{4.35}$$

admits a unique weak solution such that

$$\min_{\mathbb{R}}\{\rho_0\} \leq \rho(t, x) \leq \max_{\mathbb{R}}\{\rho_0\},\tag{4.36}$$

almost everywhere for $x \in \mathbb{R}$ and $t > 0$.

Theorem 4.3.2. Let ρ, σ be two entropy solutions of (4.32) with initial data ρ_0 and σ_0 respectively. Then, for any $T > 0$ the following L^1 contraction holds

$$\|\rho(t, \cdot) - \sigma(t, \cdot)\|_{L^1} \leq e^{\mathcal{K}T} \|\rho_0 - \sigma_0\|, \quad \forall t \in (0, T],\tag{4.37}$$

where the constant \mathcal{K} is given by

$$\begin{aligned}\mathcal{K} &= w_\eta(0) \|v'\|_\infty \left(\sup_{t \in [0, T]} \|\rho(t, \cdot)\|_{\text{BV}(\mathbb{R})} + 2 \sup_{t \in [0, T]} \|\rho_0\|_\infty \right) \\ &+ \sup_{t \in [0, T]} \|\rho_0\|_1 \left(2(w_\eta(0))^2 \|v''\|_\infty \sup_{t \in [0, T]} \|\rho_0\|_\infty + \|v'\|_\infty \|w'_\eta\|_{L^\infty([0, \eta])} \right).\end{aligned}\tag{4.38}$$

4.4 The Aw-Rascle-Zhang model

We now consider the traffic flow models that are based on first order hyperbolic systems, in contrast to the scalar conservation law (4.2),(4.30). These models were designed to capture particular characteristics of traffic flow, like stop-and-go waves. Such models are, often, referred to as "second" order models based on the number of differential equations that comprise the model. The first such model was proposed by Payne and Whitham [54, 78]. Unfortunately, as critiqued in [79], it presents some drawbacks, like the unrealistic fact that information may propagate faster than the actual velocity of cars. Taking these limitations into account, Aw, Rascle [56] and Zhang [57] proposed a new two-equations model, which is referred to as the Aw-Rascle-Zhang (ARZ) model. Here we will recall the ARZ model and its mathematical properties, and summarise the solution to the general Riemann problem.

The ARZ model is described as

$$\partial_t \rho + \partial_x \rho v = 0,\tag{4.39a}$$

$$\partial_t \rho y + \partial_x \rho y v = 0,\tag{4.39b}$$

where $\rho = \rho(t, x)$ and $v = v(\rho)$ are the averaged density and velocity of the vehicles, and $y = v + p(\rho)$ is the Lagrangian marker. The anticipation factor of the drivers is determined by a pressure-like function $p(\rho)$ which is assumed to be smooth and increasing, and it is closed by the following isentropic-like equation of state

$$p = \rho^\gamma, \quad \gamma > 0.\tag{4.40}$$

Additionally, $\rho \mapsto \rho p(\rho)$ is strictly convex, and at the vacuum, the anticipation factor is assumed to satisfy $p(\rho) \sim \rho^\gamma$, near $\rho = 0, \gamma > 0$.

Hyperbolicity of the ARZ model can easily be verified by considering (4.39) in nonconservative form

$$\partial_t \rho + \partial_x \rho v = 0,\tag{4.41a}$$

$$\partial_t v + (v - \rho p'(\rho)) \partial_x v = 0,\tag{4.41b}$$

where the eigenvalues of the system are $\lambda_1 = v - \rho p'(\rho)$ and $\lambda_2 = v$ which are real and distinct, along with corresponding right eigenvectors $r^1 = \begin{pmatrix} 1 \\ -p'(\rho) \end{pmatrix}$ and $r^2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, respectively. It is clear that $\lambda_1 < \lambda_2$ under the above assumptions, with closure (4.40), however (4.41) is weakly hyperbolic when $\rho = 0$, such as a vacuum, with $\lambda_1 = \lambda_2$. Furthermore, λ_1 is associated to a genuinely nonlinear (GNL) field and it will admit either shocks or rarefaction waves, while λ_2 is associated to a linearly degenerate field which will lead to contact discontinuities. Admissible solutions of (4.39) must satisfy the Lax entropy inequality [80]: for a convex entropy function $\eta(\mathbf{u})$ with entropy flux $q(\mathbf{u})$, solutions to (4.39) must satisfy

$$\partial_t \eta(\mathbf{u}) + \partial_x q(\mathbf{u}) \leq 0, \quad (4.42)$$

in the sense of distributions, where $\mathbf{u} = (\rho, v)$. In particular for any convex function $F(\cdot)$ of the Riemann invariant w , we may expect that

$$\partial_t \rho F(w) + \partial_x \rho F(w) u \leq 0. \quad (4.43)$$

4.4.1 Elementary waves

In order to study the Riemann problem, we must first derive the Riemann invariants and draw the Lax curves, see [80]. A scalar function w is a i-Riemann invariant (i-RI Lax) if it satisfies $\nabla w \cdot r^i \equiv 0$, where the gradient is with respect to \mathbf{u} . For the system (4.41), we have the 1-RI Lax as $w = v + p(\rho)$ and 2-RI Lax as v . We have already seen that λ_1 is GNL and λ_2 is LD, thus the waves of the first family will either lead shocks or rarefaction waves, depending on the left and right states, while the waves of the second family will lead to contacts. We now study these elementary waves for (4.41) using the Lax curves.

Rarefaction waves. Let \mathbf{u}^- be a left state then it is connected to a right state \mathbf{u} iff

$$w(\mathbf{u}) = w(\mathbf{u}^-) \quad \text{and} \quad \lambda_1(\mathbf{u}) > \lambda_1(\mathbf{u}^-). \quad (4.44)$$

We now draw the Lax-curves in the \mathbf{u} and $(\rho, \rho v) = \mathbf{m}$ -planes, see Figure 4.4, where we note that the concavity of the curves will be opposite for $\gamma > 1$. The curve $v = v^- + (\rho^-)^\gamma - \rho^\gamma$ in the \mathbf{u} -plane immediately leads to the following bounds on the velocity in the right state

$$0 \leq v \leq w(\mathbf{u}^-) = v^- + (\rho^-)^\gamma. \quad (4.45)$$

Alternatively, the curves in the \mathbf{m} -plane give a geometric interpretation of the eigenvalues. Observe that the slope of the straight line $z(\mathbf{u}) = z(\mathbf{m}^+)$ is the eigenvalue $\lambda_2(\mathbf{u})$, and $\lambda_1(\mathbf{u})$ is the slope of the tangent to the curve $m = \rho v = \rho(w(u^-) - p(\rho))$. Indeed, we have

$$\frac{dm}{d\rho} = -\rho p'(\rho) - p(\rho) + w(\mathbf{u}^-), \quad (4.46)$$

where $\left. \frac{dm}{d\rho} \right|_{\mathbf{u}^- = \mathbf{u}} = v - \rho p'(\rho) = \lambda_1(\mathbf{u})$.

As the curve $w(\mathbf{m}) = w(\mathbf{m}^-)$ is strictly concave in the \mathbf{m} -planes, the derivative $\frac{dm}{d\rho} = \lambda_1(\mathbf{u})$ is strictly decreasing with respect to ρ . Consequently, admissible solutions for the rarefaction must satisfy

$$w(\mathbf{m}) = w(\mathbf{m}^-) \quad \text{and} \quad \rho < \rho^-. \quad (4.47)$$

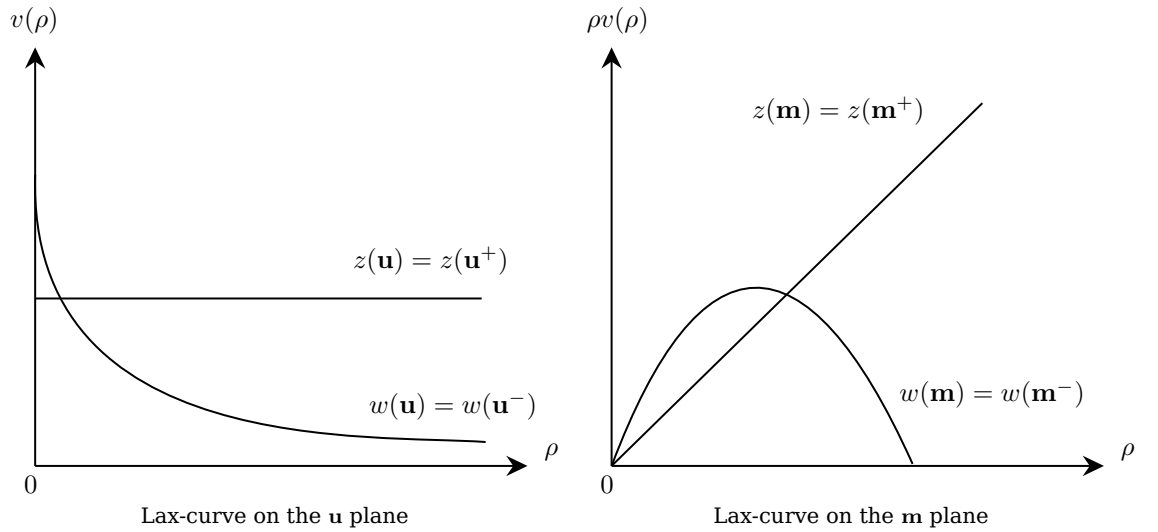


Figure 4.4: Lax-curves for $\gamma < 1$

Contact. These are very simple to comprehend from the Lax-curves in Figure 4.4. For a left state \mathbf{u} to be connected to a right state \mathbf{u}^+ through a contact would require

$$v = v^+, \quad (4.48)$$

i.e. in the m -plane the left state belongs on the line

$$m = \rho v = z(\mathbf{u}^-)\rho, \quad (4.49)$$

and no further admissibility criteria are necessary. Observe here that the waves are travelling exactly with speed of the vehicles. Classically, these contact discontinuities satisfy both the relation $z(\mathbf{u}) = z(\mathbf{u}^+)$ and the RH relations, since the solution is discontinuous through such a wave.

Shocks. In order to apply the RH condition, we must resort to conservative form (4.39). Let s be the speed of the shock wave. Weak solutions of (4.39) must satisfy the following RH condition

$$s = \frac{[[\mathbf{f}(\mathbf{Y})]]}{[[\mathbf{Y}]]}, \quad (4.50)$$

where $\mathbf{Y} = (\rho, \rho y)^\top$ and $\mathbf{f}(\mathbf{Y}) = (\rho v, \rho y v)^\top$.

Using the relation (4.50) and considering the velocity $V = v - s$ in the frame of the shock, it is a easy to arrive at

$$Q[[v + p(\rho)]] = 0, \quad (4.51)$$

where $Q = \rho^+ V^+ = \rho^- V^-$ is the mass flux.

Therefore, for (4.51) to hold, we may consider the following two cases:

1. $Q = 0$. This leads to a contact discontinuity

$$\rho^+(v^+ - s) = \rho^-(v^- - s) = 0, \quad \Rightarrow \quad v^+ = v^-, \quad (4.52)$$

which we have already discussed.

2. $Q \neq 0$. Here the model (4.39) reveals a special quality about itself as the shock curves

$$v + p(\rho) = v^- + p(\rho^-), \quad (4.53)$$

coincides with the rarefaction curve 1-RI Lax, therefore the ARZ model is a *temple* class model [81, 82]. The 1-shock waves of speed s which connects \mathbf{u} and \mathbf{u}^- are called admissible in the sense of Lax if they satisfy

$$s < \lambda_1(\mathbf{u}^-), \quad (4.54a)$$

$$\lambda_1(\mathbf{u}^+) < s < \lambda_2(\mathbf{u}^+). \quad (4.54b)$$

Consequently, a state \mathbf{u} can be connected to \mathbf{u}^- by a Lax shock of the first family iff the corresponding point in the \mathbf{m} -plane belongs to the following piece of curve

$$\rho(v + p(\rho)) = \rho(v^- + p(\rho^-)) = \rho w(\mathbf{u}^-), \quad \rho > \rho^-. \quad (4.55)$$

Remark 4.4.1. In the \mathbf{m} -plane the shock speed will be the slope of the secant joining \mathbf{m}^- to \mathbf{m}^+ . Furthermore, following (4.55), the slope of the tangent to the curve at \mathbf{m}^- will always be greater than the slope of any secant joining \mathbf{m}^- to \mathbf{m}^+ , see Figure 4.4. Therefore, the Lax inequality (4.54) can easily be verified through geometric representation.

4.4.2 The Riemann problem

Having discussed the solutions to (4.39) using elementary waves, we now consider the Cauchy problem for

$$\begin{aligned} \partial_t \rho + \partial_x \rho v &= 0, \\ \partial_t \rho y + \partial_x \rho y v &= 0, \end{aligned}$$

with Riemann initial data

$$\mathbf{u}(0, x) = \mathbf{u}_0(x) = \begin{cases} \mathbf{u}^-, & \text{for } x < 0, \\ \mathbf{u}^+, & \text{for } x > 0, \end{cases} \quad (4.57)$$

where we recall that $\mathbf{u} = (\rho, v)$. The initial data can similarly be setup in terms of the vector $\mathbf{m} = (\rho, \rho v)$. The invariant domain for the Cauchy problem is defined as

$$\Omega := \{\mathbf{u} \in \mathbb{R}^2 : \rho \geqslant, v \geqslant, w = v + p(\rho) \leqslant v_{\max}\}. \quad (4.58)$$

In order to construct solutions to the Riemann problem, we connect a \mathbf{u}^- state to a middle state \mathbf{u} by a wave of the first family, and then connect \mathbf{u} to the right state \mathbf{u}^+ by a contact discontinuity of the second family. Of course, one may also choose to connect the state in the reverse order. It is worth recalling that the 1-RI Lax is a rarefaction wave if $\rho < \rho^-$, except when there is a vacuum as the two eigenvalues coalesce $\lambda_1(\mathbf{u}) = \lambda_2(\mathbf{u}^+)$.

We now consider several cases in which the initial data (4.57) can be set, and analyse the solution to the Riemann problem with help of the Lax-curves.

Case 1: $\rho^- > 0, \rho^+ > 0, 0 \leqslant v^+ \leqslant v^-$.

It is evident from Figure 4.4 that under the condition $0 \leqslant v^+ \leqslant v^-$ the intermediate state \mathbf{u} will be a point intersecting the curves $w(\mathbf{u}) = w(\mathbf{u}^-)$ and $z(\mathbf{u}) = z(\mathbf{u}^+)$, where $\rho > \rho^-$. Thus \mathbf{u}^- is connected to \mathbf{u} by a 1-shock, whereas

the right state \mathbf{u}^+ is connected to the intermediate state by a 2-contact discontinuity. Similarly on the \mathbf{m} -plane, if \mathbf{m}^- is the initial data at any point on the curve $w(\mathbf{m}) = w(\mathbf{m}^-)$ and above the line $z(\mathbf{m}) = z(\mathbf{m}^+)$ then the slope of the secant s joining \mathbf{m}^- and the intersection of the curves will not be greater than the slope of the tangent to the curve $w(\mathbf{m}) = w(\mathbf{m}^-)$ at \mathbf{m}^- . Thus the Lax inequality (4.54) is satisfied and the solution is admissible.

Case 2: $\rho^- > 0, \rho^+ > 0, v^- \leq v^+ \leq v^- + p(\rho^-)$.

In this case, considering the solution in the \mathbf{u} -plane, the left state \mathbf{u}^- is a point on the curve $w(\mathbf{u}) = w(\mathbf{u}^-)$ which is below the line $z(\mathbf{u}) = z(\mathbf{u}^+)$. The intersection of the curves is again the intermediate state \mathbf{u} , and it is connected to the left state by a 1-rarefaction wave (as $\rho < \rho^-$). The intermediate state is then connected to the right state \mathbf{u}^+ by a 2-contact.

In the \mathbf{m} -plane, the left state \mathbf{m}^- will be a point on the curve $w(\mathbf{m}) = w(\mathbf{m}^-)$ which is below the line $z(\mathbf{m}) = z(\mathbf{m}^+)$, and it is easy to realise that the slope of the secant connecting the left state and the intermediate state \mathbf{m} , which is at the intersection of the curves, is greater than the slope of the tangent to the curve $w(\mathbf{m}) = w(\mathbf{m}^-)$ at \mathbf{m}^- . Thus the solution satisfies the admissibility criteria for the rarefaction wave.

Case 3: $\rho^- > 0, \rho^+ > 0, v^- \leq v^+ \leq v^- + p(\rho^-)$.

We now see a situation where the left and right states \mathbf{u}^- and \mathbf{u}^+ , respectively, will be connected to two separate intermediate states \mathbf{u}_1 and \mathbf{u}_2 with a vacuum state connecting \mathbf{u}_1 and \mathbf{u}_2 , see Figure 4.5. Note that for any v , $\rho \equiv 0$ is a solution. Therefore the vacuum state can be viewed as a rarefaction wave or as a discontinuous solution, either shock or rarefaction, which satisfies the RH condition.

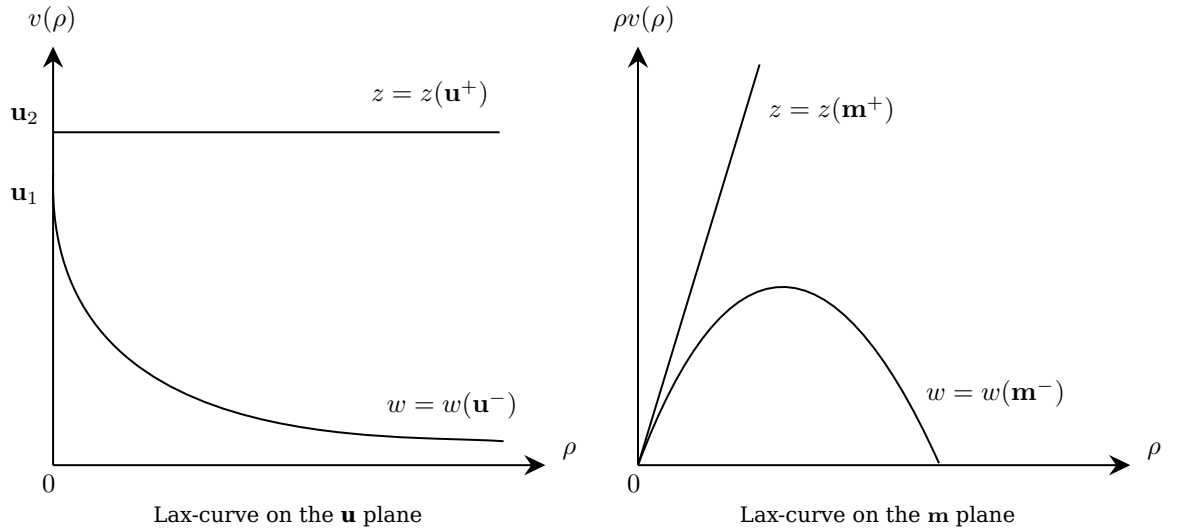


Figure 4.5: Lax curves for an intermediate vacuum state

Case 4: $\rho^- > 0, \rho^+ = 0$ or $\rho^- = 0, \rho^+ > 0$.

Finally, we consider the case where either the left state or the right state is a vacuum. We refer to Figure 4.4 and note that if the right state is a vacuum ($\rho^+ = 0$) then any point on the curve $w(\mathbf{m}) = w(\mathbf{m}^-)$ will be connected to the

left state (origin) by a 1-rarefaction wave. On the other hand, if the left state is vacuum ($\rho^- = 0$) then any right state on the line $z(\mathbf{m}) = z(\mathbf{m}^+)$ will be connected to the left state by a 2-contact wave.

4.4.3 Stability near the vacuum

In this section, we are interested in a deeper insight towards the Riemann problem where one of two constant states is a vacuum, see Case 4 above. However, before proceeding further, we first recall some fundamental properties of weak solutions of hyperbolic systems.

Let us perturb (4.39) by an artificial viscosity

$$\begin{aligned} \partial_t \rho + \partial_x \rho v &= \varepsilon \partial_x^2 \rho, \\ \partial_t y + \partial_x y v &= \varepsilon \partial_x^2 y, \end{aligned} \tag{4.59}$$

where ε is small, then there exists a travelling wave solution to (4.59) of the form

$$(\rho, y) := (\rho, y)((x - st)/\varepsilon) = (\rho, y)(\xi), \tag{4.60}$$

such that

$$(\rho, y)(t, x) \rightarrow (\rho^\pm, y^\pm) \text{ when } x \rightarrow \pm\infty. \tag{4.61}$$

Such viscous profiles are stable under small perturbations and are known to converge to a Lax shock when $\varepsilon \rightarrow 0$. Therefore, mathematically, we can consider that 2-contact discontinuity as a 1-shock that admits a viscous profile. The speed of this shock will be the slope of the line $0\mathbf{m}$, where \mathbf{m} is the point where the Lax-curves intersect in the \mathbf{m} -plane, see Figure 4.4.

Considering this notion we go back to the Riemann problem where one of the states is the vacuum, see Case 4 in the previous section, and perturb the initial data. Let us fix the right state $\mathbf{m}^+ = (\rho^+, v^+)$ and perturb the left vacuum state to get $\mathbf{m}^- = (\rho^-, v^-)$. Of course the solution to this Riemann problem can again be analyzed in a similar way as done in the previous section, depending on the values of (ρ^-, v^-) as compared to (ρ^+, v^+) . However here we consider two special cases for the initial data which were not part of the previous analysis.

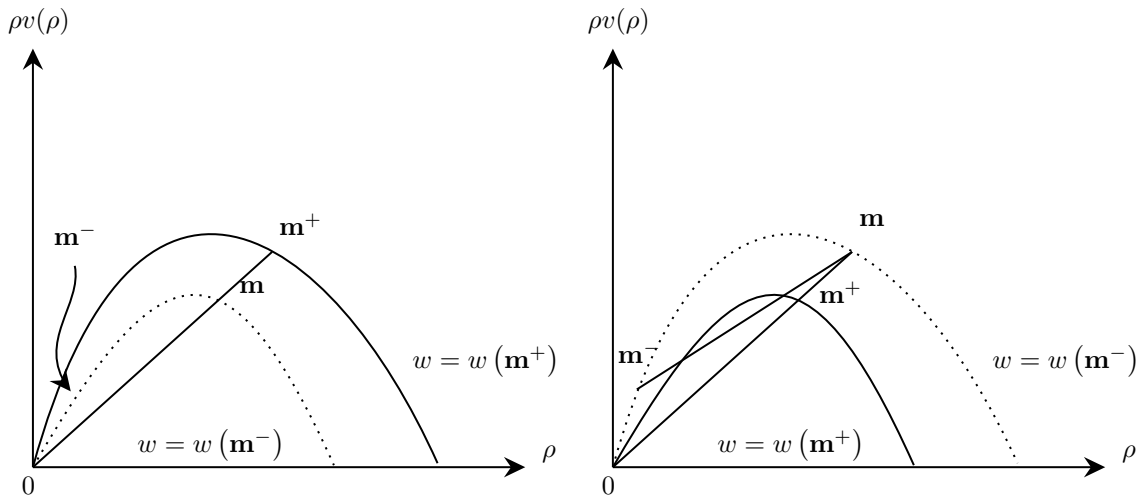


Figure 4.6: Perturbation of the vacuum left state \mathbf{m}^- .

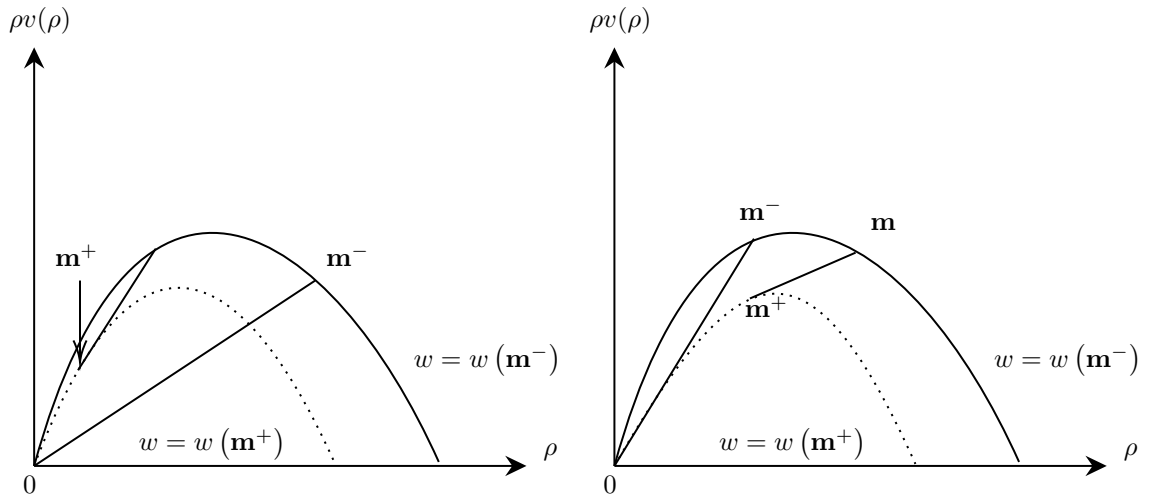


Figure 4.7: Perturbation of the vacuum right state m^+ .

- Case 1:** $v^+ < v^- < v^- + p(\rho^-) < v^+ + p(\rho^+)$. The solution now consists of a shock connecting the left state m^- to an intermediate state m with $w(m) = w(m^-) < w(m^+)$, followed by a contact discontinuity that connects m and m^+ , see Figure 4.6. In this case, the original single 2-contact discontinuity has been replaced by a 1-shock wave with speed $s \neq v^+$ followed by a contact wave.
- Case 2:** $v^+ < v^- < v^+ + p(\rho^+) < v^- + p(\rho^-)$. Here v^- is very large, and the solution structure is similar to the previous case however now $w(m) = w(m^-) > w(m^+)$. Therefore the 1-shock is of a much larger amplitude than the original 2-contact discontinuity, see Figure 4.7. Therefore, we observe a large oscillation with a small perturbation on the Riemann data which leads to instabilities.

It is clear from this analysis that the solution to (4.59) does not continuously depend on the initial data upon small perturbation and thus well-posedness cannot be claimed. A similar oscillatory behaviour of the solution was also observed when the initial data comprising of a left rarefaction wave connecting a vacuum was perturbed. Moreover, in this case, the maximal velocity v reached by the vehicles is found to be $v_{\max} = v^- + p(\rho^-)$. This implies that the maximal velocity reached by the vehicles on a empty road depends on the initial data, which is incorrect. The authors in [56, 83] suggested a fix for this problem by supplementing (4.41) with a relaxation term in the momentum equation

$$\begin{aligned} \partial_t \rho + \partial_x \rho v &= 0, \\ \partial_t v + (v - \rho p'(\rho)) \partial_x v &= \frac{V(\rho) - v}{\delta}, \end{aligned} \quad (4.62)$$

where δ is the characteristic time of relaxation and $V(\rho)$ is a suitable nonincreasing velocity corresponding to each density ρ .

An immediate question that follows from (4.62) is the convergence towards the LWR model (4.2) upon instantaneous relaxation and this claim holds as was shown recently in [84].

4.5 Discretization of the Aw-Rascle-Zhang model

In the previous sections, we have described the ARZ model in substantial detail and highlighted that it is superior to the LWR model (4.2) in resolving traffic flows. It therefore motivates us to embark upon the numerical discretization of the ARZ model, where the numerical scheme allows for sharp resolution of the contact discontinuity and shock (without numerical diffusion). In our recent developments, we study this problem through the work done in [85] where the numerical scheme is mixed and resolves contact discontinuities through a random sampling technique, while for the solution of the first family are resolved using the Godunov method. For the numerical fluxes in the Godunov scheme, we design HLL fluxes [86]. The scheme ensures L^2 and L^∞ stability and we demonstrate this through a series of Riemann problems.

We are interested in the Cauchy problem for the ARZ model (4.39) which can be written in the following form:

$$\begin{aligned} \partial_t \mathbf{u} + \partial_x \mathbf{f}(\mathbf{u}) &= 0, \\ \mathbf{u}(0, x) &= \begin{cases} \mathbf{u}^-, & \forall x \leq 0, \\ \mathbf{u}^+, & \forall x > 0, \end{cases} \end{aligned} \quad (4.63)$$

where $\mathbf{u} = (\rho, \rho w)^\top$ and $\mathbf{f}(\mathbf{u}) = \mathbf{u}v$. The Jacobian of the system (4.63) admits two eigenvalues $\lambda_1(\mathbf{u}) = v - \rho p'(\rho)$ and $\lambda_2(\mathbf{u}) = v$, where λ_1 is associated to a GNL field, while λ_2 is associated to a LD field. Depending on the Riemann data, solutions to (4.63) will admit waves of the first family (rarefaction or shocks) or of the second family (contact) or both. If the solution admits waves of both the families then the two waves are connected through an intermediate region $\mathbf{u}^* = (\rho^*, \rho^* w^*)$, which, under the closure law (4.40) is defined as:

$$\rho^* = (w(\mathbf{u}^-) - v^+)^{1/\gamma}, \quad w^* = v^+ + p(\rho^*), \quad \gamma > 0. \quad (4.64)$$

It is also worth mentioning that the solution to (4.63) obeys a maximum principle for the Riemann invariants $w = v + p(\rho)$ and v . Let $(t, x) \rightarrow \mathbf{u}_r(\frac{x}{t}; \mathbf{u}^-, \mathbf{u}^+)$ be the self-similar solution of (4.63), then we have

$$\begin{aligned} \inf_{\mathbf{u}} \{w(\mathbf{u}^-), w(\mathbf{u}^+)\} &\leq w\left(\mathbf{u}_r\left(\frac{x}{t}; \mathbf{u}^-, \mathbf{u}^+\right)\right) \leq \sup_{\mathbf{u}} \{w(\mathbf{u}^-), w(\mathbf{u}^+)\}, \\ \inf_{\mathbf{u}} \{v^-, v^+\} &\leq v\left(\mathbf{u}_r\left(\frac{x}{t}; \mathbf{u}^-, \mathbf{u}^+\right)\right) \leq \sup_{\mathbf{u}} \{v^-, v^+\}. \end{aligned} \quad (4.65)$$

Computational domain and approximate solution. Let the physical domain be discretized using a mesh Ω_h comprising of non-overlapping cells j , of uniform size h , such that the cell centers are defined as $x_j = (j + 1/2)h$ and the cell interfaces as $x_{j+1/2} = jh$ for $j \in \mathbb{Z}$. The time is discretized using n step such that the computational time $t^n = n\Delta t$, where Δt is each time-step. A piecewise constant approximate solution $(t, x) \mapsto \mathbf{u}_j^n$ is given by $\mathbf{u}(t^n, x_j) = \mathbf{u}_j^n, \forall j \in \mathbb{Z}, n \in \mathbb{N}$. The initial condition is defined as

$$\mathbf{u}_j^0 = \frac{1}{h} \int_{x_{j+1/2}}^{x_{j+1/2} + h} \mathbf{u}_0(x) dx, \forall j \in \mathbb{Z}.$$

Failure of the Godunov scheme. The Godunov scheme is written as

$$\mathbf{u}_j^{n+1} = \mathbf{u}_j^n - \frac{\Delta t}{h} (\mathbf{h}_{j+1/2}^n - \mathbf{h}_{j-1/2}^n), \quad \forall j \in \mathbb{Z}, n \in \mathbb{N}, \quad (4.66)$$

where $\mathbf{h}_{j+1/2}^n = \mathbf{f}(\mathbf{u}_r(0^-; \mathbf{u}_j^n, \mathbf{u}_{j+1}^n))$ is the numerical flux and $\mathbf{u}_r(0^-; \mathbf{u}_j^n, \mathbf{u}_{j+1}^n)$ is the Riemann problem at $j + 1/2$. The time-step in (4.66) is restricted by the CFL condition

$$\frac{\Delta t}{h} \max_{\mathbf{u}} \{|\lambda_i(\mathbf{u})|, i = 1, 2\} \leq \frac{1}{2}. \quad (4.67)$$

Note that following Green's theorem, (4.66) is equivalent to

$$\mathbf{u}_j^{n+1} = \frac{1}{2} (\mathbf{u}_{j-1/2,R}^{n+1} + \mathbf{u}_{j+1/2,L}^{n+1}), \quad (4.68)$$

where

$$\mathbf{u}_{j-1/2,R}^{n+1} = \frac{2}{h} \int_{x_{j-1/2}}^{x_j} \mathbf{u}_r \left(\frac{x - x_{j-1/2}}{\Delta t}; \mathbf{u}_{j-1}, \mathbf{u}_j \right) dx = \mathbf{u}_j^n - \frac{2\Delta t}{h} (\mathbf{f}(\mathbf{u}_j^n) - \mathbf{h}_{j-1/2}^n) \quad (4.69)$$

and

$$\mathbf{u}_{j+1/2,L}^{n+1} = \frac{2}{h} \int_{x_j}^{x_{j+1/2}} \mathbf{u}_r \left(\frac{x - x_{j+1/2}}{\Delta t}; \mathbf{u}_j, \mathbf{u}_{j+1} \right) dx = \mathbf{u}_j^n - \frac{2\Delta t}{h} (\mathbf{h}_{j+1/2}^n - \mathbf{f}(\mathbf{u}_j^n)). \quad (4.70)$$

As a result, under the condition (4.67), the solution \mathbf{u}_j^{n+1} is only affected by the solutions in the grid $G_j = [x_{j-1}, x_{j+1}]$, see Figure 4.8.

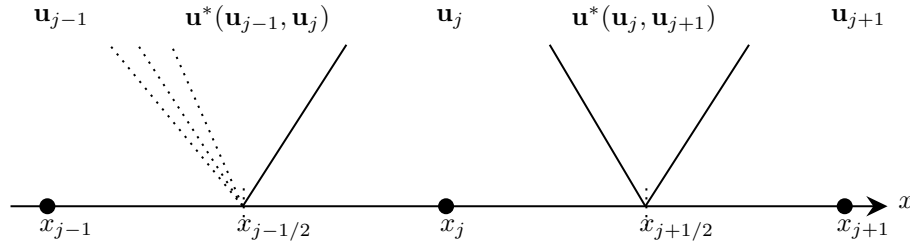


Figure 4.8: Grid $G_j = [x_{j-1}, x_{j+1}]$ with 1-rarefaction and 2-contact at $x_{j-1/2}$ and 1-shock and 2-contact at $x_{j+1/2}$.

Furthermore, note that under the Godunov scheme the update in time is based on the cell-averaged solutions. Let the Riemann data be set as

$$\mathbf{u}(0, x) = \begin{cases} \mathbf{u}^-, & \forall x \leq j, \\ \mathbf{u}^+, & \forall x > j, \end{cases}$$

where $v^- = v^+$, then the solution will result in a single contact discontinuity travelling at speed $v = v^- = v^+$. The first update of the solution vector $\mathbf{u} = (\rho, \rho w)^T$ will be

$$\rho_j^{\Delta t} = \bar{\rho}, \quad (\rho w)_j^{\Delta t} = \bar{\rho w}, \quad (4.71)$$

where the over-bar indicates the following

$$\bar{\alpha} = \frac{1}{h} \int_0^h \alpha(\Delta t, x) dx.$$

If, at the first time-step, the velocity is computed from (4.71), then a quick calculation will result in

$$v_j^{\Delta t} = v + \frac{\overline{\rho p(\rho)} - \bar{\rho} p(\bar{\rho})}{\bar{\rho}},$$

where $v_j^{\Delta t} \geq v$ as $\rho \mapsto \rho p(\rho)$ is convex and Jensen's inequality is invoked. To avoid this, in our work we implement the transport-equilibrium scheme [85], where

the contact discontinuity is resolved using a random sampling technique, and the solution is then glued to the solution of the 1-wave by a Godunov update. Note, that the ARZ model (4.39) is not a thermodynamic model, so the techniques proposed in Here we summarize the essential ideas of the scheme.

Resolving contacts. The velocity v in the ARZ model is a nonnegative function, therefore the contact discontinuity arising from the Riemann problems placed at the interfaces $x_{j-1/2}$ and $x_{j+1/2}$ will only affect $[x_{j-1/2}, x_j]$ and $[x_{j+1/2}, x_{j+1}]$. Furthermore, there is an intermediate region at the wake of the contact discontinuity that connects the 1-wave, see Figure 4.8. The solution to the Riemann problems for the grid G_j admits the following solution:

$$\mathbf{u}(t, x) = \begin{cases} \mathbf{u}_{j-1}^n & \text{if } x \in [x_{j-1}, x_{j-1/2}), \\ \mathbf{u}^*(\mathbf{u}_{j-1}^n, \mathbf{u}_j^n) & \text{if } x \in [x_{j-1/2}, x_{j-1/2} + v_j^n(t - t^n)), \\ \mathbf{u}_j^n & \text{if } x \in [x_{j-1/2} + v_j^n(t - t^n), x_{j+1/2}), \\ \mathbf{u}^*(\mathbf{u}_j^n, \mathbf{u}_{j+1}^n) & \text{if } x \in [x_{j+1/2}, x_{j+1/2} + v_{j+1}^n(t - t^n)), \\ \mathbf{u}_{j+1}^n & \text{if } x \in [x_{j+1/2} + v_{j+1}^n(t - t^n), x_{j+1}). \end{cases} \quad (4.72)$$

According to Glimm's random sampling strategy, an algorithm selects random on the grid G_j a solution state from the possible states of (4.72) based on the degree of presence of any solution state within the grid. Let a_n be a well distributed random sequence, the algorithm amounts to:

$$\mathbf{u}(x, t^n + \Delta t) = \begin{cases} \mathbf{u}_{j-1}^n & \text{if } x \in [x_{j-1}, x_{j-1/2}) \\ \mathbf{u}_j^{n+1/2} & \text{if } x \in [x_{j-1/2}, x_{j+1/2}) \\ \mathbf{u}_{j+1,L}^{n+1/2} & \text{if } x \in [x_{j+1/2}, x_{j+1}), \end{cases} \quad (4.73)$$

where

$$\mathbf{u}_j^{n+1/2} = \begin{cases} \mathbf{u}^*(\mathbf{u}_{j-1}^n, \mathbf{u}_j^n), & \text{if } a_{n+1} \in (0, \frac{\Delta t}{h} v_j^n), \\ \mathbf{u}_j^n & \text{if } a_{n+1} \in [\frac{\Delta t}{h} v_j^n, 1), \end{cases} \quad (4.74)$$

and

$$\mathbf{u}_{j+1,L}^{n+1/2} = \begin{cases} \mathbf{u}^*(\mathbf{u}_j^n, \mathbf{u}_{j+1}^n), & \text{if } a_{n+1} \in (0, \frac{2\Delta t}{h} v_j^n), \\ \mathbf{u}_{j+1}^n & \text{if } a_{n+1} \in [\frac{2\Delta t}{h} v_j^n, 1), \end{cases} \quad (4.75)$$

Here a_n is computed using the well-known van der Corput random sequence

$$a_n = \sum_{k=0}^m i_k 2^{-(k+1)},$$

where m is the m^{th} time iteration. For a detailed study of the random choice method, including convergence to the correct solution, we direct the reader to [87, 88].

Resolving 1-wave. Once the contact wave has been resolved, the 1-wave is glued through the following Godunov update

$$\begin{aligned} \mathbf{u}_{j+1/2,L}^{n+1} &= \frac{2}{h} \int_{x_j}^{x_{j+1/2}} \mathbf{u} \left(\frac{x - x_{j+1/2}}{\Delta t}; \mathbf{u}_j^{n+1/2}, \mathbf{u}_{j+1}^n \right) dx \\ &= \mathbf{u}_j^{n+1/2} - \frac{2\Delta t}{h} \left(\mathbf{h}(\mathbf{u}_j^{n+1/2}, \mathbf{u}_{j+1}^n) - \mathbf{f}(\mathbf{u}_j^{n+1/2}) \right), \end{aligned} \quad (4.76)$$

$$\begin{aligned} \mathbf{u}_{j-1/2,R}^{n+1} &= \frac{2}{h} \int_{x_{j-1/2}}^{x_j} \mathbf{u} \left(\frac{x - x_{j-1/2}}{\Delta t}; \mathbf{u}_{j-1}^n, \mathbf{u}_j^{n+1/2} \right) dx \\ &= \mathbf{u}_j^{n+1/2} - \frac{2\Delta t}{h} \left(\mathbf{f}(\mathbf{u}_j^{n+1/2}) - \mathbf{h}(\mathbf{u}_{j-1}^n, \mathbf{u}_j^{n+1/2}) \right). \end{aligned} \quad (4.77)$$

Note that if $\mathbf{u}^*(\mathbf{u}_{j-1}^n, \mathbf{u}_j^{n+1/2}) = \mathbf{u}_j^{n+1/2}$ then

$$\mathbf{u}_{j-1/2,R}^{n+1} = \frac{2}{h} \int_{x_{j-1/2}}^{x_j} \mathbf{u} \left(\frac{x - x_{j-1/2}}{\Delta t}; \mathbf{u}_j^{n+1/2}, \mathbf{u}_j^{n+1/2} \right) dx = \mathbf{u}_j^{n+1/2}.$$

The resulting scheme is

$$\mathbf{u}_j^{n+1} = \frac{1}{2} \left(\mathbf{u}_{j+1/2,L}^{n+1} + \mathbf{u}_{j-1/2,R}^{n+1} \right) = \mathbf{u}_j^{n+1/2} - \frac{\Delta t}{h} \left(\mathbf{h}_{j+1/2}^{n+1/2,L} - \mathbf{h}_{j-1/2}^{n+1/2,R} \right), \quad \forall j \in \mathbb{Z} \quad (4.78)$$

where

$$\mathbf{h}_{j+1/2}^{n+1/2,L} = \mathbf{h}(\mathbf{u}_j^{n+1/2}, \mathbf{u}_{j+1}^n), \quad \mathbf{h}_{j-1/2}^{n+1/2,R} = \begin{cases} \mathbf{h}(\mathbf{u}_{j-1}^n, \mathbf{u}_j^{n+1/2}) & \text{if } \mathbf{u}^*(\mathbf{u}_{j-1}^n, \mathbf{u}_j^{n+1/2}) = \mathbf{u}_j^{n+1/2}, \\ \mathbf{f}(\mathbf{u}_j^{n+1/2}), & \text{otherwise.} \end{cases} \quad (4.79)$$

For the choice of the numerical fluxes in the Godunov scheme, we design a HLL approximate solver [86, 89] for the ARZ model (4.39) that bounds the 1-wave in the exact entropy weak solution of the Riemann problem and ensures that a local minimum entropy principle holds. The numerical flux is defined as

$$\mathbf{h}_{j+1/2}^{n+1/2,L} = \begin{cases} \mathbf{f}(\mathbf{u}_j^{n+1/2}) & \text{if } 0 \leq s_L, \\ \frac{s_R (\mathbf{f}(\mathbf{u}_j^{n+1/2}) - s_L \mathbf{u}_j^{n+1/2})}{s_R - s_L} & \text{if } s_L \leq 0 \leq s_R, \\ \mathbf{f}(\mathbf{u}_{j+1}^n) & \text{if } s_R \leq 0, \end{cases} \quad (4.80)$$

where $s_L = \min\{\lambda_1(\mathbf{u}_j^{n+1/2}), \frac{\rho^* v^+ - \rho^- v^-}{\rho^* - \rho^-}\}$ and $s_R = v^+$.

Numerical tests. We will now demonstrate the qualities of the transport-equilibrium scheme through a series of Riemann problems for ARZ model (4.39), and compare the numerical solution to the solutions obtained by using the Godunov scheme as well as the exact solution. All numerical experiments performed in this work are on a computational mesh $\Omega_h = [0, 1]$ which is divided into 100 elements, with Dirichlet condition on the boundaries. The time integration is performed using forward Euler method and the computed solutions were obtained at a final time $T_{\max} = 0.15$. For all numerical experiments the pressure is computed using (4.40), where $\gamma = 1.4$.

The Riemann initial data comprises of the left and right states \mathbf{u}^- and \mathbf{u}^+ , respectively, which are split at $x = 0.5$. The set of initial data are given below and the numerical results can be found in Figure 4.9.

(a) 2-contact:

$$\rho^- = 0.8, \quad v^- = 1.0, \quad \rho^+ = 0.2, \quad v^+ = 1.0.$$

(b) 1-shock

$$\rho^- = 0.2, \quad v^- = 0.8, \quad \rho^+ = 0.5244, \quad v^+ = 0.5.$$

(c) 1-rarefaction

$$\rho^- = 0.8, \quad v^- = 0.2, \quad \rho^+ = 0.23502, \quad v^+ = 0.8.$$

(d) 1-shock, 2-contact

$$\rho^- = 0.2, \quad v^- = 0.8, \quad \rho^+ = 0.8, \quad v^+ = 0.2.$$

(e) 1-rarefaction, 2-contact

$$\rho^- = 0.8, \quad v^- = 0.2, \quad \rho^+ = 0.8, \quad v^+ = 0.7.$$

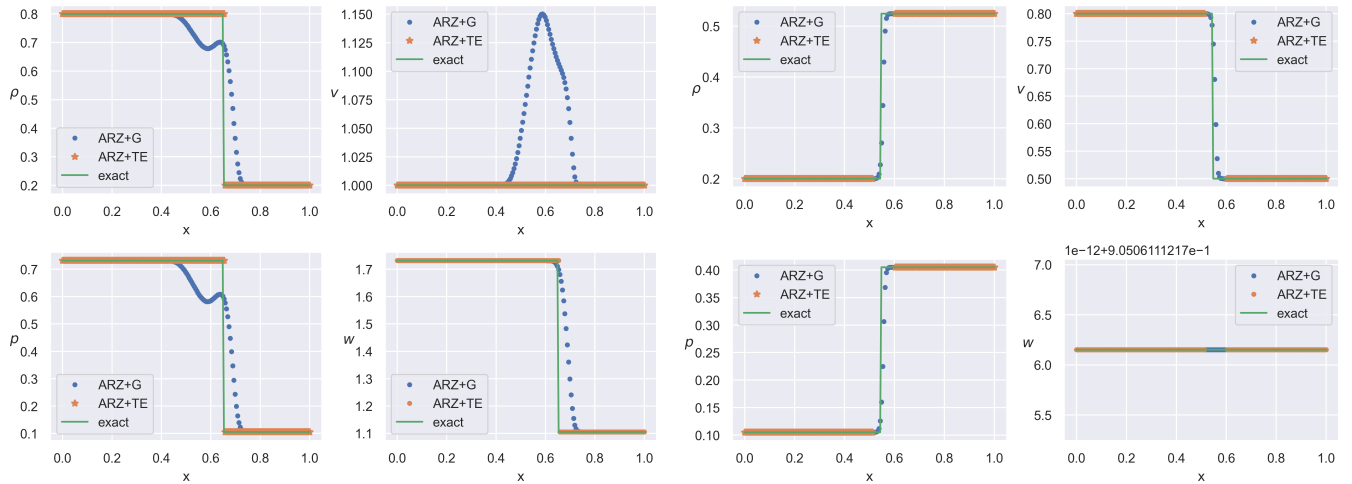
(f) 1-rarefaction, vacuum, 2-contact

$$\rho^- = 0.2, \quad v^- = 0.2, \quad \rho^+ = 0.8, \quad v^+ = 0.4.$$

4.6 Conclusion

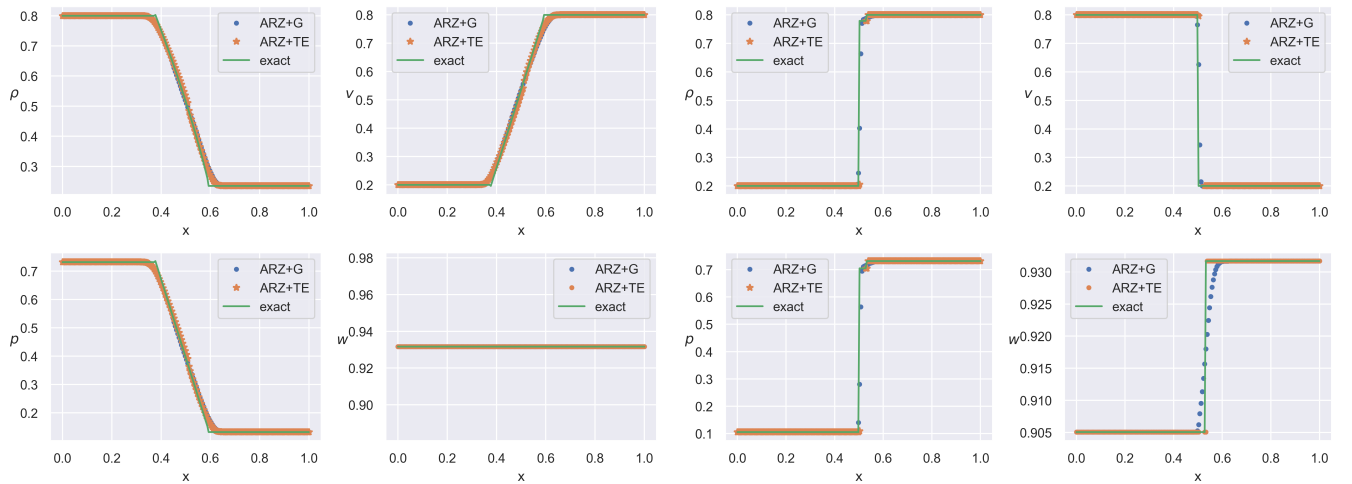
Through this contribution, we provide an overview of different mathematical models for macroscopic traffic flow and provide numerical techniques for their accurate resolution. The models discussed in this work are the LWR model [52], the non-local flux model [67] and the ARZ model [56]. We provide an in-depth mathematical analysis of these models, which in turn enables a robust comparison of the advantages and disadvantages of these models. During the course of this work, we conclude that the ARZ model is far more superior in predicting traffic as compared to the LWR model, where the complex wave structure results in more realistic prediction of traffic. However, numerical discretisation of the ARZ model is quite challenging as the numerical scheme needs to preserve the wave structure and also satisfy stability criteria. Towards this goal, we improve on the work done in [85] and provide a discretization strategy that preserves the maximum principle while also satisfying the entropy stability criteria. As an ongoing work, we are further improving the numerical scheme to stabilize it for Riemann initial data with vacuum.

In our future work, the planned course of action is twofolds: first, we plan to analyse the macroscopic model with non-local fluxes [90, 91, 92]. Similar to the ARZ model, the non-local flux model also predicts the local traffic conditions based on the traffic downstream, however unlike the ARZ the non-local flux models are scalar conservation laws. Therefore a rigorous comparison between the ARZ model and the non-local flux model is of considerable importance. Second, we plan to extend the numerical scheme for the ARZ model to flow on networks [62] and setup benchmark test cases that highlight the robustness and stability for flow on traffic networks.



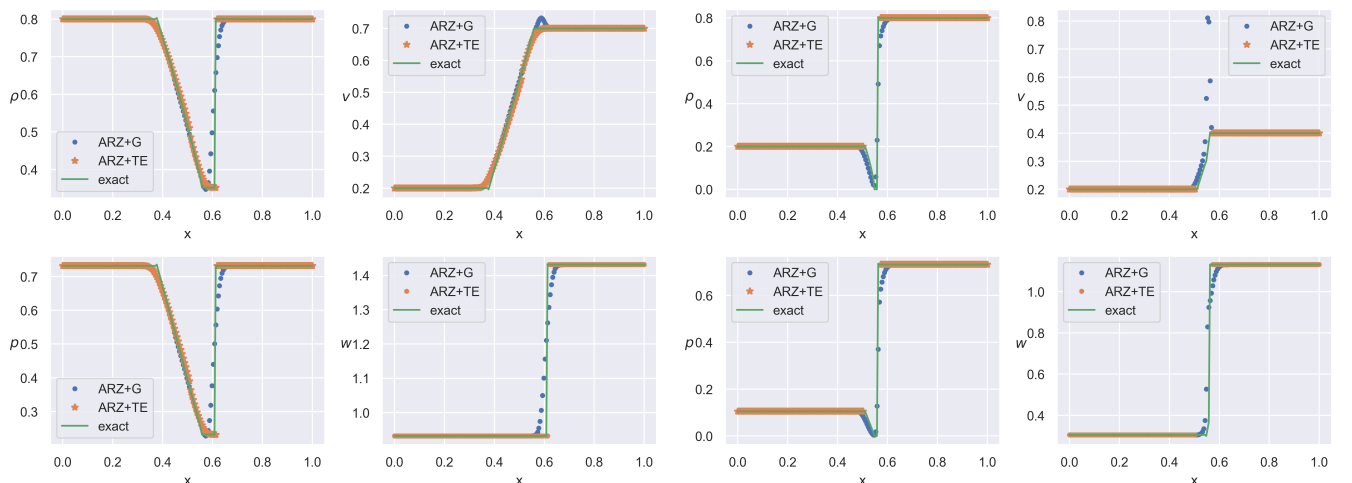
(a) 2-contact

(b) 1-shock



(c) 1-rarefaction

(d) 1-shock, 2-contact



(e) 1-rarefaction, 2-contact

(f) 1-rarefaction, vacuum, 2-contact

Figure 4.9: Riemann problems for the ARZ model using the transport-equilibrium scheme. Within the legends, ARZ+G indicates solution using the standard Godunov scheme, ARZ+TE indicates solution using transport-equilibrium scheme, and exact is the exact solution.

Chapter 5

Gamified interfaces design (CLLE)

5.1 An overview of gamification and mobility

This section proposes an overview of gamification and mobility and concerns more specifically how using game mechanics in mobility and transport.

Gamification is an emerging practice inspired by video games and is increasingly used in the field of human-computer interaction. It consists of inserting game elements and mechanics into an activity whose context is not initially related to the game [93]. Whether in the medical context through health [94] [95], education [96], e-commerce [97] or mobility and transportation [98], this new approach is adopted by designers primarily for its benefits. Indeed, gamification would influence users' engagement, play on their motivation to perform an activity, but above all would increase users' satisfaction and pleasure [99]. This approach can be implemented through a multitude of elements borrowed from games: for example, the attribution of rewards and prizes, points, challenges, rankings between players, levels or steps, etc. (see [100]).

In the context of mobility and transportation, gamification is used in different situations and for different reasons. During driving activity, gamification is often applied to the topic of eco-driving. For example, [101] used gamification through the dashboard interface to encourage drivers to adopt more eco-friendly driving styles. An acceptable trip Eco-score was calculated and allowed to earn badges. Collecting points improved overall driving skills. [102], on the other hand, tested the effect of give challenges during the driving situation to reduce driver monotony-induced fatigue and the accidents that can result. In the context of road safety, [103] highlighted the possible inability of children to effectively and actively analyze their environment, sometimes leading them to perform inconsistent behaviors. In order to act on their perception of danger, they implemented gamified self-learning modules on traffic awareness, situational awareness, risk detection, and risk management. The gamification elements helped the children maintain their motivation to learn and achieve higher scores on the knowledge modules. Concerning mobility, gamification has been used in the Mobility-as-a-Service (MaaS) application. The MaaS app is a new concept that combines different modes of transportation into a single mobility platform accessible through a single interface on a smartphone or tablet [104]. These applications are part of a societal desire to improve the environmental impact of travel through sustainable transportation and to optimize urban traffic. This desire is reflected in initiatives such as encouraging a company's employees to use sustainable modes of travel by offering different challenges [105]. Publications on MaaS applications often take the form of case studies where prototype applications are tested on a specific population, as each city has its own constraints and offerings in terms of mobility and transportation [106]. For example, [107] studied the implementation of a MaaS application in the city of Madrid; [108] in the city of Hertogenbosch in the Netherlands; in the city of Madeira [109] or in Stockholm [110]. When gamification is integrated into MaaS applications, the objectives are multiple and complementary.

The primary goal is to motivate and engage users to use the MaaS app, but also to maintain this usage over time [111]. [112] showed that users spent more time using public transportation and walking during the reward period, but this effect was not demonstrated over time. Studies combining gamification and mobility application, sometimes rely on psychological models of motivation such as self-determination theory [113], or behavior such as the theory of reasoned action [114]. The issues behind the use of gamification as motivator and engagement factor may be, for example, to reduce parking problems by motivating users to feed the real-time information of the MaaS application [115]. For example, [115] tested giving rewards to users contributing to the sharing of parking

availability information. Thus, users contributed more to information sharing when they obtained rewards.

The second objective is to influence or change the behavior of the users of the application. Designers can implement gamification to meet specific needs. This can be to encourage users to change their mode of transportation to reduce road congestion issues by switching from using a private car to a shared mode of transportation [116], or to adopt sustainable behaviors by using better transportation [117], [118], [119]. [120], for example, implemented earning points and badges when the user made a certain type of trip or explored certain mobility alternatives (e.g., when testing the use of a bike share for the first time).

Whether they are children or adults, pedestrians, drivers, cyclists or using public transport, all citizens can be involved in using mobility applications. Understanding their needs and constraints is a first central point. Gamification is a promising new approach that is still little studied in the context of MaaS situations. The objectives of the designers are multiple and the insertion of the different gamification elements, as well as their impact on human behaviors, are still little investigated and depend on each situation. Although studies already exist, they remain specific case studies on targeted populations that can sometimes be criticized for lacking experimental rigor. Thus, their conclusions remain fragile to allow a direct transposition of the results to other situations. Studying gamification in the specific context of MaaS applications through controlled experimental studies in both laboratory and field settings is therefore necessary. This will contribute to a clear lack of knowledge about the impact of gamification when using a MaaS application.

5.2 Heuristic analysis of eight MaaS applications

As a way to establish what was already in practice in the MaaS world, we decided to focus on 8 applications and evaluate them with the criterias developed by [121]. To those criterias were added four suggested by [122] as a way to not only assess the user experience in general but also to evaluate the emotional design and the means of persuasion used by developers in order to engage their customers.

As per stated, 8 applications were chosen for this heuristic analysis. We first went through different possibilities, assessing MaaS in France and around the world and then decided to choose a panel representing MaaS development in different cities. We started with the MaaS developed by the city of Toulouse (Tisseo), then the one of Bordeaux (TBM), Montpellier (TaM), Saint Etienne (Moovizy) and Paris (Bonjour RATP). Then we added a few MaaS that worked in different cities or even countries such as Whim, Moovit and CityMapper (fig.5.1).

This analysis was meant to assess the usability of those apps, as well as seeing if any of them were using means such as gamification or persuasive design to engage their users. It was established that several such as TBM, Moovizy or CityMapper use the presentation of CO₂ or calories as a way to influence the behaviors of their users. As well, we found some mechanisms such as counting points or gaining titles and avatars that can be considered as gamification according to the definition of [123], in three of the eight applications (fig.5.2).

Nevertheless, it seems like the use of gamification in transport applications is scarce compared to its use in other applications such as fitness or health.

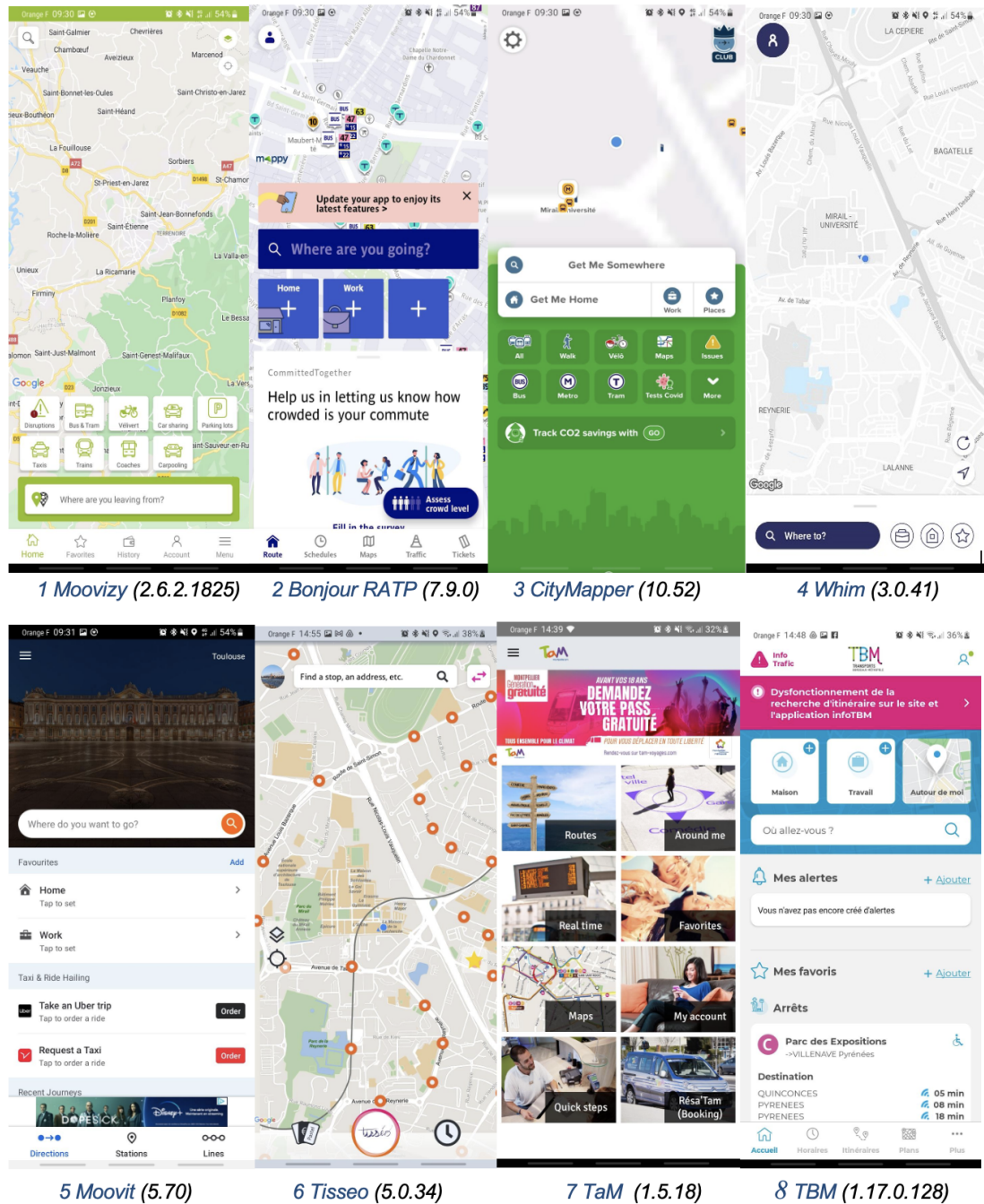


Figure 5.1: Screenshot of the eight MaaS applications selected for the heuristic analysis

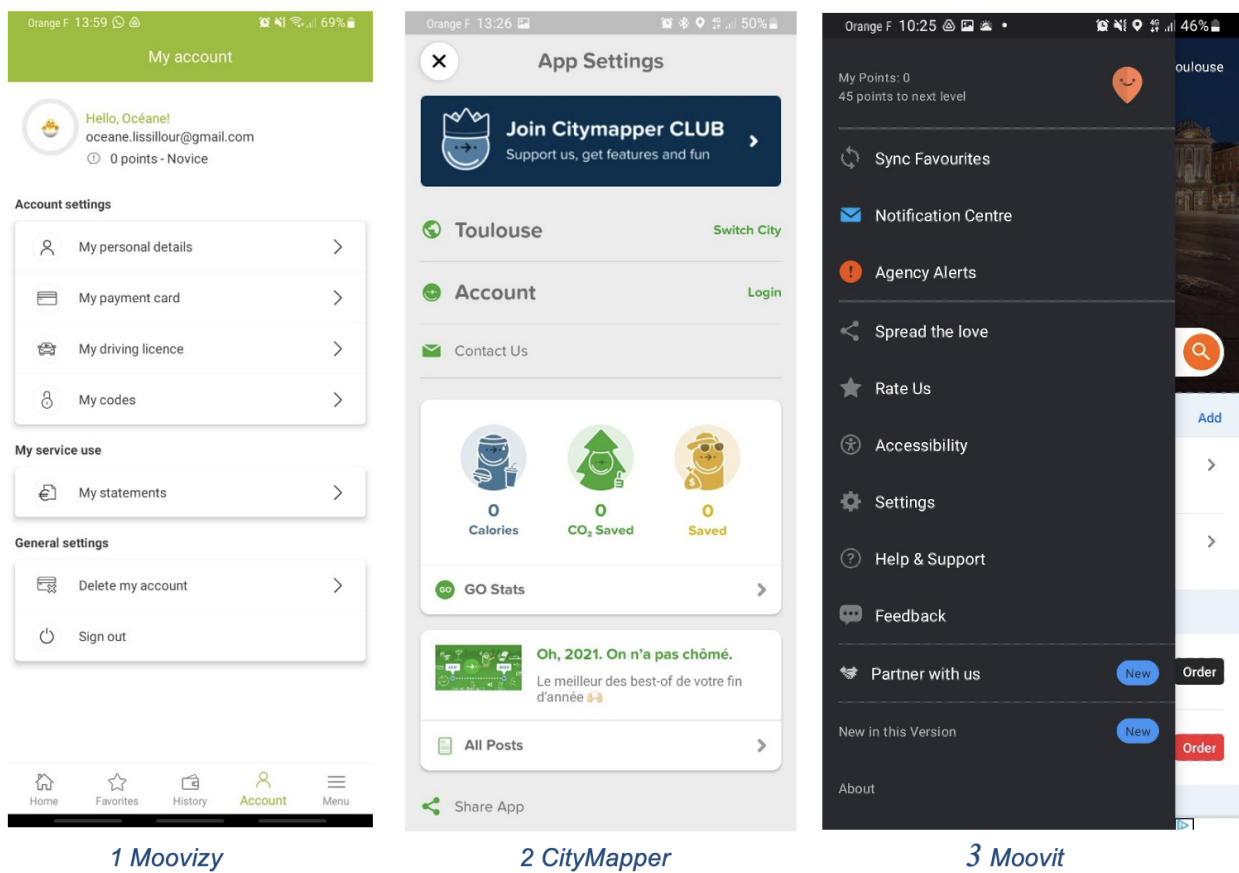


Figure 5.2: Screenshot of the three gamified MaaS applications selected for the heuristic analysis

5.3 Conclusion

In conclusion, gamification can be used for different purposes. Through the research action "Citizen Participation", we want to study the user experience when integrating gamification in a MaaS application. We want to understand the cognitive processes involved when a user interacts with a gamified system. A first step will be to make a state of the art of the literature to better understand the concept and its use in other domains than mobility. Thus, an exhaustive list of theoretical models used in studies involving gamification is necessary before developing experiments. Second, an inventory of existing MaaS applications is necessary, especially when they incorporate gamification. Their heuristic evaluation allows us to compare the design of existing MaaS applications and to highlight design flaws. We have chosen eight MaaS applications in which three applications are gamified. This analysis allows us to note that the gamification of MaaS applications is limited in number and game elements. Third, experimental and field studies will be conducted to obtain a better understanding of the cognitive processes in the context of mobility and especially with the use of MaaS applications. These studies will allow us to provide recommendations for the design of the MaaS applications. The objective is to design an application that offers the best user experience and that is accepted by the user. Thus, the design will have to take into account the needs, expectations and capabilities of the users to obtain usable and acceptable systems with which the users will be engaged with which they will engage in the long term and with which they will experience fun.

Chapter 6

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ASR - Architecture, Systems and Networks

- RMES - Networks, Mobile, Embedded, Wireless, Sattellites
- SEPIA - Operating systems, distributed systems, from Middleware to Architecture
- SIERA - Service IntEgration and netwoRk Administration
- T2RS - Real-Time in networks and systems
- TRACES - Trace stands for research groups in architecture and compilation for embedded systems

CISO - HPC, Simulation, Optimization

- APO - Parallel Algorithms and Optimisation
- REVA - Real Expression Artificial Life

FSL - Reliability Systems and Software

- ACADIE - Assistance for certification of distributed and embedded applications
- ARGOS - Advancing Rigorous Software and System Engineering
- ICS - Interactive Critical Systems
- SM@RT - Smart Modeling for softw@re Research and Technology

GD - Data Management

- IRIS - Information Retrieval and Information Synthesis
- PYRAMIDE - Dynamic Query Optimization in large-scale distributed environments
- SIG - Generalized information systems

IA - Artificial Intelligence

- ADRIA - Argumentation, Decision, Reasoning, Uncertainty and Learning methods
- LILaC - Logic, Interaction, Language and Computation
- MELODI - Methods and Engineering of Language, Ontology and Discourse

ICI - Interaction, Collective Intelligence

- ELIPSE - Human computer interaction
- SMAC - Cooperative multi-agents systems
- TALENT - Teaching And Learning Enhanced by Technologies

SI - Signals and Images

- MINDS - coMputational Imaging anD viSion
- SAMoVA - Structuration, Analysis, Modeling of Video and Audio documents
- SC - Signal and Communications
- STORM - Structural Models and Tools in Computer Graphics