

# Simultaneous Reflectionless Transient Excitation of Modes Using Virtual Critical Coupling for a Single-channel Resonant Microwave Cavity

Théo Delage\*, Valentin Mazières†, Olivier Pascal\*, Jérôme Sokoloff\*

\*LAPLACE, Université Fédérale de Toulouse, 31062 Toulouse Cedex 9, France, theo.delage@laplace.univ-tlse.fr\*

†ISAE-SUPAERO, Université Fédérale de Toulouse, 31055 Toulouse, France

**Abstract**—In this paper, we propose to extend the application of our time domain method for the extraction of S-matrix zeros from a single-channel cavity. The first step consists in measuring the S-parameters of a multimodal cavity in order to extract the complex frequencies of several modes to be excited under virtual critical coupling. Then, overcoupled resonances are identified from this measurement in order to apply the method on relatively isolated resonances (weak modal overlapping regime). We show the simultaneous excitation of several modes under virtual critical coupling.

**Index Terms**—virtual critical coupling, S-matrix zeros, scattering singularities, microwave cavity, multimode, measurements.

## I. INTRODUCTION

The concept and demonstration of plasma ignition by time reversal microwave excitation has paved the way for the search for microwave plasma sources controlled by the careful time shaping of the excitation signal [1], [2]. Usually, it is the structure hosting the plasma that is designed to be efficient during the stages of the ignition process - plasma off / plasma on [3]. We have recently put forward the idea of considering specific scattering phenomena in order to modulate and match the excitation signal of the plasma source to make the energy transfer efficient in both stages of the ignition [4]. The plasma off stage could be achieved under virtual critical coupling, i.e. by exciting a S-matrix zero typically located in the complex frequency plane [5], [6]. In this case, during excitation, all the incident energy will be transmitted to the cavity and stored, and no reflection will be observed. We have proposed a time domain method to extract this S-matrix zero from measurements, in the case of a single-mode, single-channel microwave cavity. The mode studied in this case must be overcoupled [4]. The proposed approach is an alternative to those using the harmonic inversion technique [7] or the Wigner time delay [8].

While our early work focused on a single-mode single-channel cavity, we propose in the present paper to extend our method to a multimodal single-channel cavity. In order to use this method, it is still necessary to consider overcoupled and relatively isolated resonances. Here, thanks to the linearity of the system, we show the simultaneous excitation of several transient reflectionless modes under virtual critical coupling.



Fig. 1. Photograph of the cavity.

## II. EXTRACTION OF OVERCOUPLED S-MATRIX ZEROS FROM A MULTIMODAL $S_{11}$ MEASUREMENT

We consider the cavity shown in Fig. 1. This parallelepipedal aluminium cavity is coupled to an excitation signal through a 30 mm monopole. Inside, it has two metal rods facing each other, forming a gap in which the electric field is concentrated and can facilitate the ignition of a plasma discharge [9]. Using the Keysight E5071C Vector Network Analyser (VNA), we measure the  $S_{11}$  of the cavity over a frequency band between 1.6 and 5 GHz. Fig. 2a shows the modulus and expanded phase measurement of this spectrum. The fundamental mode of the cavity is located around 1.77 GHz.

We wish to extract some S-matrix zeros from this measured spectrum. According to what was shown in the original paper on virtual critical coupling [5], and from the temporal coupled mode theory [10] applied to a single-channel, single-mode system, the angular frequency associated with a S-matrix zero is given by ( $e^{j\omega t}$  convention):

$$\omega_{zero} = \omega_0 + j(\gamma_{int} - \gamma_{ext}) \quad (1)$$

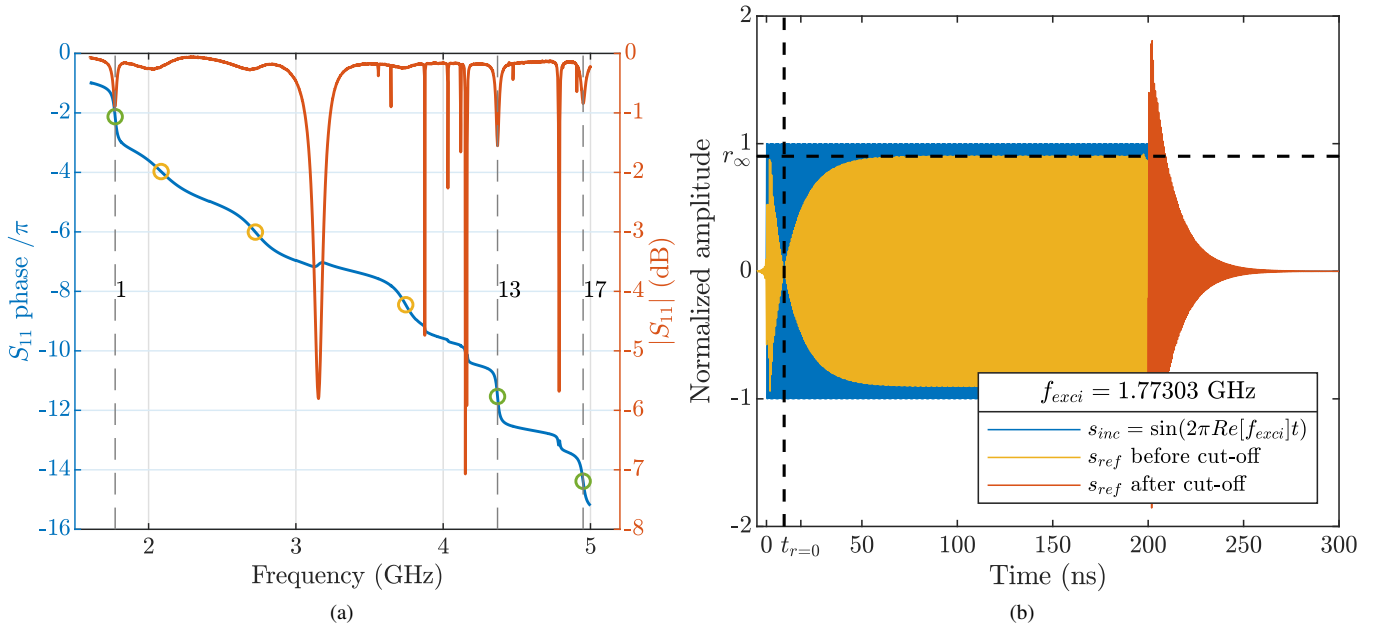


Fig. 2. (a)  $S_{11}$  of the cavity measured with VNA between 1.6 and 5 GHz. Circles indicate phase variations that allow identification of overcoupled modes. In yellow, the phase frequency dynamics are slow, reflecting fast time dynamics. In green, the selected modes have fast phase frequency dynamics, and therefore slower time dynamics. In the paper, we study the S-matrix zeros of modes 1, 13 and 17. (b) Harmonic response of the cavity fundamental mode. Measurement points for calculating decay rates are identified by dotted lines.

with  $\omega_0$  the real pulsation of a normal mode,  $\gamma_{int}$  the decay rate associated with intrinsic losses (e.g. energy dissipation at the cavity walls), and  $\gamma_{ext}$  the external decay rate indicating the energy decay mechanisms through couplings (at the access here). The S-matrix zero can then be excited via a signal of the form  $e^{j\omega_{zero}t}$ . The case of critical coupling is found if  $\gamma_{int} = \gamma_{ext}$ , leading to excitation of the zero by a stationary signal at the real angular frequency  $\omega_0$ . The case where  $\gamma_{int} > \gamma_{ext}$  requires an excitation signal whose amplitude decreases in an exponential way to excite the zero. The mode is then said to be undercoupled, and such an excitation is of no particular interest for the present application. However, if  $\gamma_{int} < \gamma_{ext}$ , a signal of the form  $e^{(\gamma_{ext}-\gamma_{int})t} e^{j\omega_0 t}$ , with exponentially growing amplitude, will excite the S-matrix zero of the mode, then qualified as overcoupled. These are the modes we are interested in.

Several modes can be identified on the plot of  $S_{11}$  modulus and phase (Fig. 2a). In the absence of a critically coupled resonance (a deep peak of the modulus - a S-matrix zero on the real axis of the frequency plane), modes can be over- or under-coupled in this case. Then, spectral signatures of the overcoupled modes are characterized by a continuous phase jump with an inflection point and an "inverted f" shape (circles on Fig. 2a). Indeed, overcoupling implies that more energy leaks from the cavity to the coupled channel than is directly reflected at the coupling access. In the  $S_{11}$  complex plane, this results in a complete phase sweep between 0 and  $2\pi$ . For an undercoupling, the phase sweep is reduced to a portion of the plane (with a round trip around the same phase). Modulus is also signed by a more or less deep peak.

To confirm the nature of modes coupling, we can excite them via a stationary signal and observe the harmonic response of the cavity, particularly during the transient regime. Fig. 2b illustrates the harmonic response of the cavity to fundamental mode excitation (excitation signal  $s_{inc}(t)$  in blue, reflected signal  $s_{ref}(t)$  during excitation in yellow). According to the temporal coupled mode theory, this response is the one expected for an overcoupling (see also references [4] and [5]). Harmonic response is obtained by convolution product of the excitation signal with the  $S_{11}$  in time (becomes a simple product in spectral domain).

The method we presented in our article applies to identify precisely S-matrix zeros of a single-channel cavity for overcoupled modes [4]. Moreover, to identify them clearly, isolated resonances must be considered. We therefore treat each mode to identify S-matrix zeros as if the spectrum was monomodal. Six modes meet these criteria in the measured spectrum (orange and green circles on Fig. 2a). However, three of them are strongly overcoupled (around 2.08 GHz, 2.72 GHz and 3.68 GHz - orange circles on Fig; 2a). Indeed, their frequency dynamics are slow, leading to fast temporal dynamics, i.e. their transient period is short. The increasing exponential amplitude required for their excitation is too sharp and the excitation signal has a very high amplitude in a few periods. This kind of signal can be difficult to generate in practice. For instance, the bandwidth of the arbitrary waveform generator of our experimental setup [4] is lower than the width of these modes. We therefore propose to study only the three zeros identified in Tab. I (green circles on Fig; 2a). In this case, the frequency dynamics of the phase are fast, and therefore the

TABLE I  
S-MATRIX ZEROS EXTRACTED WITH THE TIME DOMAIN METHOD

Mode 1	$1.77303 - j0.01156$ GHz
Mode 13	$4.36885 - j0.00982$ GHz
Mode 17	$4.95103 - j0.01692$ GHz

time dynamics are slower.

Identification of the zero associated with mode 1 by our method, is based on analysis of the harmonic response of Fig. 2b. The reflected signal after excitation cut-off (in red on the figure) decays mainly through the complex frequency associated with the pole of mode 1. Fourier transform of this signal then gives us this complex frequency. However, temporal coupled mode theory applied to a single-mode resonator shows that real parts of the pole and the zero are identical [5], [4]. Then, measurements of the zero reflection instant during transient of the excitation  $t_{r=0}$  and of the amplitude of the reflection coefficient in steady state  $r_\infty$  (envelope of the yellow signal on the figure), allow us to calculate the intrinsic and external decay rates of the cavity ( $\gamma_{int}$  and  $\gamma_{ext}$ ). We then calculate the imaginary part of the zero (see Eq. 1). Zeros associated with modes 13 and 17 are identified in the same way.

Finally, it should be noted that this method will not work directly in the case of strong coupling between modes. At least one should try to reconsider the reflection coefficient from the temporal coupled mode theory describing a multimodal system.

### III. SIMULTANEOUS EXCITATION OF MODES UNDER VIRTUAL CRITICAL COUPLING

Fig. 3 illustrates temporal responses of the cavity associated with individual and simultaneous excitations of the three S-matrix zeros identified in Tab. I. The different responses are calculated as before from numerical definition of excitation signals and measured  $S_{11}$  spectrum. In blue we find the excitation signals with respective exponentially increasing amplitude. Signals reflected during excitations are coloured in yellow. We observe the absence of a reflected wave during excitation, and thus the realisation of virtual critical coupling. In red, we have coloured the signals associated with the decay of the energy after excitation cut-off. As can be seen from a spectral study of the signal, this occurs across poles of the system. Note that reflected signal over time is calculated at once, the colours being used only to distinguish them for illustration purposes.

The excitation signal of the three S-matrix zeros simultaneously is shaped by the sum of signals allowing independent

excitation of S-matrix zeros. Signals are synchronized so that the three excitations reach their maximum amplitude at the same time. We observe the absence of a reflected wave during excitation, while at cut-off, energy decreases across poles of the system.

### IV. CONCLUSION

In this communication, we have proposed the application of our time domain method for extracting S-matrix zeros for a single-channel, multi-mode cavity. Although this method is limited to isolated and overcoupled resonances, we have constructed the response of the cavity to simultaneous excitation of multiple overcoupled S-matrix zeros, in other words, the excitation of multiple modes under virtual critical coupling.

In the final paper, we could establish more precisely benefits of these approaches in the context of plasma ignition, in particular on the spatial focusing of energy via a temporal adaptation of the excitation of the virtual critical coupled cavity modes. Indeed, it seems possible to synchronise the different excitation signals in such a way that they build up particular interferences in the cavity. This seems to offer further prospects for the matching understanding of microwave plasma sources in cavities controlled by time reversal.

### REFERENCES

- [1] V. Mazières, R. Pascaud, L. Liard, S. Dap, R. Clergereaux, and O. Pascal, "Plasma generation using time reversal of microwaves," *Applied Physics Letters*, vol. 115, no. 15, p. 154101, 2019.
- [2] V. Mazières, O. Pascal, R. Pascaud, L. Liard, S. Dap, R. Clergereaux, and J.-P. Boeuf, "Space-time plasma-steering source: Control of microwave plasmas in overmoded cavities," *Phys. Rev. Applied*, vol. 16, p. 054038, Nov 2021.
- [3] F. Silva, K. Hassouni, X. Bonnin, and A. Gicquel, "Microwave engineering of plasma-assisted CVD reactors for diamond deposition," *Journal of Physics: Condensed Matter*, vol. 21, no. 36, p. 364202, Aug 2009.
- [4] T. Delage, O. Pascal, J. Sokoloff, and V. Mazières, "Experimental demonstration of virtual critical coupling to a single-mode microwave cavity," *Journal of Applied Physics*, in press.
- [5] Y. Ra'di, A. Krasnok, and A. Alù, "Virtual critical coupling," *ACS Photonics*, vol. 7, no. 6, pp. 1468–1475, 2020.
- [6] A. Krasnok, D. G. Baranov, H. Li, M.-A. Miri, F. Monticone, and A. Alù, "Anomalies in light scattering," *Adv. Opt. Photon.*, vol. 11, no. 4, pp. 892–951, Dec 2019.
- [7] U. Kuhl, R. Höhmann, J. Main, and H.-J. Stöckmann, "Resonance widths in open microwave cavities studied by harmonic inversion," *Phys. Rev. Lett.*, vol. 100, p. 254101, Jun 2008.
- [8] L. Chen, S. M. Anlage, and Y. V. Fyodorov, "Generalization of wigner time delay to subunitary scattering systems," *Phys. Rev. E*, vol. 103, p. L050203, May 2021.
- [9] B. Fragne, J. Sokoloff, O. Rouzaud, O. Pascal, and M. Orain, "A versatile set-up to study plasma/microwave sources for liquid fuel ignition," *Eur. Phys. J. Appl. Phys.*, vol. 92, no. 3, p. 30903, 2020.
- [10] H. A. Haus, *Waves and fields in optoelectronics*. Englewood Cliffs, NJ: Prentice-Hall, 1984.

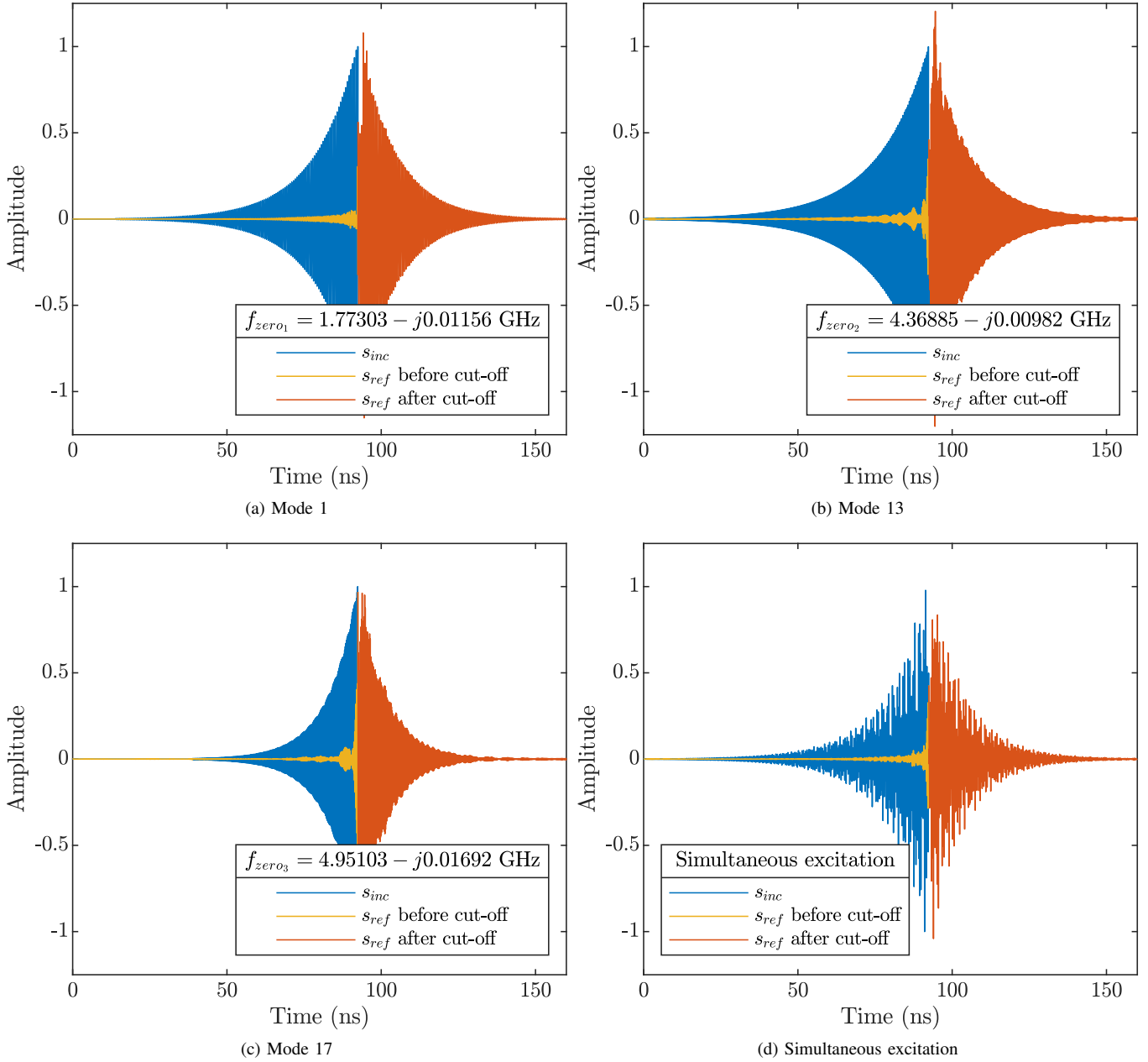


Fig. 3. Independent and simultaneous excitation of three modes under virtual critical coupling. Amplitudes are normalised to the incident signal.