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Stability and dynamics of the flow past of a bullet-shaped blunt body moving in a pipe

Paul Bonnefis, David Fabre[†] & Christophe Airiau.

IMFT, Université de Toulouse, UMR 5502 CNRS / INPT / UPS,
Allée du Pr Camille Soula, 31400 Toulouse, France

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The flow past a bullet-shaped blunt body moving in a pipe is investigated through global linear stability analysis (LSA) and direct numerical simulation (DNS). A cartography of the bifurcation curves is provided thanks to LSA, covering the range of parameters corresponding to Reynolds number $Re = [50 - 110]$, confinement ratio $a/A = [0.01 - 0.92]$ and length-to-diameter ratio $L/d = [2 - 10]$. Results show that the first bifurcation is always a steady bifurcation associated to a non-oscillating eigenmode with azimuthal wavenumber $m = \pm 1$ leading to a steady state with planar symmetry. For weakly confined cases ($a/A < 0.6$) the second bifurcation is associated to an oscillating mode with azimuthal wavenumber $m = \pm 1$, as in the unconfined case. On the other hand, for the strongly confined case ($a/A > 0.8$), a destabilization of non-oscillating modes with $|m| = 2, 3$ and a restabilization of the $m = \pm 1$ eigenmodes are observed. The aspect ratio L/d is shown to have a minor influence for weakly confined cases and almost no influence for strongly confined cases. DNS is subsequently used to characterize the nonlinear dynamics. The results confirm the steady bifurcation predicted by LSA with excellent agreement for the threshold Reynolds. For weakly confined cases, the second bifurcation is a Hopf bifurcation leading to a periodic, planar-symmetric state in qualitative accordance with LSA predictions. For more confined cases, more complex dynamics is obtained, including a steady state with $|m| = 3$ geometry and aperiodic states.

1. Introduction

The flow past blunt bodies is a problem of practical importance, with obvious engineering applications to transport. In such applications it is necessary to estimate and predict the lift and drag forces exerted on the body as well as to assert the influence of the geometry on these forces for a shape optimization procedure. Wake flows in the transitional regime (with Reynolds numbers of order $10^2 - 10^3$) is also a problem of fundamental interest where global stability theory and bifurcation theory have been particularly successful to explain complex nonlinear dynamics. The most documented case corresponds to the wake of a cylindrical body placed perpendicularly to the flow (Bénard 1908; Von Karman 1912; Provansal *et al.* 1987). This case is characterised by a Hopf bifurcation for $Re \approx 47$ giving rise to the well-known Bénard-Von Kaàrmàn vortex street. Secondary bifurcations occurring in the range $Re \approx 200$ and leading to three-dimensional states have also been investigated by stability analysis of the periodic solution and bifurcation theory (Thompson *et al.* 1996). Among three-dimensional geometries, spheres and disks have been particularly considered as canonical cases. Linear stability analysis (LSA) (Natarajan & Acrivos 1993; Meliga *et al.* 2009a)

[†] Email address for correspondence: david.fabre@imft.fr

provides a powerful framework allowing to tackle this class of problems. This approach predicts that the first unstable modes is a non-oscillating mode (i.e. with purely real global eigenvalue) characterised by azimuthal wavenumbers $m = \pm 1$. It leads to a steady state (*SS*) solution with planar symmetry, a presence of a pair of longitudinal vortices and finally a non-zero lift force exerted on the body. The LSA study also predicts the onset of a secondary eigenmode which is oscillating (i.e. a pair of complex conjugated eigenvalues) and also associated to an azimuthal wavenumber $m = 1$. Comparisons with direct numerical simulations and application of normal form theory (Fabre *et al.* 2008; Auguste *et al.* 2010) and weakly nonlinear analysis (WNLA) (Meliga *et al.* 2009a) showed that this secondary mode is responsible for the onset of an oscillating state which is either reflection-symmetry preserving (RSP) for spheres and thick disks or reflection-symmetry breaking (RSB) for thin disks. Effects of motion of the body have also been considered. First, the effect of imposed rotation on the wake of a sphere has been analysed. In the case the axis of rotation is aligned with the flow, rotation breaks the symmetry between $m = +1$ and $m = -1$ modes and modifies the bifurcation scenario leading to the onset of quasiperiodic states (Pier 2013). In the case the axis is transverse, weak rotation stabilizes the RSP mode but strong rotation gives rise to a new oscillating mode with a smaller frequency (Citro *et al.* 2016; Fabre *et al.* 2017). Secondly, Tchoufag *et al.* (2014) have demonstrated the influence of wake dynamics on the motion of bodies in free movement submitted to buoyancy force. In that case, the destabilization of the base flow field may result in a path deviation of the buoyancy-driven disk or sphere leading to a variety of states including zig-zag paths, steady-oblique paths, etc.

Another canonical blunt body geometry which was selected by a number of studies in the literature is the bullet-shaped body, consisting in a half-ellipsoidal nose glued to a cylindrical blunt rear. It has the advantage to have a shape closer to real industrial applications as trains for instance. Experiments performed by Brücker (2001) revealed a stabilizing effect of the presence of the ellipsoidal nose, in comparison with the flow past disks. An extensive study presented by Bohorquez *et al.* (2011) uses three approaches, DNS, LSA and experiments. This study reveals that the bifurcation sequences and wake patterns are globally similar to the case of a sphere, and that increasing the length of the body generally delays the bifurcations towards larger Reynolds numbers. A base-bleed flow control has also been tested and its stabilizing effect was demonstrated. The sequence of bifurcations occurring in the wake has been examined by Bury & Jardin (2012) using DNS, from the laminar axisymmetric wake to the onset of chaotic behaviour. In Jiménez-González *et al.* (2014), the effect of spinning of this blunt body around its axis of symmetry is shown to have stabilizing effect, promoting the second most amplified mode and widening the range of existence of a stable axisymmetric wake. Note that a series of recent works have also been conducted for the wake of axisymmetric bodies for much larger ranges of Reynolds number and using a mean-flow based approach (Rigas *et al.* 2014, 2015; Callahan *et al.* 2021; Rigas *et al.* 2017), indicating that linear stability and low-order nonlinear modeling is also relevant in this range.

The present study investigates the effect of confinement on wake dynamics. The effect of confinement on axisymmetric wakes has already been analyzed from a fundamental point of view by Juniper (2006) considering local stability analysis of a family of parallel flow models. We also focus on the bullet-shaped blunt body (cf fig. 1). Many industrial issues raise in the case of an object travelling in a confined environment. A good example is a high-speed train passing through a tunnel, how it enters the tunnel and how the tunnel influences the aerodynamics of the train (Mok & Yoo 2001; Kwon *et al.* 2003; Baron *et al.* 2001). The issue encountered relies more on the pressure wave created by the train nose and its interaction with the tunnel than the wake itself, but the drag is still

of interest. In another study [Choi & Kim \(2014\)](#) is investigated the optimisation of the nose shape of the high-speed Korean subway and the tunnel cross-sectional area influence on the total drag. Of course, with velocities of several hundred kilometers per hour, the Reynolds numbers are of order 10^8 and characterisation of nonlinear dynamics in the transitional range may be irrelevant. The situation changes considering new technologies in train transportation such as evacuated tube transportation system where a capsule travels at high velocity in a near vacuum network of pipes. Numerous studies describe the limitations and opportunities arising in such configurations ([Opgenoord & Caplan 2018](#); [Oh et al. 2019](#); [Braun et al. 2017](#)) and highlight differences in aerodynamics compared to standard trains. The expected operating pressure for such system is in the range $1 - 100$ Pa, leading to Reynolds numbers in the range $10^3 - 10^5$. Hence, characterisation of dynamics in the transitional range using a combination of LSA, bifurcation theory, and DNS, may be relevant. The identification of non-axisymmetric bifurcations giving rise to a lift force may be of practical interest in the operation of such devices. Such applications also operate in the transonic regime, so that for an accurate modelling compressibility and rarefied gas effects should also be taken into account. However, as a first approach towards these problems, it might be interesting to stick to an incompressible flow and target the effect of confinement regardless of additional effects. Our current investigations on a slender axisymmetric blunt-based body moving in a tube is inspired by such industrial applications. In order to pave the way to such complicated problems, the study has been limited to incompressible flows and to a Reynolds number Re lower than 1500.

The paper is organized as follows. The configuration, the governing equations and the Linear Stability Analysis equations and resolution methods are presented in section II. Section III is devoted to the characterisation of instability properties thanks to LSA. A parametric study of the linearly unstable modes is obtained as function of the confinement ratio, of the length-to diameter ratio and of the Reynolds numbers. Section IV is dedicated to direct numerical simulations (DNS) and to comparisons with the results of the LSA analysis. DNS is used to confirm the predictions of LSA regarding the first bifurcation threshold and to explore the nonlinear behaviour arising away from this threshold. The paper ends by some concluding remarks. Two appendixes have been added, the first one on the analytical solution of the annular Couette-Poiseuille flow and the last one on the mesh convergence study.

2. Methodology

2.1. Configuration and parameters

The geometry of the bullet-shaped blunt body moving in a tube and a the main geometrical parameters are shown in figure 1. The body consists of a half-ellipsoid nose glued to a cylindrical rear. The diameter of the cylinder is referred as d . The ellipsoid of revolution is defined by its major axis with $a_x = 2a_y = d$ and its minor axis which fits with the cylindrical section by imposing $a_y = a_z = d/2$.

The diameter of the pipe is noted D , so that the effect of confinement will be defined by either a diameter ratio $\xi = d/D$ or an area ratio $a/A = \xi^2$ with $a = \pi d^2/4$ the frontal area of the body and $A = \pi D^2/4$ the area of the tube.

The origin of the frame is taken at the junction between half-spheroidal and cylindrical parts, so that the body spans from $x = -d$ (nose) to $x = L - d$ (base).

The object moves with a velocity U in the direction $-\mathbf{e}_x$ and the wall of the pipe is fixed. Assuming the flow is incompressible and isothermal, the nondimensional parameters of this problem are the Reynolds number $Re = \frac{\rho U d}{\mu}$, the diameter aspect ratio $\xi = d/D$

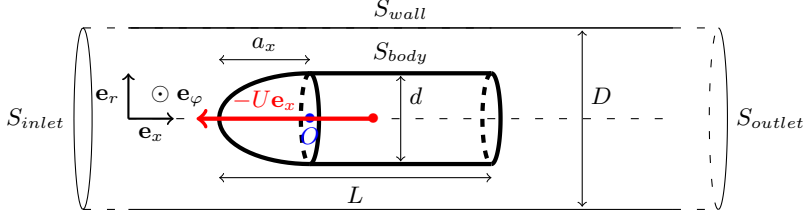


Figure 1: Geometry of the axisymmetric blunt body in a pipe.

and the length aspect ratio L/d . ρ and μ are respectively the constant density and dynamic viscosity of the fluid. In most cases the parameter L/d will be set to 2, except in sec. 3.3 where the effect of this parameter will be investigated.

The study will be conducted in the frame of reference associated to the body. The boundaries of the computational domain are given in figure 1. It is limited by respectively an inlet section S_{inlet} and an outlet section S_{outlet} . In this frame, the body is fixed, and placed within an incoming flow U of direction $+\mathbf{e}_x$ and the tube wall also moves at the same velocity with respect to the body. Hence the dimensionless incompressible Navier-Stokes equations and associated boundary conditions are:

$$\partial_t \mathbf{u} = \mathcal{N}\mathcal{S}([\mathbf{u}, p]) \equiv -\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \frac{2}{Re} \nabla \cdot \mathbf{D}(\mathbf{u}) \quad (2.1a)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (2.1b)$$

$$\mathbf{u}|_{S_{body}} = \mathbf{0} \quad (2.1c)$$

$$\mathbf{u}|_{S_{inlet} \cup S_{wall}} = \mathbf{e}_x \quad (2.1d)$$

$$\left[-p \mathbf{e}_x + \frac{2}{Re} \mathbf{D}(\mathbf{u}) \cdot \mathbf{e}_x \right]_{S_{outlet}} = \mathbf{0} \quad (2.1e)$$

where \mathbf{u} is the relative velocity, and $\mathbf{D}(\mathbf{u}) = (\nabla \mathbf{u} + \nabla^T \mathbf{u})/2$ is the rate-of-strain tensor. The divergence formulation for the viscous terms is related to the choice of the Finite Element Method (FEM) to solve the equations.

The governing equations are non-dimensionalized by the body velocity U , the fluid density ρ and the body diameter d . The last boundary condition is written as a no-stress condition on the outlet section, which is a convenient choice for outlet condition with FEM approach.

2.2. Global linear stability analysis

2.2.1. Equations

The global linear stability approach is performed in the line of the now classical approach described for instance in Sipp & Lebedev (2007); Fabre *et al.* (2018). Within the LSA framework, the velocity and pressure are decomposed into a base flow and a small perturbation as follows:

$$\mathbf{u}(r, \varphi, x, t) = \mathbf{u}_b(r, x) + \epsilon \hat{\mathbf{u}}(r, x) e^{im\varphi + \lambda t}, \quad p(r, \varphi, x, t) = p_b(r, x) + \epsilon \hat{p}(r, x) e^{im\varphi + \lambda t} \quad (2.2)$$

Here $[\mathbf{u}_b; p_b]$ is the so-called "base flow", namely the solution of the axisymmetric, time-independent version of equations (2.1);

$$\mathcal{NS}([\mathbf{u}_b, p_b]) = \mathbf{0}; \quad \nabla \cdot \mathbf{u}_b = 0. \quad (2.3)$$

In (2.2), a small-amplitude perturbation of the base-flow is assumed in the form of an eigenmode $[\hat{\mathbf{u}}, \hat{p}]$ associated to an eigenvalue $\lambda = \lambda_r + i\lambda_i$. The real part of the eigenvalue is the growth rate. A positive value indicated here an amplification. The imaginary part is a nondimensional frequency (time oscillation), which is most conveniently represented by the Strouhal number $St = \lambda_i/(2\pi)$ thanks to the nondimensionalization choices. Fourier decomposition in the azimuthal direction is possible with the axisymmetric invariance and an azimuthal wavenumber $m \in \mathbb{Z}$ can be added in the exponential wave-like part of the perturbation. Introducing the decomposition (2.2) into the Navier-Stokes equations and linearizing leads to an eigenvalue problem written as:

$$\lambda \hat{\mathbf{u}} = \mathcal{LNS}_{\mathbf{u}_b}([\hat{\mathbf{u}}, \hat{p}]) \quad (2.4a)$$

$$\nabla_m \cdot \hat{\mathbf{u}} = 0 \quad (2.4b)$$

$$\hat{\mathbf{u}}|_{S_{inlet} \cup S_{wall} \cup S_{body}} = \mathbf{0} \quad (2.4c)$$

Where \mathcal{LNS}^m is the Linearized Navier-Stokes operator defined as

$$\mathcal{LNS}_{\mathbf{u}_b}^m([\hat{\mathbf{u}}, \hat{p}]) = -\mathbf{u}_b \cdot \nabla_m \hat{\mathbf{u}} - \hat{\mathbf{u}} \cdot \nabla_m \mathbf{u}_b - \nabla_m \hat{p} + \frac{2}{Re} \nabla_m \cdot \mathbf{D}_m(\hat{\mathbf{u}}) \quad (2.5)$$

Here quantities ∇_m and \mathbf{D}_m are the gradient and rate-of-strain operators with $\partial_\varphi(\cdot)$ replaced by $i \times m(\cdot)$.

2.2.2. Resolution methods

The resolution methods employed here are essentially similar to the one used in recent papers such as Tchoufag *et al.* (2014); Fabre *et al.* (2019) considering stability analysis of axisymmetric incompressible flows.

Thanks to axisymmetry, the base-flow velocity is searched in the cylindrical frame $[\mathbf{e}_x, \mathbf{e}_r, \mathbf{e}_\varphi]$, as $\mathbf{u}_b = [u_{b,x}(x, r), u_{b,r}(x, r), 0]$ so that only two components of velocity are kept. On the other hand, for eigenmodes three components of velocity are needed, i.e. $\hat{\mathbf{u}} = [\hat{u}_x(x, r), \hat{u}_r(x, r), \hat{u}_\varphi(x, r)]$. Within this assumption it is enough to consider a two-dimensional numerical domain (Ω) corresponding to a meridian plane (x, r) .

For both base-flow equations and linear stability equations, a Finite-Element method is used. For this sake, the equations are first turned into a weak form by introducing test functions \mathbf{v} and q and a scalar product $\langle \varphi_1, \varphi_2 \rangle = \int_\Omega \bar{\varphi}_1 \cdot \varphi_2 \, d\Omega$. For instance, the weak form of the base-flow equations 2.5 are written as

$$\forall (\mathbf{v}, q), \quad \langle \mathbf{v}, \mathcal{NS}([\hat{\mathbf{u}}_b, \hat{p}]) \rangle + \langle q, \nabla_0 \cdot \hat{\mathbf{u}}_b \rangle = 0. \quad (2.6)$$

An integration by parts of the viscous terms is afterward performed and their derivation order is reduced. Dirichlet boundary conditions are incorporated by penalization while the stress-free outlet condition is directly satisfied thanks to integration by parts. The nonlinear problem is then solved using an iterative Newton method. The developed form of the base-flow equations in cylindrical coordinates and details about the Newton method can be found, for instance, in Tchoufag *et al.* (2014); Fabre *et al.* (2019).

Similarly, the weak form of stability problem yields

$$\forall (\mathbf{v}, q), \quad \lambda \langle \mathbf{v}, \hat{\mathbf{u}} \rangle = \langle \mathbf{v}, \mathcal{LNS}_{\mathbf{u}_b}^m([\hat{\mathbf{u}}, \hat{p}]) \rangle + \langle q, \nabla_m \cdot \hat{\mathbf{u}} \rangle, \quad (2.7)$$

Mesh	λ	N_v	t_c [s]
Adapted	$1.1685 + 1.09952i$	6936	44
Split	$1.1686 + 1.09957i$	137374	2139

Table 1: Comparison of results found with a mesh build with the adaptation strategy and a refined one obtained by splitting. The table reports the number the leading eigenvalue λ , the number of mesh vertices N_v and the computational time t_c (on a Macbook, 2Ghz, 32Go RAM, single processor) required for computing 10 modes with the shift-invert strategy, for the set of parameters $Re = 320$, $a/A = 0.75$, $L/d = 2$.

which after discretization leads to a generalized eigenvalue problem :

$$\lambda B\hat{X} = A\hat{X} \quad (2.8)$$

A shift-and-invert method is applied to obtain a collection of eigenvalues (typically 10) located closest to a "shift" value taken as a guess of the searched eigenvalues. As usual in such approaches, several values of the "shift" were systematically tested to explore the complex plane and to ensure that no unstable eigenvalue was missed in the study.

The weak form of the stability equations, details about the integration by parts and the construction of matrices A and B can again be found in Tchoufag *et al.* (2014); Fabre *et al.* (2019).

2.2.3. Numerical implementation

All numerical operations (generation and juniper of a mesh, building of matrix operators, Newton iteration for base-flow problem and shift-invert method for eigenvalue problem) are handled thanks to the Finite-Element software FREEFEM++ (Hecht 2012). The FEM discretization is built with the classical Taylor-Hood elements for all computations.

First, a triangular mesh is built using the well-known Delaunay-Voronoi algorithm, and a preliminary base flow and some modes are computed. An adaptive mesh strategy is adopted in order to ensure convergence of results with respect to mesh refinement, as it is described in Fabre *et al.* (2018). Namely, after first computing a base flow and an adjoint eigenmode, the mesh is adapted to both these fields, and the process is repeated twice. To demonstrate the efficiency of this method, we report in table 1 the eigenvalues computed with the adapted mesh and with an even more refined mesh obtained by adapting to both direct and adjoint modes and subsequently splitting all triangular cells in four. The results demonstrate that the mesh adaptation method provide converged results with a very reasonable mesh density and computational cost.

The whole computational chain, including mesh generation, adaptation, loop over parameters, and post-processing, is monitored in the Matlab environment thanks to the STABFEM suite (Fabre *et al.* 2018) which is a set of MATLAB/OCTAVE drivers/wrappers specifically designed to perform such studies. A sample script reproducing a selection of results from the present study is available through the website of the STABFEM project[†].

[†] <https://stabfem.gitlab.io/StabFem>

Section	L/d	a/A	Re	$ m $
III - A	2	0.01, 0.75, 0.81	320, 200 - 1200	1
III - B	2	0.01 - 0.92	320 - 1130, 110 - 400	1
III - C	2 - 10	0.01 - 0.75	110 - 140 - 200	1
III - D	2	0.6 - 0.92	80 - 1100	2-3

Table 2: Ranges of parameters investigated and corresponding sections of the paper.

2.3. Direct numerical simulations

To validate and to extent the results of the stability analysis, some full direct numerical simulations are performed with the open source computational fluid dynamics software package, OpenFOAM®. Time varying solutions of the equation (2.1) are computed with its incompressible finite volume solver, *pimpleFOAM* built with a second-order spatial derivative schemes and an Euler temporal scheme. A fixed Courant number set to $Co = 0.5$ ensures the stability of these schemes. The meshes are built with the cfMesh software provided with OpenFOAM. A typical mesh and a mesh convergence study are presented in the appendix B. Results and comparison with LSA are discussed in section IV.

3. Linear Stability Analysis: Results

With the objective of building an exhaustive cartography of instability properties, four parameters will be varied. The two first ones are geometrical parameters, namely the aspect ratio L/d and the confinement parameter a/A . The third input parameter is the Reynolds number and the fourth one is the azimuthal wavenumber m . Table 2 indicates the range of parameters defined for this study, and the concerned sections. Regarding the azimuthal wavenumber, it is known for open flows past blunt bodies that the most unstable modes are found for $m = \pm 1$ (Natarajan & Acrivos 1993; Auguste *et al.* 2010; Jiménez-González *et al.* 2014). This fact justifies that our study will primarily focus on this value. Other values of m are postponed in Sec. III-D.

3.1. $m = \pm 1$ modes for sample values of the section aspect ratio a/A

In this section and the next one we set the length-to-diameter aspect ratio $L/d = 2$ and we focus on the effect of the confinement ratio a/A .

3.1.1. Weakly confined flow

A weakly confined case is first considered, corresponding to section aspect ratio $a/A = 0.01$ (or diameter aspect ratio $d/D = 0.1$). Figure 2 displays the base flow around the blunt body for a Reynolds number $Re = 320$. The axisymmetric base flow field exhibits a standing eddy which has approximately the same length as the body itself. The boundary layer present on the body surface is made visible through generation of negative azimuthal vorticity. Overall, this structure seems to be very similar to the one found in Jiménez-González *et al.* (2014) for the same object and conditions in an unconfined flow.

For the same base flow, a part of spectrum found using LSA approach is shown on

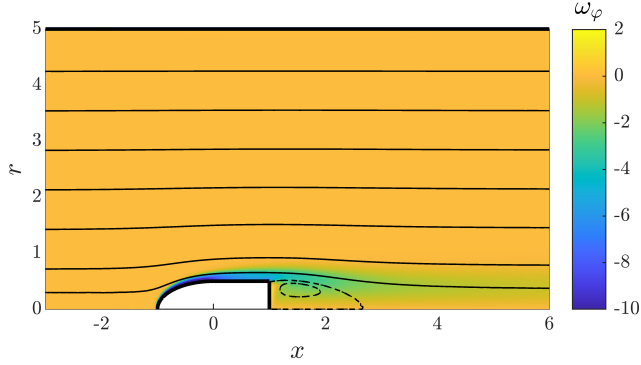


Figure 2: Azimuthal vorticity ($\omega_\varphi = \nabla \times \mathbf{u}_b \cdot \mathbf{e}_\varphi \equiv \partial_x u_r - \partial_r u_x$) and streamlines of the base flow for $Re = 320$, $L/d = 2$, $a/A = 0.01$.

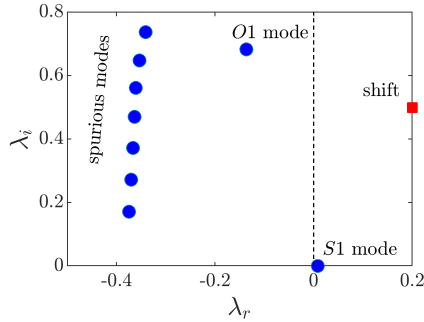


Figure 3: Example of a spectrum found with the shift and invert algorithm ($Re = 320$, $L/d=2$, $a/A=0.01$). The 10 eigenvalues closest to the "shift" value indicated in red are computed.

figure 3. It reveals two physical modes, the first one called $S1$ is non-oscillating (often referred as stationary) and unstable ($\lambda_r > 0$). The second one called $O1$ is oscillating and damped. The other modes quasi-aligned are some spurious modes of non-physical nature and come only from the numerical discretization. Similar results are observed by Jiménez-González *et al.* (2014) for a non-spinning object in unconfined space. In their study, the onset of the first instability (the $S1$ mode) was detected at a critical Reynolds number $Re_{c,S1} = 325.2$ whereas its value is $Re_{c,S1} = 312.2$ in the present study, leading to less than four percent of difference. It can be concluded that the confinement produces a small influence over the onset of the first instability in this case.

Figure 4 displays the four most amplified eigenvalues as function of Re , again for $a/A = 0.01$. The first unstable mode appearing is non-oscillating and remains the most amplified mode over the whole range of Re studied. The second most amplified mode, $O1$, becomes unstable at $Re_{c,O1} = 478.3$ and $St_c = 0.103$. This Strouhal value is very

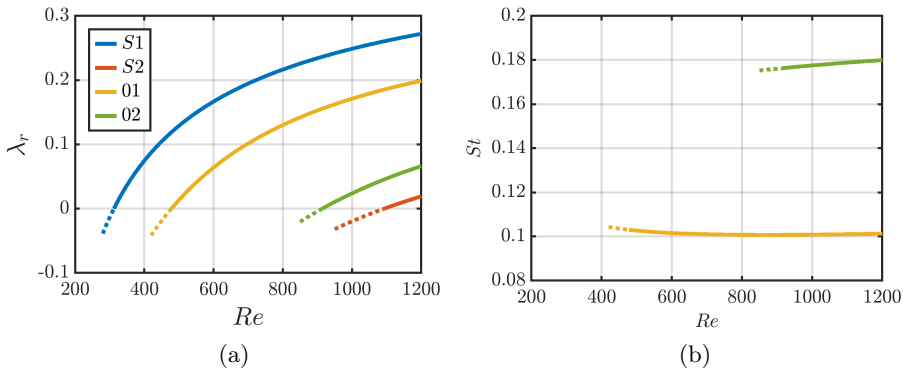


Figure 4: amplification rate (a) and Strouhal number (b) as a function of the Reynolds number for the first unstable modes, $a/A = 0.01$, $L/d = 2$.

close to the one found by [Bohorquez *et al.* \(2011\)](#) who reported $St = 0.102$. But the latter authors found a somehow larger value of the critical Reynolds number, namely $Re_{c,O1} = 518$.

In addition to the effect of confinement, this gap between critical Reynolds number may be explained by the fact that the computational domain defined for the stability analysis included only the cylindrical rear of the body and excluded the nose in [Bohorquez *et al.* \(2011\)](#). The critical Reynolds is also notably higher than the one given by the reference case of a thin disk. The geometry of the nose of the blunt body changes the amount of vorticity produced at its surface, as pointed out by [Brücker \(2001\)](#), and it is known that this vorticity production is responsible for triggering the instabilities ([Magnaudet & Mougín 2007](#)). Having a profiled nose diminishes such production of vorticity and pushes back the onset of instabilities to larger Reynolds. For instance, in figure 2, the vorticity intensity for $Re = 320$ is $|\omega_\varphi| \approx 10$, which is comparable to the intensity observed for a thin disk with aspect ratio $\chi = 3$ for $Re \approx 150$ as studied in [Auguste *et al.* \(2010\)](#).

Up to here, the main scenario revealed in the present configuration by the linear stability analysis is a first non-oscillating mode $S1$ amplification followed by an oscillating one $O1$. It is the same encountered for all axisymmetric bodies considered in literature ([Natarajan & Acrivos 1993](#); [Meliga *et al.* 2009b](#); [Tchoufag *et al.* 2014](#)). When pushing the Reynolds number towards higher values, two additional modes are found, an oscillating one and a non-oscillating one termed $O2$ and $S2$. These higher modes arise at much larger Reynolds numbers, in the range $Re \in [900 - 1200]$. They are less likely to be observed experimentally or numerically because in such regimes the mean flow is already very far from the axisymmetric base-flow analysed with the linear stability theory. Nevertheless, when the confinement effect will be increased, these higher modes will turn to be relevant to obtain a consistent picture of the bifurcation scenario. Hence they will be kept in the analysis and their critical Reynolds $Re_{c,S2}$ and $Re_{c,O2}$ will be tracked.

The structures of those modes are illustrated in figure 5. The plots display both the azimuthal velocity (colors) and pressure (lines) levels in a meridional (x, r) -plane (left plots) and the axial vorticity levels in a transverse (y, z) -plane corresponding to a cut at location $x = 2$ (right plots). The first unstable mode, $S1$ has a rather simple vorticity structure. In the meridional plane, the azimuthal vorticity of the eigenmode is positive in the region of the shear layer. Recalling that the vorticity of the base flow shear layer

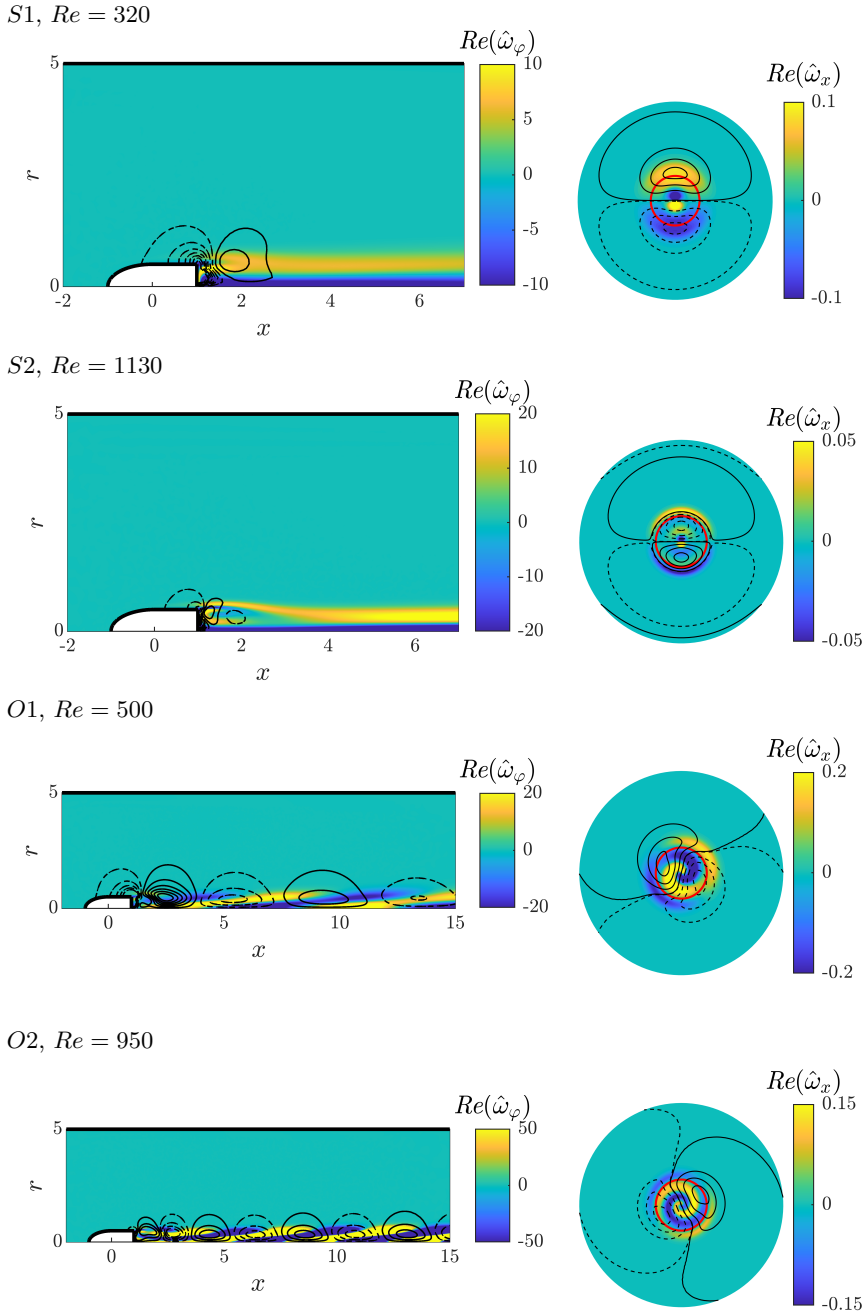


Figure 5: Eigenmodes found for $a/A = 0.01$, real parts of the vorticity with iso-levels of pressure. Slices are defined by $x = 2$ and $r \leq 2$.

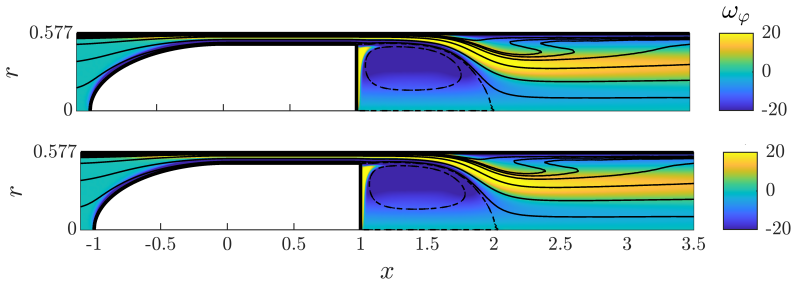


Figure 6: azimuthal vorticity and streamlines of the base flow in the moving frame attached to the body for $L/d = 2$ and $a/A = 0.75$. Top, $Re = 110$, bottom $Re = 320$

is negative (see figure 2), the effect of the eigenmode is to decrease in this region the net vortical intensity in the shear layer. Owing to the antisymmetry of $m = \pm 1$ modes, the azimuthal vorticity of the eigenmode is negative in the opposite side, meaning that the shear layer is enhanced. The axial vorticity, on the other hand, reveals a pair of counter-rotating streamwise vortices, as already noticed for disk and spheres (Meliga *et al.* 2009b; Natarajan & Acrivos 1993) and similar blunt bodies in open flow (Jiménez-González *et al.* 2014). The $S2$ mode shows a similar structure with a small additional pair of counter-rotating vortices in the vicinity of the blunt body rear, and a more complex pressure field than the $S1$ mode.

Let us now consider the oscillating modes $O1$ and $O2$ displayed in the bottom part of figure 5. One can observe in an azimuthal plane an alternation of positive and negative streamwise vorticity which is the signature of unsteady vortex shedding. A shorter spatial wake length scale can be seen for the $O2$ mode, related to its higher shedding frequency (*i.e.* larger St). Note that when observed in a transverse plane, these modes display a characteristic spiral structure. This does not imply that if these modes are present in a nonlinear solution, a spiral structure will necessarily be observed. Indeed, it is known that due to the degeneracy associated to mirror symmetry, $m = +1$ and $m = -1$ eigenmodes are mirror-images of each other and can lead to two kind of nonlinear states (Fabre *et al.* 2008): (i) "rotating waves" corresponding to a pure $m = +1$ (or $m = -1$) eigenmodes with a spiral structure, and (ii) "standing waves" corresponding to superposition of $m = 1$ and $m = -1$ eigenmodes characterized by a symmetry plane.

3.1.2. Confined flow with $a/A = 0.75$

Let us now consider a more confined flow with a section ratio of $a/A = 0.75$ or a diameter ratio of $d/D = 0.87$. The structure of the base flow and the influence of the tube wall are displayed in figure 6. Compared to the unconfined or weakly confined flow, the recirculation length is shorter as the confinement becomes stronger. For Reynolds number $Re = 320$, the flow changes direction close to the pipe wall and goes downstream but without setting a closed recirculation zone attached to the wall, even for large values $Re > 320$. The presence of separation in this area is accompanied with a production of negative vorticity. This structure reveals the presence of a confined wall jet. Within the small gap between the body and pipe walls the flow can be seen as parallel and one may expect the flow to be well approximated by a parallel-flow solution called "annular Couette-Poiseuille flow". This classical solution is reproduced in appendix A. The theoretical analytical nondimensional velocity profile is compared in figure 7 to the actual axial flow profile extracted from the base-flows represented in figure 6. At location $x = 0.75$ (in the

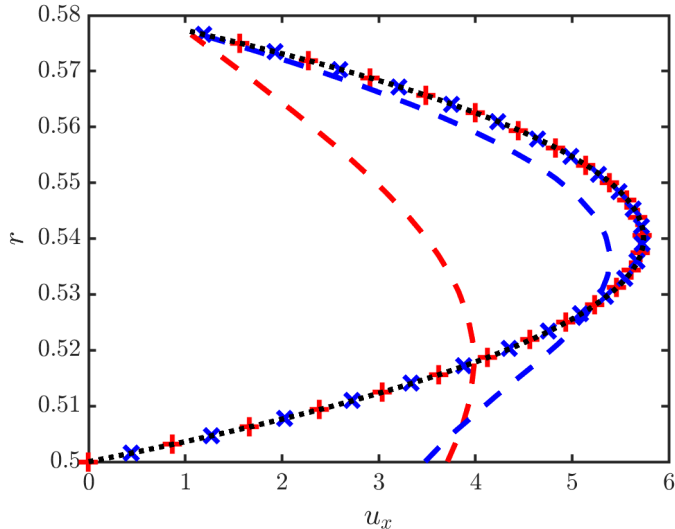


Figure 7: Axial velocity u_x as function to r for $Re = 110$ (red) and $Re = 320$ (blue) at locations $x = 0.75$ (symbols) and $x = 1.25$ (dashed lines); Comparison with annular Poiseuille solution (black dotted line).

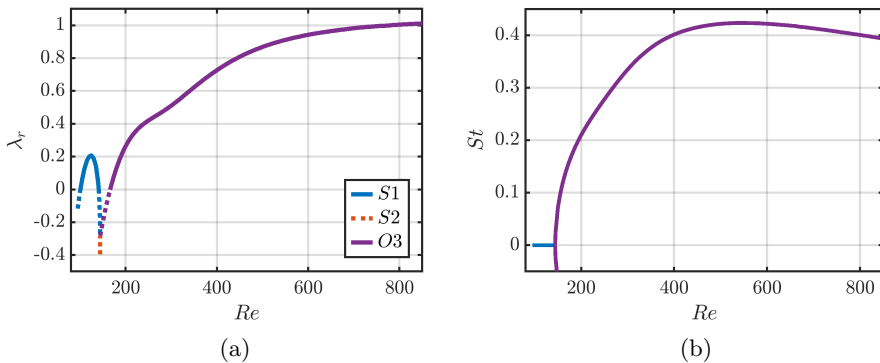


Figure 8: Amplification rate λ_r (a) and Strouhal number (b) versus the Reynolds number for the first unstable modes, $a/A = 0.75$. The horizontal range of the figure corresponds to $Re \in [80 - 1000]$.

rear part of the afterbody), the observed velocity profile is indistinguishable from the theoretical solution, both for Reynolds numbers $Re = 110$ and 320 .

The velocity profile at location $x = 1.25$, slightly behind the body is also plotted in the figure. The curves show that the velocity profile turns into an annular jet, affected by some diffusion, especially for $Re = 110$.

The curves giving the amplification rate and the Strouhal number versus the Reynolds number as computed by the linear stability theory are displayed in figure 8, for a the base flow solved with the section aspect ratio $a/A = 0.75$.

Three branches are found as the Reynolds number varies corresponding to two non-oscillating (called again $S1$ and $S2$) and one oscillating mode. The latter is of a distinct

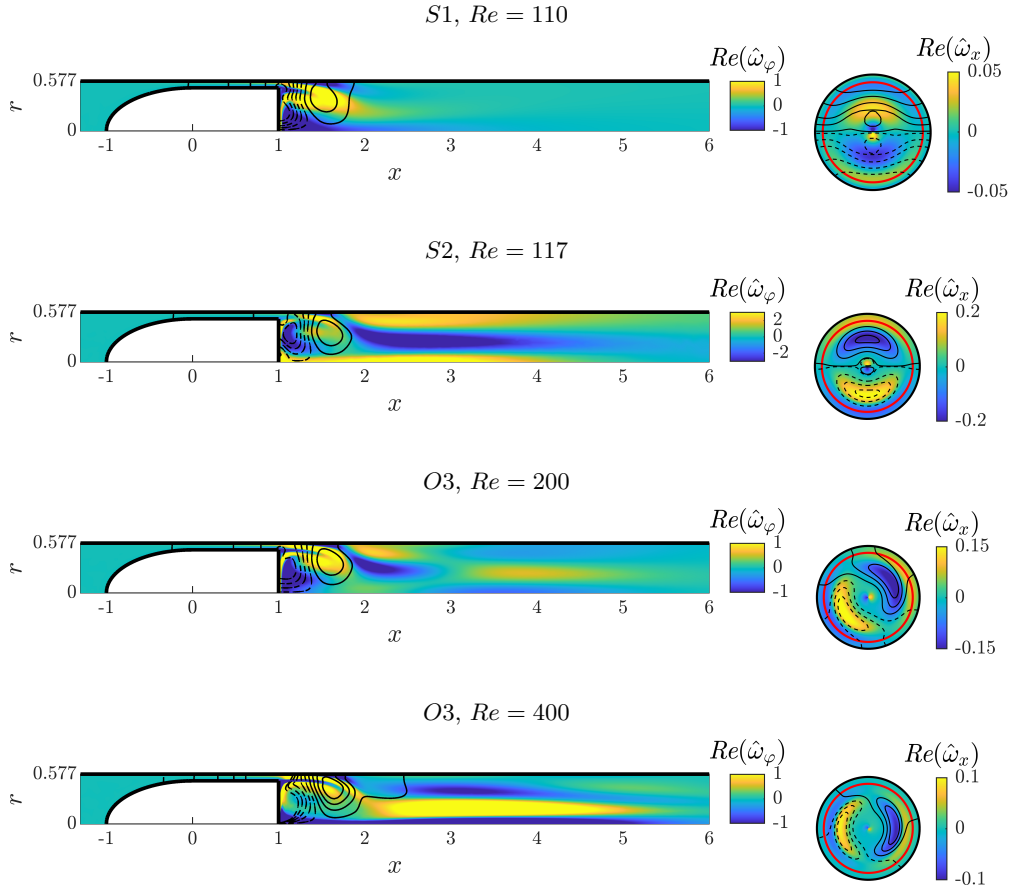


Figure 9: Eigenmodes found for $a/A = 0.75$, real parts of the streamwise vorticity with iso-levels of pressure.

nature comparing to the modes $O1$ and $O2$ previously encountered. It is characterized by a Strouhal number in a lower range, and it is thus called $O3$. The amplification curves of the $S1$ mode follows an inverted parabola: as Re value increases, the amplification of the $S1$ mode raises, reaches a maximum and decreases. The decreasing $S1$ branch meets the rising $S2$ branch and both branches collide at the Reynolds number $Re = 180.4$. Above this value, the collision gives rise to a pair of complex conjugate eigenvalues corresponding to the $O3$ oscillating mode. The symmetry of the problem entails that this oscillatory branch is twofold, for each eigenvalue λ found, $\bar{\lambda}$ is also an eigenvalue. The Strouhal number of the $O3$ raises strongly after the collision of $S1$ and $S2$ from $St = 0$ to $St = 0.1897$ at $Re = 490$ and then slightly decreases.

Figure 9 displays the vortical structure and some iso-pressure contours of the unstable eigenmodes for different values of the Reynolds number. The $S1$ and $S2$ mode exhibit the same behaviours found for low confinement configuration, with a negative pressure zone at the rear of the blunt body followed by a positive pressure one.

Nevertheless, these pressure contours are distorted by the proximity of the pipe wall as the extrema get closer to axis of symmetry. The vorticity of these two modes goes through the same changes and is much important close to to body.

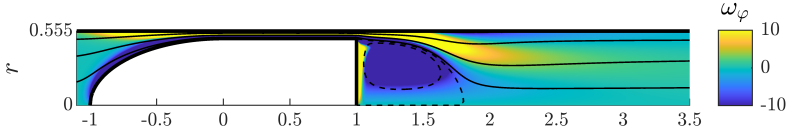


Figure 10: Azimuthal vorticity and streamlines of the base flow in the moving frame attached to the body for $L/d = 2$ and $a/A = 0.81$, $Re = 110$.

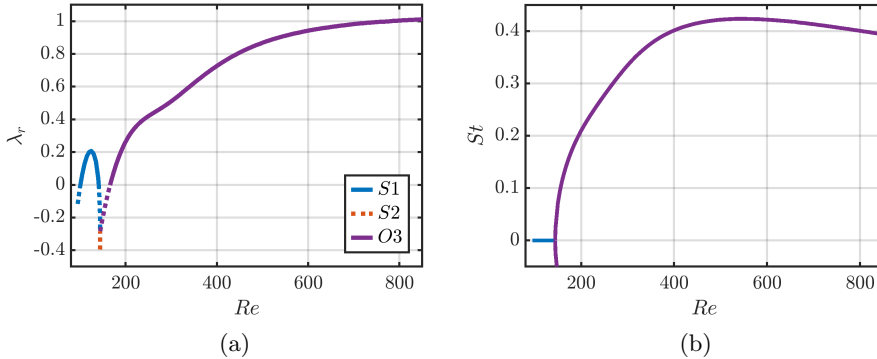


Figure 11: Amplification rate and Strouhal number for the three modes found for $L/d = 2$ and $a/A = 0.81$. The horizontal range of the figure corresponds to $Re \in [80 - 850]$.

The $S1$ mode is more active in the recirculation zone whereas for the $S2$ mode the azimuthal vorticity is higher in the region where the streamlines of the base flow expand, around $x = 2.5$, suggesting a different instability mechanism. At last, the structure of the $O3$ mode seems to be a mix of the $S1$ and $S2$ mode. The pattern of the vorticity and the pressure is very similar to the $S1$ mode in the recirculation zone. The downstream region ($x > 2$) is similar to the same region of the $S2$ mode but the temporal mode oscillation results in production of vorticity of alternated sign. For $Re = 400$, the $O3$ mode is similar, the influence of a larger recirculation zone can be noticed. Alternate values of vorticity in the streamwise direction are still present but they are pushed downstream, outside the scope of the plot. Stronger vorticity is also observable because of an important contraction of the base flow due to its local reversal.

3.1.3. Strongly confined flow with $a/A = 0.81$

Consider now an even more strongly confined flow with a section ratio $a/A = 0.81$ or a length ratio $d/D = 0.9$.

The base flow (fig. 10) remains very similar to the previous case but the bifurcation scenario revealed by LSA approach seems different. Figure 11 displays the computed amplification rate and Strouhal number of the first three modes as function of the Reynolds number. Again, two non-oscillating eigenmodes $S1$, $S2$ and an oscillating mode $O3$ are found. Here, the $S1$ mode becomes unstable between the Reynolds numbers $Re_c = 101.1$ and $Re = 141.1$, the amplification rate plot keeps its inverted parabola shape. The $S2$ mode is observed as a stable mode up to $Re \approx 150$ where a collision with the $S1$ mode gives rise to a pair of complex eigenvalues corresponding to the $O3$ mode. The latter, first arises as a stable mode, and subsequently becomes destabilized through a Hopf bifurcation at $Re = 166.2$. Then, the $O3$ remains the predominant mode over

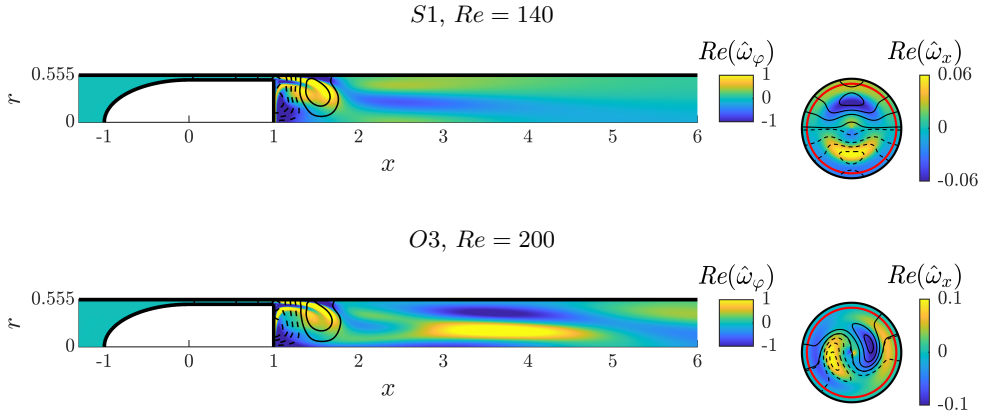


Figure 12: Eigenmodes found for $a/A = 0.81$, real parts of the vorticity with iso-levels of pressure. Slices are given for $x = 2$

the range of parameter studied. A stable pocket is present between the appearance of the $S1$ and $O3$ mode where the $S1$ and $S2$ branches collide, as they are both stable, forming the $O3$ branch. Note that the value of the dimensionless frequency of the $O3$ mode is twice the value of the previous case, $St = 0.4227$ at $Re = 550$. This fact is not really surprising since the maximum axial velocity in the jet (as predicted by the annular Couette-Poiseuille solution given in appendix A) is about also twice the value of the previous case.

Considering the differences between the present case and the previous one, one can postulate the existence of an intermediate value of the confinement ratio where the collision of the $S1$ and $S2$ modes and the destabilization of the $O3$ mode will occur simultaneously. This situation, characterised by the existence of two simultaneous neutral modes with zero eigenvalues, corresponds to a codimension-two bifurcation of Takens-Bogdanov type. This point will be confirmed in the parametric study of Sec. 3.2.

To end up with characterisation of the $a/A = 0.81$ case, figure 12 reveals the structure of the unstable modes $S1$ and $O3$. Observations made in the previous sub-section, for $a/A = 0.75$, apply here. We can add that the influence of the confinement is noticeable in the $S1$ mode as the maximum of velocity of the base flow is higher compared to the previous case. The $O3$ mode also possesses patch of alternated sign of azimuthal vorticity exhibiting higher extrema than the previous case, for the same reason cited just above.

3.2. Cartography of $m = \pm 1$ modes in the $a/A - Re$ plane for $L/d = 2$.

A first exploration of the stability picture has been carried out for some selected values of a/A . The study is now extended continuously to a larger range of the confinement parameter with $a/A \in [0.01, 0.92]$. The azimuthal wavenumber of the perturbation and the length-to-diameter aspect ratio are kept respectively to $m = \mp 1$ and to $L/d = 2$.

The neutral stability curve for any mode is the location of zero amplification perturbations. For each value of the ratio a/A are obtained a critical Strouhal number St_c and a critical Reynolds number which are the limit of the instability for any mode. The neutral curves are displayed in figure 13 for the six modes of interest. The map is built by following the branches in the parameter space with a step in confinement ratio of $\Delta(a/A) = 0.001$ and/or a Reynolds number increment of $\Delta Re = 1$. A strategy has been developed to ensure a continuous and accurate values of the curve. A first sweep of

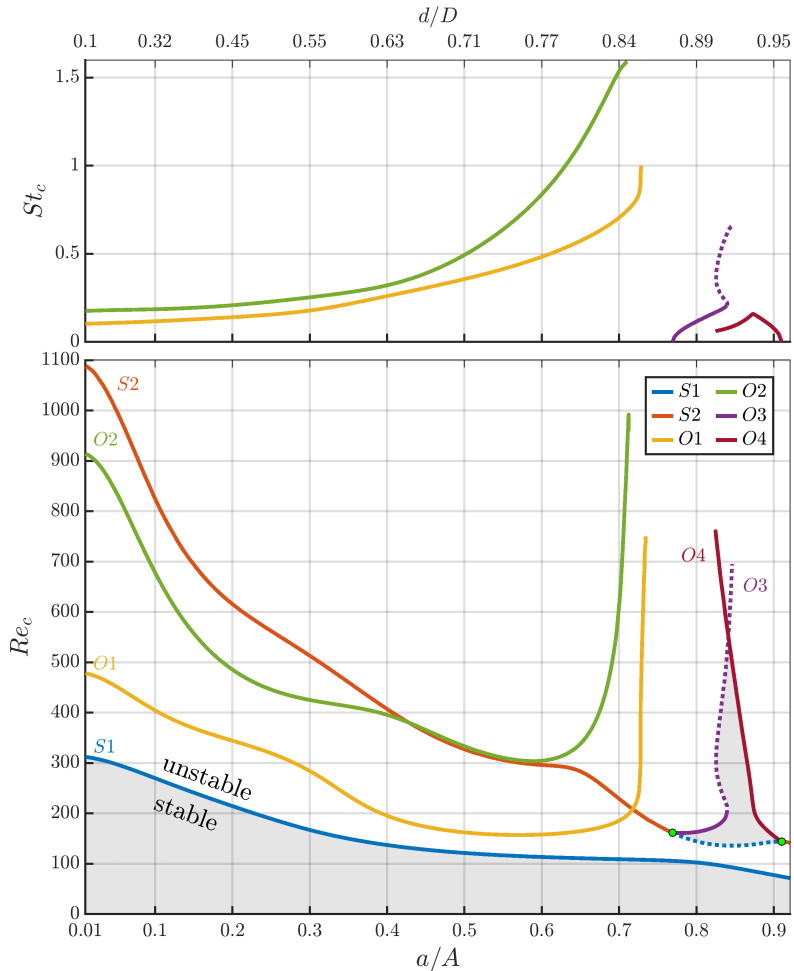


Figure 13: Neutral curves in the $Re - a/A$ plane, and critical Strouhal number as a function of the confinement ratio. The body length is kept constant, $L/d = 2$. The dashed portions of the blue and purple curves are part of the neutral curves corresponding to restabilization of the $S1$ and $O3$ modes.

the $(Re, a/A)$ plane have been initially performed to save computational time. Then a thorough computations are conducted following unstable branches. For each confinement value, Re is increased in order to find lower and upper bounds of it critical value Re_c , and a linear interpolation is completed to get a more accurate value such as $\lambda_r(Re_c) = 0$.

The first important result observed from this figure is about the destabilization of the axisymmetric base flow. It is always caused by the same $S1$ mode for all area ratio a/A in the range 0.01 to 0.92. The loss of axial symmetry always occurs through a stationary bifurcation.

The deep stability analysis has also revealed the existence, for the secondary modes, of two different regimes and a transition zone. First, in the weakly confined regime, up to $a/A < 0.7$, the secondary dominant mode is the $O1$ mode, and higher modes ($S2, O2$) arise in a much larger range of Reynolds number which makes their physical relevance unlikely. In addition the sequence of instabilities with a non-oscillating $S1$ mode followed

by an oscillating $O1$ mode is thus the same as observed for other blunt bodies in free stream flow (Natarajan & Acrivos 1993; Meliga *et al.* 2009b; Fabre *et al.* 2008; Auguste *et al.* 2010). The confinement is also found to be destabilizing for both these modes, as the critical Reynolds thresholds decrease as the confinement ratio a/A grows. Large confinement also increases the frequency of the oscillating $O1$ mode, in consistence with the fact that the mean velocity of the annular jet formed past the body increases with the confinement (for a given flow rate, decreasing section increases velocity).

A transition regime is observed in the interval $0.7 < a/A < 0.76$. The threshold of the $S2$ mode first decreases after $a/A \approx 0.6$ to approach that of the $O1$ mode. The latter is then strongly and abruptly stabilized, and it is no longer found for $a/A > 0.72$. In the range $0.72 < a/A < 0.76$ the stationary $S2$ mode is become the dominant secondary mode.

The strongly confined regime occurs for a/A greater than 0.76. At this regime, the oscillating modes $O1$ and $O2$ are not longer present and new ones ($O3$, $O4$) appear with low or moderate dimensionless frequency. Let us follow in figure 13 the mode evolution along a vertical line at a/A close to 0.86 and consider the evolution when increasing Reynolds number. It can be seen that the $S1$ initially stable becomes unstable on a short range of Re , then it is unstable in a larger range of Re , and finally the flow instability is generated by the appearance of the mode $O3$ and $O4$. In a short area, coloured in grey in the figure, it is found a pocket of stability. It can be noticed that the neutral curve of the $O3$ mode also displays two turning points close to $Re_c \approx 200$. So in a narrow range around $a/A = 0.84$, the destabilization / restabilization sequence occurs twice as Re is raised. The complexity of the stability diagram for very strong confinement is a translation of the real physics complexity in this region with fast annular wall jet, separated flows and vortical interactions.

As already discussed, the emergence of the stable pocket is expected to be associated to a codimension-two bifurcation of Takens-Bogdanov type, where both $S1$ and $S2$ modes are simultaneously neutral. This statement is confirmed in figure 13, as indicated by the green point with coordinates $(a/A, Re_c)_{TB}^{O3} = (0.769, 161.57)$ from which the $O3$ neutral curve emerges. Note that a second Takens-Bogdanov point is observed at coordinates $(a/A, Re_c)_{TB}^{O4} = (0.91, 143.96)$. The latter bounds the stable pocket on the other side and is associated to the emergence of the $O4$ mode. As indicated in the upper plot, the critical Strouhal number of $O3$ and $O4$ modes is zero at the codimension-two points, as expected for a Takens-Bogdanov bifurcation. The Strouhal numbers of these modes raise as one moves away from these points.

3.3. Effect of the L/d aspect ratio

The effect of the length-to-diameter ratio L/d of the blunt body is now investigated keeping again the restriction to $m = \pm 1$ modes. This geometrical parameter is found to modify the stability properties only in the weakly confined regime at $a/A < 0.7$ identified above. Consequently, only the neutral curves of the modes $S1$, $S2$, $O1$ and $O2$ relevant to this regime are tracked. The neutral curves of these modes are shown in figure 14 for different values of $L/d = \{46, 8, 10\}$. They are compared to the results of the reference case with $L/d = 2$ presented in the previous paragraph (in green in figure 14). For low confinement, $a/A < 0.4$, the increase of the body length stabilizes the flow as pointed out by Brücker (2001) in his experiments. He suggests a larger boundary thickness caused by a longer body is responsible for this stabilizing effect. To verify this argument, figure 15 (left plot) shows the vorticity at the blunt body surface for different body lengths. On the ellipsoidal nose ($x < 0$), the plots are superposed indicating the generation of the same amount of vorticity. Then, on the cylindrical surface of the blunt body ($x < 0$), the

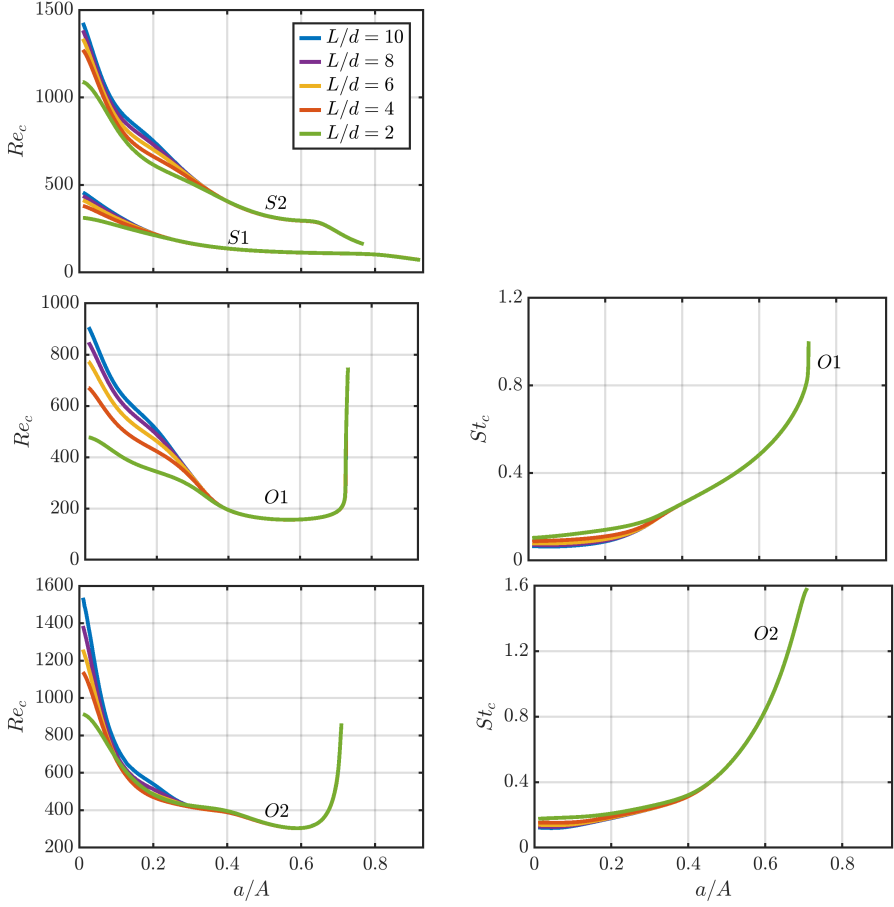


Figure 14: Neutral curves of the stationary modes for different body lengths.

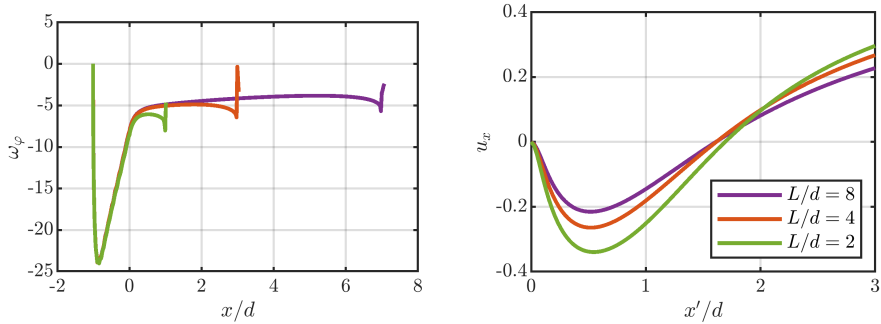


Figure 15: Left, azimuthal vorticity of the base flow at the blunt body surface for $Re = 330$. Right, axial velocity downstream the blunt body, the frame has been shifted in order to set the rear of the bodies at the same location. Only the base flow for $L/d = 2$ has an unstable mode $S1$.

vorticity reaches a higher value for short objects. Indeed, a streamline along the body and its recirculation zone is more curved for short objects, accumulating therefore more vorticity feeding the separated flow in the rear.

In conclusion, for shorter objects, the recirculation zone generates stronger reverse velocities (see fig. 15, right plot), promoting wake instabilities at lower Reynolds number compared to the case of longer objects. We can also note that even if the vorticity intensities are quite different, their sizes do not differ so much.

Back to figure 14, as the area ratio increases, all curves tend to collapse into one, either the Re_c or the St_c . It means, for $a/A > 0.4$, that the body length does have any influence on the onset of the four investigated instability modes. This is consistent with the fact that, as verified in figure 7, once a certain confinement is reached and whatever the length of the body, the velocity profile is the same and corresponds to the annular Couette-Poiseuille solution recalled in appendix A.

3.4. Higher azimuthal wavenumber modes

To complete the parametric study, we now consider eigenmodes with azimuthal wavenumbers other than ± 1 . No axisymmetric ($m = 0$) unstable mode is found, but numerous unstable modes with $|m| > 1$ are detected. Most of them occur in ranges of Reynolds number far above the primary threshold of $m = \pm 1$ modes so they are not likely to be observed in any real flow. Only two modes were detected with critical Reynolds number in the same range as $m = \pm 1$ modes. Both of them are non-oscillating, with respectively $m = 2, 3$ azimuthal wavenumbers, and will be referred to as $S_{m=2}$ and $S_{m=3}$. These modes arise in strongly confined regime $a/A > 0.6$ where length of the body has negligible effect. In this section we keep the body aspect ratio $L/d = 2$ but conclusions given in this paragraph actually hold for all values of L/d .

The structure of these new eigenmodes are illustrated in figure 16. Their geometry is best understood by looking at the views in a transverse x -plane (plots in the right column). Mode $O2$ is characterised by the existence of two orthogonal symmetry planes and displays four main structures of axial vorticity of alternated signs, while mode $O3$ has three planes of symmetry and six main vorticity structures. Secondary vorticity structures of opposed signs are also visible near the symmetry axis. The views in a vertical plane (left column) give a complementary picture. One can notice that compared to $m = \pm 1$ eigenmodes the present ones are more localized in the close wake and do not extend in the far wake.

The neutral curves in the $(Re_c, a/A)$ plane of the modes computed for $m = \{1, 2, 3\}$ are plotted in figure 17 which completes the results of figure 13 with additional azimuthal wave numbers. The Strouhal number are not displayed because the newly considered modes are stationary, $St_c = 0$. For a low confinement, the stationary modes $S_{m=2}$ and $S_{m=3}$ have critical Reynolds number much higher than the first unstable mode and they are rather unlikely to be observed in real experiments. However, as the confinement increases, their critical Reynolds number Re_c decreases, and they alternatively become the second and third mode to be unstable for $a/A > 0.7$. Interestingly, in this strongly confined regime, these two modes become unstable for Reynolds number values very close to those corresponding to restabilization of the $S1$ mode. Hence, in such ranges they are the only unstable modes to exist. So non-axisymmetric flow characterised by azimuthal wavenumber $m = 2$ or 3 (or a superposition of both) are expected to be observed without the presence of any $m = 1$ component in experiments or simulations. Such structures are characterised by the absence of lift forces exerted on the body, as justified for instance in (Tchoufag *et al.* 2014). Only $m = \pm 1$ modes can contribute to a lift force.

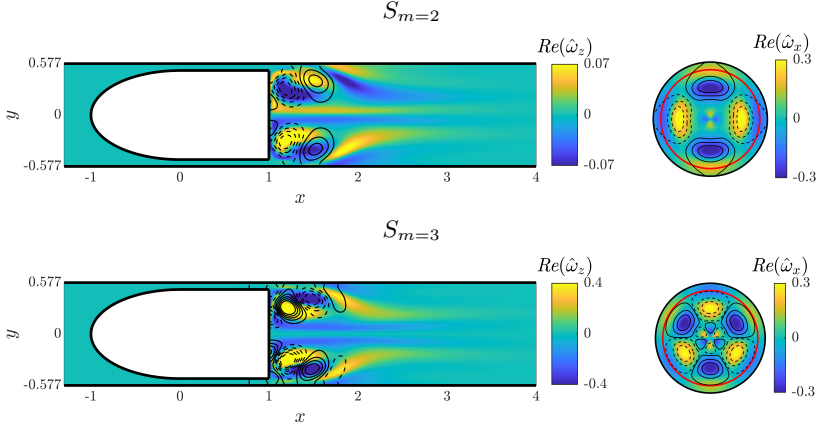


Figure 16: Unstable non-oscillating modes at $Re = 200$ and $a/A = 0.75$. Left: z -component of the vorticity and iso-level of pressure. Right: slice in the transverse plane at $x = 2$, streamwise component of the vorticity and iso-levels of pressure, the red circle represents the projection of the base of the blunt body in this plane.

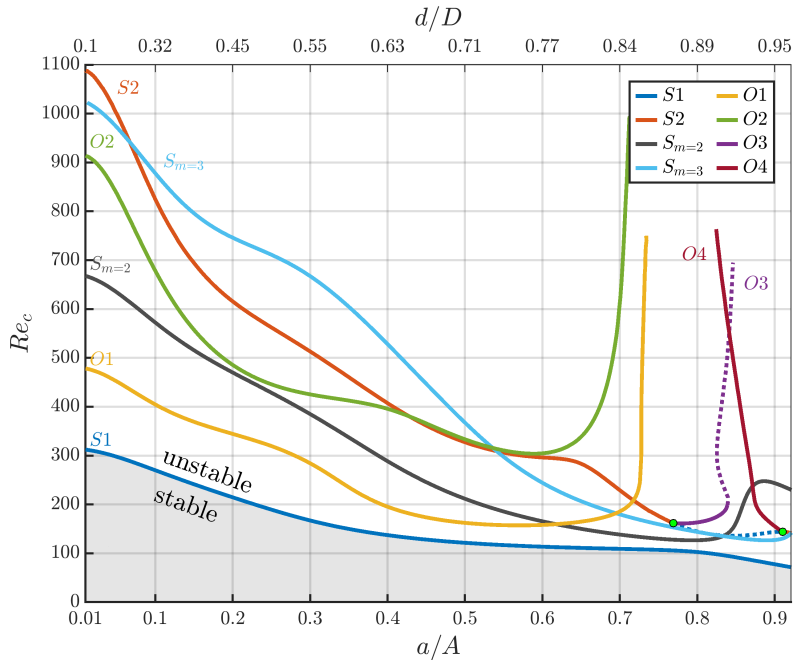


Figure 17: Critical Reynolds number as a function of the confinement ratio for all modes considered. The body length is kept constant, $L/d = 2$.

4. Exploration of nonlinear dynamics using DNS

In the previous section an exhaustive mapping of the linear stability characteristics of the flow with respect to the aspect ratios and the Reynolds number has been performed. In this section the nonlinear dynamics is now explored using Direct Numerical Simulations (DNS). The aim is both to confirm the LSA predictions regarding the primary

instability threshold and to investigate the nonlinear dynamics arising away from this threshold.

4.1. Dynamical regimes detected by DNS and comparison with LSA

In the numerical exploration we selected five values of the confinement ratio a/A covering the different regimes indicated by LSA, and ranges of Re from slightly below the primary threshold found by LSA to about twice this value. The conducted simulation runs are given by their coordinates in the parametric plane ($Re - a/A$) in figure 18.

Five general kinds of solutions are observed and are displayed using different symbols. The first kind (white squares) is an axially symmetric state corresponding to a stable configuration with zero lift \mathcal{L} i.e. the lift coefficient $C_\ell = \frac{\mathcal{L}}{1/2\rho U_\infty \pi r^2}$ is measured lower than 10^{-4} . The second (black squares) is a 3D steady state characterised by a constant lift and a symmetry plane. This state is noted $SS1$ as its structure is a strong indication of the direct effect of a steady $|m = \pm 1$ eigenmode. The third (black circles) is a reflection-symmetry preserving (RSP) state. This mode is defined by an oscillatory lift around a non-zero mean value, the wake still displaying a planar symmetry. Aperiodic behaviours (black stars) have also been observed. Finally, the fourth kind of solutions (black triangle, noted SS_3) are steady states with a structure characterised by an $m = 3$ component.

The LSA predictions are reproduced in figure 18 to allow a comparison with DNS results. The transition from the axisymmetric state and the steady non-axisymmetric state $SS1$ revealed by DNS is observed to be well predicted by the marginal stability curve $Re_{c,S1}(a/A)$ indicating destabilization of the $S1$ modes. This fully confirms that the nonlinear state $SS1$ is effectively directly due to a supercritical nonlinear saturation of the $S1$ mode.

On the other hand, in the computed cases, the observed secondary bifurcations (leading either to a periodic RSP state or to an aperiodic state), does not directly match with any secondary bifurcation curve found by LSA. This is not really surprising, since the secondary bifurcation occurs along the bifurcated steady state mode (SS) which differs from the axisymmetric solution used as a base-flow for the linear stability analysis. However, the nature of the secondary modes revealed by LSA may still be relevant to fully explain the nonlinear dynamics, as it will be demonstrated by a deeper exploration of few cases in the next section.

4.2. Towards nonlinear behaviour, low confinement flow at $a/A = 0.39$

The temporal evolution of the wake for different Reynolds number and for a fixed area ratio are now analysed with the DNS. Figure 19 displays the Q-criterion for the three Reynolds numbers $Re = 130, 145, 175, 200$, with two plots for each case corresponding to view in two orthogonal directions.

For $Re = 130$ (figure 19a), the flow corresponds to the axially symmetric state, in accordance with LSA prediction. The wake is axially symmetric and the view is identical in both orthogonal directions. The structure behind the blunt body is stationary and it consists in a toroidal recirculation bubble. For $Re = 160$ (fig. 19b), is observed the $SS1$ steady, non-axisymmetric state. The loss of axisymmetry results in a tilting of the toroidal structure attached to the body, the latter expanding in one direction and retracting in the opposite one.

The next states for $Re = 175$ and $Re = 200$ (fig. 19c – d) correspond to the reflective symmetry preserving (RSP) oscillatory state. The toroidal recirculation gets destabilized and hairpin vortices are periodically advected in the streamwise direction. The interaction

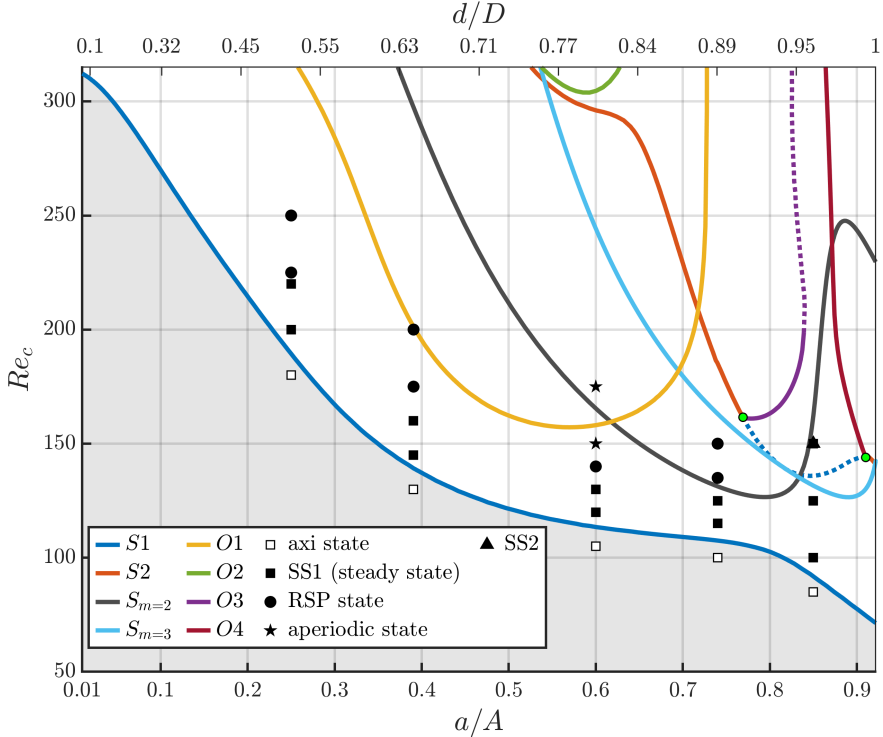


Figure 18: Neutral curves computed previously using LSA and DNS results represented by the symbols.

between those vortices and the wall is visible through wall shaped vortices which merge with the hairpin vortices as they move downstream.

Up to here, the sequence of bifurcations and the structure of the observed states are identical to the unconfined case (Bury & Jardin 2012; Bohorquez *et al.* 2011). The main difference is that due to confinement, the bifurcations arise at much lower Reynolds number value. For instance Bohorquez *et al.* (2011) report the first bifurcation for $Re = 319$ and the second for $Re = 413$.

Figure 20 displays the time history of the lift and drag coefficients (noted C_l and C_d) characterising forces exerted on the body calculated from DNS, again for $a/A = 0.39$.

For $Re = 130$ (case not displayed), the lift converges to a zero value. For all other cases, after a short transient (not shown), the simulations first seem to converge towards a steady state with zero lift, approximately in the range $t \in [30 - 50]$. The later evolution shows however that this state is not stable, and a phase of linear instability characterised by exponential growth of both coefficients is seen. In this linear phase the observed behaviour of the lift coefficient corresponds to a purely exponential growth with non-oscillating behaviour ($\approx e^{\sigma t}$ with real σ). This is a clear signature of the emergence of the non-oscillating mode $S1$ which is effectively the only unstable one detected by LSA for the values of considered Re .

For $Re = 145$ (fig. 20a) and for $Re = 160$ (fig. 20b), the subsequent nonlinear evolution is a saturation towards the $SS1$ steady state with non-zero lift. On the other hand, for $Re = 175$ (fig. 20c), this steady state seems to be transiently approached by the solution, but then a second phase of linear instability follows, this time with oscillating behaviour

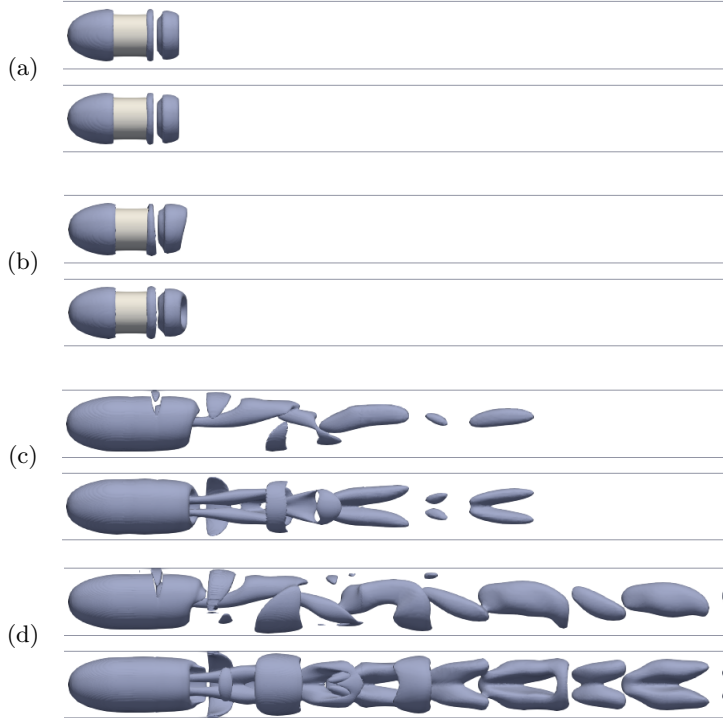


Figure 19: Iso-contour of Q-criterion for $a/A = 0.39$, two perpendicular views are represented for each case. From top to bottom, $Re = 130, 145, 175, 200$.

($\approx e^{\sigma t}$ with complex σ) is observed. This trend is the signature of the existence of a non-oscillating mode related to the $O1$ mode. For $Re = 175$, the saturated state ultimately observed is the periodic, RSP state characterised by a lift force oscillating around a non-zero mean value. For $Re = 200$ (fig. 20d), the initial behaviour and ultimate state are similar, but transient towards the RSP state displays a low-frequency modulation which is eventually damped.

The dimensionless frequency spectra of the two oscillating cases are presented on figure 21. They are performed from the C_ℓ signal of figure 20 and the transitional behaviours have been excluded. It has been verified that the sample is large enough and does not influence the spectra. Both spectra lead to similar observations. The two peaks can be interpreted as a fundamental frequency mode and its first harmonic. The amplitude of the first harmonic is much lower than the fundamental and therefore it is not visible to the naked eye on the signal which is very close to a pure sinusoid. As shown in the Table found in figure 21c, the influence of the Reynolds number on the Strouhal number St is weak with this Re range, the DNS give a 2% variation between $Re = 175$ and 200. This behaviour is substantiated by the quasi-constant St values found using the LSA for low confinement (see fig. 4b). The St values given by the DNS and LSA approaches are comparable even if a 16% relative difference is measured between the DNS ($Re = 200$) and LSA at the threshold ($Re = 201.2$). The discrepancy between those results can be explained by the fact that the $O1$ mode is obtained using an axially symmetric base flow whereas this base flow is no longer present in the DNS for $Re \geq 175$, the RSP state oscillates around steady state which is non-axisymmetric.

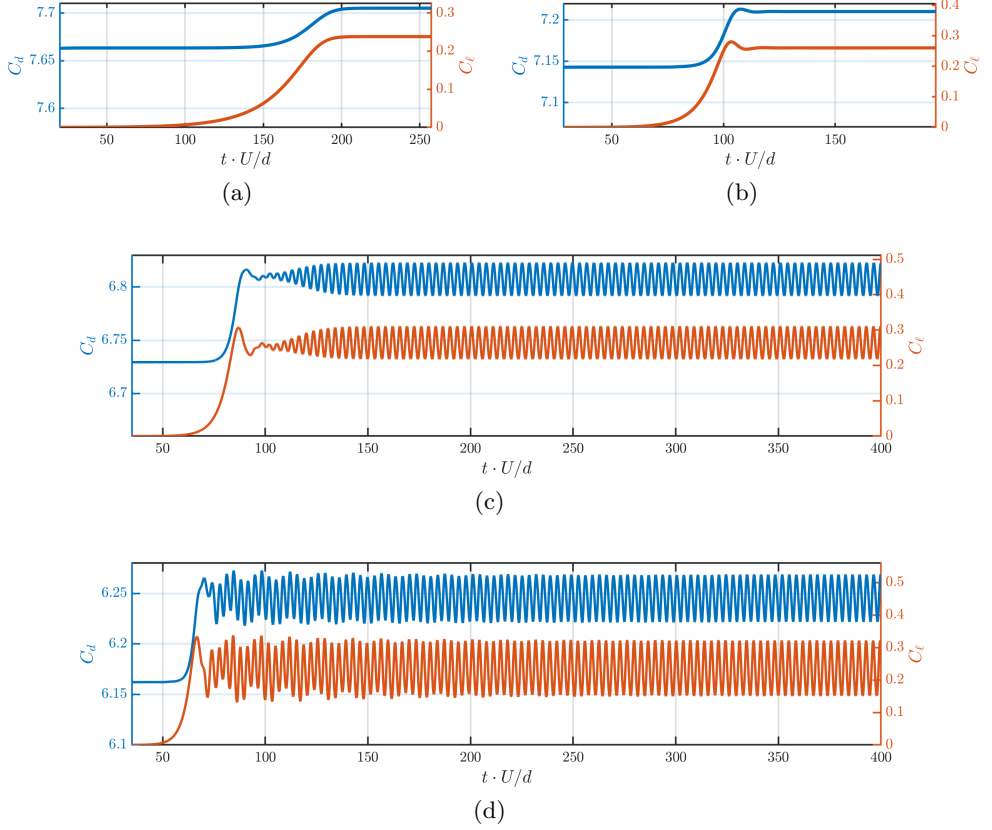


Figure 20: Lift and drag coefficients from DNS simulations ($L/d = 2$ and $a/A = 0.39$) versus dimensionless time, for (a) $Re = 145$, (b) $Re = 160$, (c) $Re = 175$ and (d) $Re = 200$.

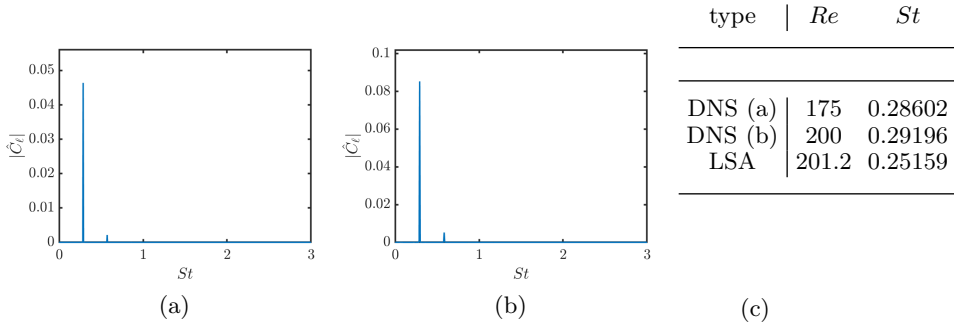


Figure 21: Frequency spectra for DNS case $L/d = 2$, $a/A = 0.39$, $Re = 175$ (a) $Re = 200$ (b) and table of St values of the higher peaks (DNS) with corresponding St_c for LSA (c).

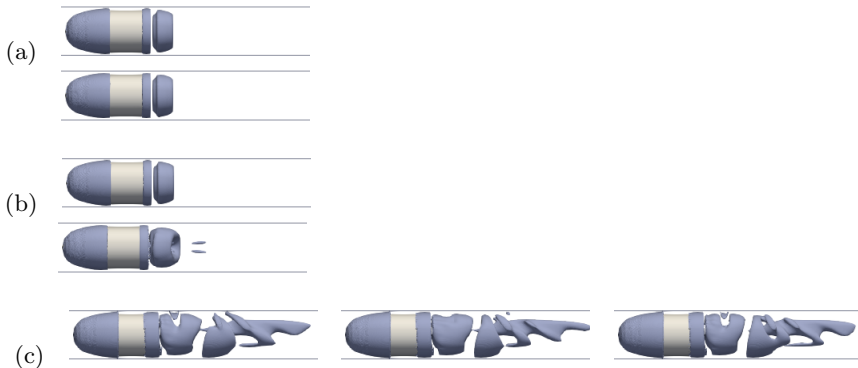


Figure 22: Iso-contour of Q-criterion for moderately confined cases; two perpendicular views are represented for each case. (a) $a/A = 0.74$, $Re = 100$, (b) $a/A = 0.74$, $Re = 115$, (c) $a/A = 0.74$, $Re = 160$. For the last case, instantaneous representations for three different instants are displayed.

4.3. Towards nonlinear behaviours, moderately confined cases ($a/A = 0.6$ and 0.74)

Consider, now, the flow structures revealed by DNS in the range of moderately confined cases. The beginning of the bifurcation sequence is the same as described in the previous paragraph. With an initially symmetric state, followed by a steady, non-axisymmetric state. Figure 22 (a – b) displays these two states observed respectively for $a/A = 0.74; Re = 100$ and $a/A = 0.74; Re = 115$. Similar structures are obtained for $a/A = 0.6$ and the same values of Re and are not displayed.

When raising the Reynolds number to $Re = 150$ in this range of moderately confined cases, nonlinearities lead to richer dynamics compared to the previous cases. Consider, first, the flow obtained for $a/A = 0.74$ (fig. 22c). Although the flow symmetries still indicate the (RSP) oscillatory state, the flow has a more complex structure than previously observed. Two main oscillating regions can be seen: the first one is the upper part of toroidal recirculation, close to the body, where a separated structure periodically appears. The second one is formed by a more distant structure, a 45° -inclined protrusion which is advected downstream. Sticking to the case $a/A = 0.74$, $Re = 150$, figure 23(a) displays the time-history of the lift and drag coefficients. The lift force shows a temporal modulation where both a low-frequency component (with dimensionless period of order 7.5) and a high-frequency component (with a period about 10 times shorter) can be discerned. The drag coefficient displays similar patterns but the amplitude of oscillations are extremely small (less than 0,5% of the average value).

Although the time-series may suggest a quasi-periodic behaviour, analysis of the Fourier-transform of the lift force (fig. 23b) indicates that the behaviour is actually strictly periodic, as demonstrated by the existence of a fundamental frequency, $St_1 = 0.1308$ along with its harmonics. The spectrum also shows that apart from the fundamental, a high-frequency content is centred around the harmonic number 10, corresponding to $St_{10} = 1.308$. This matches with the high-frequency component detected in the time-series with a period about 10 times shorter compared to the low-frequency component.

Trying to relate these dynamics to the LSA results is a bit puzzling ; going back to figure 13 shows that no unsteady modes exist for $a/A = 0.74$ since the $O1$ and $O2$ are only detected for $a/A < 0.73$ and the low-frequency $O3$ mode only arises for $a/A > 0.75$. However, again, the results obtained from LSA considering the axisymmetric base flow are only indicative here since the bifurcations arise from the steady non-axisymmetric

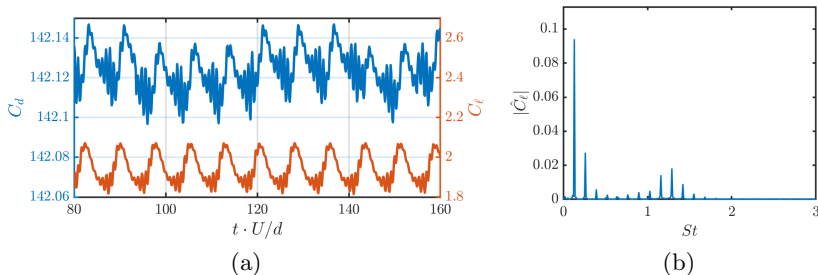


Figure 23: Characterisation of time-dependent forces exerted on body for $Re = 150$, $a/A = 0.74$: (a) time-history of lift and drag coefficients, and (b) Frequency spectrum of the lift coefficient.

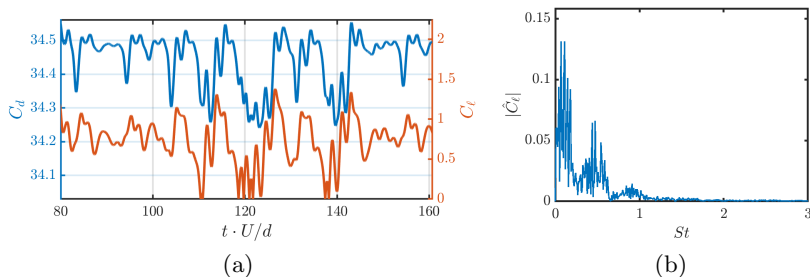


Figure 24: Characterisation of time-dependent forces exerted on body for $Re = 150$, $a/A = 0.6$: (a) time-history of lift and drag coefficients, and (b) Frequency spectrum of lift coefficient.

state. The order of magnitude of the Strouhal number St_1 characterising the low-frequency oscillation is in the same range as the $O3$ mode which exists for $St \approx 0.1 - 0.2$, suggesting that the $O3$ mode actually plays a role in the nonlinear solution given by the DNS.

Consider, now, the flow obtained for $a/A = 0.6$ and $Re = 150$. The time-series of the exerted forces (fig. 24a) show that periodicity is clearly lost and indicate a chaotic behaviour. This is confirmed by examining the Fourier transform of the lift force (fig. 24b) which newshows a broadband spectrum.

4.4. Towards nonlinear behaviours, high confinement flow at $a/A = 0.85$

To end up the exploration of nonlinear dynamics, consider now a highly confined case with $a/A = 0.81$. The first bifurcation again leads to the steady, non-axisymmetric state and is well explained by the onset of mode $S1$. On the other hand, when raising the Reynolds number, the next bifurcation does not lead to time-dependent vortex-shedding. Instead, the flow remains stationary, but it acquires a structure characterised by the shedding of six vortical structures compared to only two as in the $SS1$ state, as shown in figure 25(a) for $Re = 150$. This structure is a strong indication of the presence of an eigenmode with azimuthal wavenumber $m = 3$, and it is fully newrelated with the LSA results which indeed predict the existence of the steady mode $S3$ in the same range of parameters. Plotting the pressure in a transverse slice just behind the body in figure 25(b) indeed



(a)

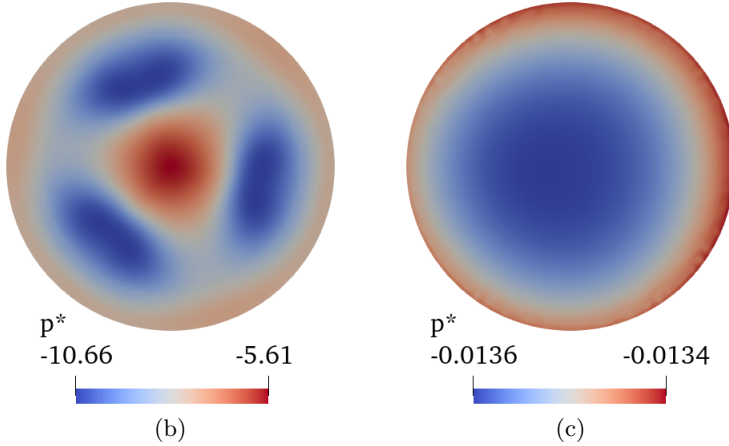


Figure 25: Illustration of the wake for $a/A = 0.85$ and $Re = 150$. Iso-contour of Q-criterion (a), side and rear diagonal views. Slices of the dimensionless pressure $p^* = p/(\rho U^2)$ in the wake of the body, at $x/D = 1.2$ (b) and at $x/D = 9$ (c).

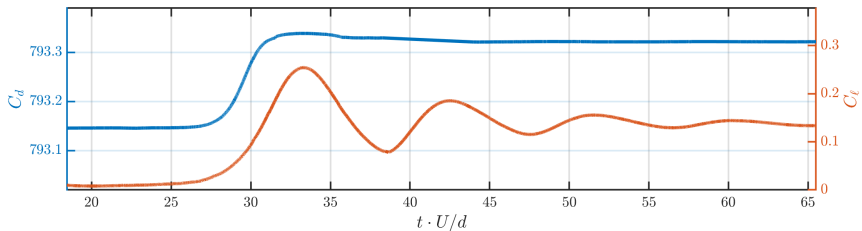


Figure 26: Lift and drag coefficient for DNS case $L/d = 2$, $Re = 150$ and $a/A = 0.85$ versus dimensionless time.

indicates a symmetry of order 3 (i.e. 3 symmetry planes). However this symmetry is not perfect. Indeed, plotting the pressure in a slice located farther downstream in figure 25(c) rather demonstrates a symmetry of order one because the region of largest pressure (blue levels) is slightly displaced towards the left. The presence of a $m = 1$ component in the flow also manifests by the existence of a non-zero lift force, as indicated by the time-series in figure 26. This suggest that the observed flow structure actually results from the presence of both $S1$ and $S3$ modes.

5. Summary and discussion

In this study, the stability of the wake induced by a bullet-shaped blunt body moving at constant velocity in moderate and strong confinement conditions has been investigated by the mean of two different numerical approaches. The first one is the global linear

stability analysis and it has been performed on a rather exhaustive set of parameters (geometrical aspect ratios and Reynolds numbers), more especially the $(a/A, Re)$ plane has been widely explored. One of the main conclusions arising from this first study is that the first destabilization of the axially symmetric is always associated to stationary mode with azimuthal wave number $m = 1$. In the low-confinement regime ($a/A < 0.6$), it is observed a sequence similar to the one seen in the unconfined case, characterised by the successive emergence of two stationary ($S1$ and $S2$) modes and two oscillatory modes ($O1$ and $O2$), all with azimuthal wavenumber $m = 1$. Increasing the confinement leads to in a decrease of the associated critical Reynolds numbers and an increase of the frequencies of the unsteady mode. The length of the body also influences the results and tends to delay the instability.

On the other hand, in the highly confined regime ($a/A > 0.75$), although the primary mode remains the $S1$ mode, the next ones to emerge are steady modes associated to wavenumbers $m = 2$ and $m = 3$. This range is also associated to a restabilization of most $m = 1$ modes: the oscillating modes $O1$ and $O2$ completely disappear, and the primary stationary mode $S1$ restabilizes, and new unsteady modes called $O3$ and $O4$ characterised by very low frequencies emerge. Interestingly, between these two latter events, there exists a range of Reynolds number where all eigenmodes with $m = 1$ are stable and only unstable modes with $m = 2, 3$ are present. Interestingly, in this high-confinement regime, the results become independent upon the length of the body. This is explained by the fact that a parallel flow of Couette-Poiseuille type establishes within the annular gap between the body and the wall.

The second part of this paper is a numerical exploration of the nonlinear dynamics. For this, direct numerical simulations are performed for various points of the $(a/A, Re)$ plane in order to confront the linear stability findings with numerical experiments. The results of the DNS agrees well with the LSA approach close to the first instability threshold as expected. For low confinement, the bifurcation scenario remains the same as the one observed for bullet-shaped blunt bodies. First the loss of axial symmetry occurs through a stationary bifurcation implying a non-zero lift, and then an oscillatory behaviour is exhibited via the reflection-symmetry preserving (RSP) state. As the confinement raises, the scenario is no longer valid and other states emerge due to the wall presence. For instance, aperiodic behaviour can be observed for intermediary confinement, $a/A = 0.6$. The nonlinearity effects increase with the confinement as it is illustrated for the RSP state when the section ratio is $a/A = 0.74$. This state differs greatly from the one found for a low confinement: the wake oscillation gathers a large number of harmonics of the same frequency as it has been demonstrated on spectra of the lift and drag coefficients.

We conclude this paper by two last remarks. First, the influence of confinement can be thought of as two ingredients : *(i)* the effect of the domain restriction by itself and *(ii)* the effect of an additional shear associated to the boundary layer at the lateral walls. To check their respective influence, we conducted a few tests by replacing the boundary condition at the lateral wall by a "slip condition", hence cancelling the second ingredient. Preliminary tests considering the case $a/A = 0.75; L/d = 2$ lead to contrasted results. On the one hand, the steady $S1$ mode is found to be promoted by a slip condition, with critical a critical Reynolds detected at $Re_{S1} \approx 63$ compared to 110 with the no-slip condition. On the other hand, an unsteady mode more akin to the $O1$ mode is detected to emerge for $Re_{S1} \approx 200$ with slip condition, in contrast with the fact that this mode vanishes for strong confinement considering no-slip. A complete exploration of this issue is left for future work.

Finally, it is interesting to compare the observation of global instabilities detected by LSA with the prediction of local approaches. Juniper (2006) conducted a comprehensive

study of the spatio-temporal stability properties of a family of axisymmetric wake/jets, and identified the ranges of existence of global instabilities as function of a co-flow parameter $A^{-1} = (U_1 + U_2)/(U_1 - U_2)$ where U_1 and U_2 are the characteristic velocities of the central and peripheral zones, and of a confinement parameter h which in our case can be identified with $D/d - 1$. Our own results for the base flow indicate a co-flow parameter of order $A^{-1} \approx 0.2$ in the region located in the near wake ($x/d \approx 1$), and the range we have explored corresponds to confinement parameter in the range $h \in [0.05 - 10]$. Considering figure 12(d) of the aforementioned paper, the flow is locally absolutely unstable in this whole range. Hence, predictions of local analysis are consistent with the onset of global mode. However, as already identified in [Bohorquez *et al.* \(2011\)](#), spatio-temporal local analysis only predicts oscillating modes with non-zero λ_i , so it may only explain the onset of oscillating modes such as $O1, O2, \dots$, but not the steady $S1$ mode which is always the first to emerge.

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Appendix A. Annular Couette-Poiseuille flow

For strongly confined cases, the flow may be approximated by a parallel flow $\mathbf{u} = u_x(r)\mathbf{e}_x$. With these assumptions the Navier-Stokes equations written in the body frame can be reduced to:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_x}{\partial r} \right) = \mu \frac{dp}{dx} \quad (\text{A } 1)$$

where the axial pressure gradient can be shown to be constant. The volume flow rate is given from the product of the front section of diameter D and of the body velocity i.e the external wall velocity u_w in the body frame. Let us use for convenience the aspect ratio $\xi = d/D$, which is lower than 1. The reference length and reference velocity are respectively set to $d/2$ and u_w . The nondimensional analytical solution (referred now as u_x) is easily found by integration, with the help of the no slip-velocity on walls ($u_x(1) = 0$, $u_x(1/\xi) = 1$) and of the conservation of the volume flow rate $q_v = \xi^{-2} u_w \pi d^2/4$. The solution reads

$$u_x(r) = \frac{\xi^2(\eta^2 - 1) - (1 + \xi^2) \log \eta}{(1 + \xi^2) \log \xi + 1 - \xi^2}, \quad \eta = \frac{2r}{d}, \quad 1 \leq \eta \leq 1/\xi \quad (\text{A } 2)$$

In addition the nondimensional pressure gradient and the Reynolds number $Re_{d/2}$ are related to ξ by:

$$\frac{\xi^2}{(1 + \xi^2) \log \xi + 1 - \xi^2} = \frac{1}{4} \frac{dp}{Re_{d/2} dx}$$

Obviously we can see that $Re_{d/2} = Re/2$. 29

Mesh ref.	Cells ($\times 10^6$)	n_c/D	n_{BL}	$\overline{C_d}$	σ_{C_d}	$\overline{C_\ell}$	σ_{C_ℓ}	St
A	0.28	17	5	6.8043	0.013311	0.26079	0.034502	0.28439
				0.46%	30%	2.3%	5.6%	0.77%
B	1.29	35	3	6.8075	0.010950	0.26237	0.032857	0.28603
				0.42%	6.8%	1.7%	2.0%	0.20%
C	1.55	35	5	6.8292	0.011845	0.26390	0.034830	0.28711
				0.01%	15%	1.2%	7.0%	0.18%
D	3.13	40	5	6.8381	0.011418	0.26505	0.034094	0.28813
				0.03%	11%	0.73%	5.8%	0.54%
E	8.46	70	3	6.8270	0.009906	0.26628	0.031337	0.28617
				0.13%	3.2%	0.27%	2.6%	0.14%
F	9.50	70	5	6.8381	0.010255	0.26700	0.032210	0.28659
				ref.	ref.	ref.	ref.	ref.

Table 3: Comparison of global quantities for different meshes ($Re = 175$, $a/A = 0.39$, $d/D = 0.625$, $L/d = 2$).

Appendix B. Mesh convergence for DNS simulations

Mesh convergences have been verified to trust the OpenFoam simulations. The output parameters of the convergence analysis are some global quantities as the time average and variance of respectively the drag coefficient ($\overline{C_d}$, σ_{C_d}) and the lift coefficient ($\overline{C_\ell}$, σ_{C_ℓ}). The last output quantity is a Strouhal number evaluated in the body wake. Three main parameters can qualify the mesh quality and they are described in the following.

The first parameter is the number of cells per blunt body diameter n_c/D . In this study it is chosen in $\{17, 35, 40, 70\}$. The refinement levels of the boundary layer developed along the body, referred as r_{BL} chosen in $\{3, 5\}$ is the second parameter. The last parameter is the resulting number of cells (automatically generated). During the mesh processing, the cells in contact with a solid wall are divided in layers tangentially to this wall to ensure a better capture of the boundary layer. The r_{BL} parameter is simply the number of layers defined by the user.

The lengths of the computational domain have been carefully chosen. The extent of each mesh in the streamwise direction is $82 \times d$. The distance between the inlet and the body nose is set to $20 \times d$ and the distance between the body rear and the outlet is $60 \times d$. The convergence study has been performed for a Reynolds number of $Re = 175$. The parameters and the values of the output quantities are reported in table 3. The mesh referred as F is the finest one and the results from simulations are considered as the reference. Relative errors to the data of the F mesh case are also added in the table. It can be seen that the values of the drag coefficient $\overline{C_d}$ and of the Strouhal St are converged for all meshes, even for the mesh A the coarsest one which could be assumed of bad quality. The relative errors are lower than 0.46% for C_d coefficients and lower than 0.77 for the nondimensional frequencies St . The lift coefficient C_ℓ can still be considered as converged for all meshes with a maximal relative error up to 2.3% for mesh

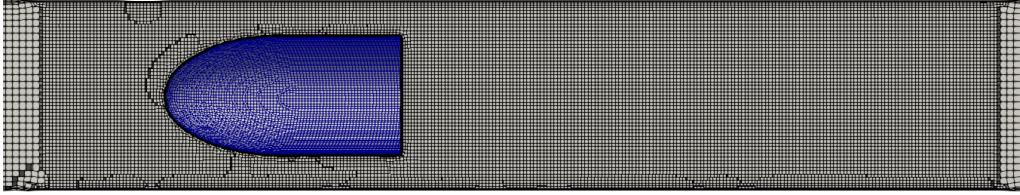


Figure 27: Axial cut-of of the mesh B

A. Nevertheless, it seems to be more difficult to reach convergence for the variances of C_d and C_ℓ . The explanation can be found in the very low level of these variances compare to the mean value of C_d and C_ℓ coefficients. It indicates a very low amplitude of the fluctuations and that numerically some large dense meshes are always required in such a case.

Finally, mesh B (see figure 27) has been selected for the main computations in this paper because it is the best compromise between numerical accuracy, mesh size and computer resources.

The variances are quite low respectively for the C_ℓ and C_d to 6.8% and 2%. For other Reynolds number Re than 175, the number of cells in the boundary layer has been kept by using the scaling given by the law relative to the laminar boundary layer thickness and the Reynolds number $\delta_{BL}/d \propto Re^{-1/2}$.

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