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## An introduction to cognitive planning

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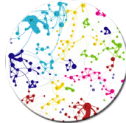
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# An introduction to cognitive planning

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# From classical and epistemic planning to cognitive planning

- Classical planning
  - Actions: ontic (physical) actions
  - Goal: modifying the environment
- Epistemic planning
  - Actions: epistemic/informative actions
  - Goal: affecting/changing the target's beliefs



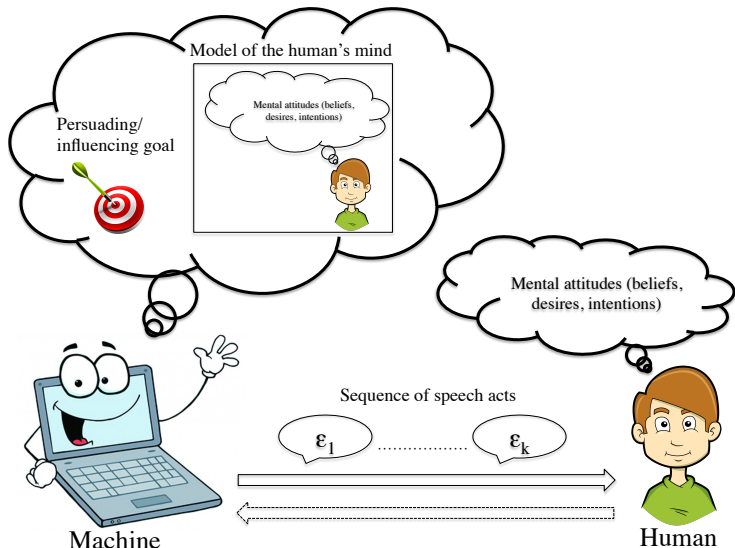
# From classical and epistemic planning to cognitive planning

## ■ Cognitive planning

- Actions: **speech acts**
- Goal: affecting/changing the target's **cognitive state**
  - beliefs
  - intentions and behaviour
- It relies on a theory of the influence of beliefs and desires (reasons) on intentions/behaviors
- ANR project CoPains (“Cognitive Planning in Persuasive Multimodal Communication”) 2019-2023:  
⇒ Webpage: <https://www.irit.fr/CoPains/>



# Cognitive planning





# Cognitive planning for persuasion and influence

**Influence:** inducing an agent to perform a certain action

- 1 by impeding or enabling some of her actions/choices (**regimentation**)
- 2 by changing her payoffs (**incentive/deterrent**)
- 3 by changing her beliefs (**persuasion**)



# Cognitive planning for persuasion and influence

## Example (Influence through regimentation)

The gouvernement influences people to vaccinate against covid-19 by precluding access to public spaces (restaurants, museums, etc) in the absence of a vaccination certificate.

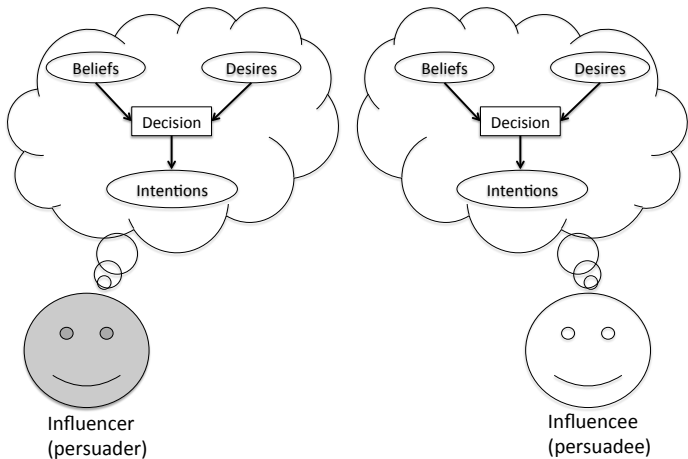
## Example (Influence through incentive creation)

The gouvernement influences people to buy electric cars by offering electric car tax cuts.

## Example (Influence through persuasion)

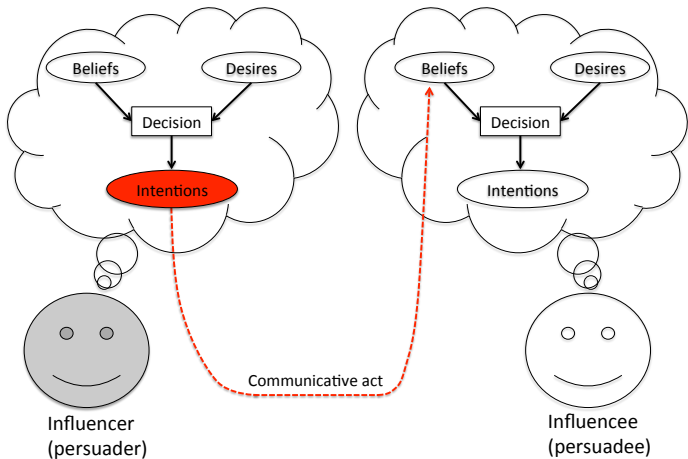
Bob influences Ann to not vaccinate against covid-19 by convincing her that covid-19 mRNA vaccines modify human DNA.

# Cognitive planning for persuasion and influence

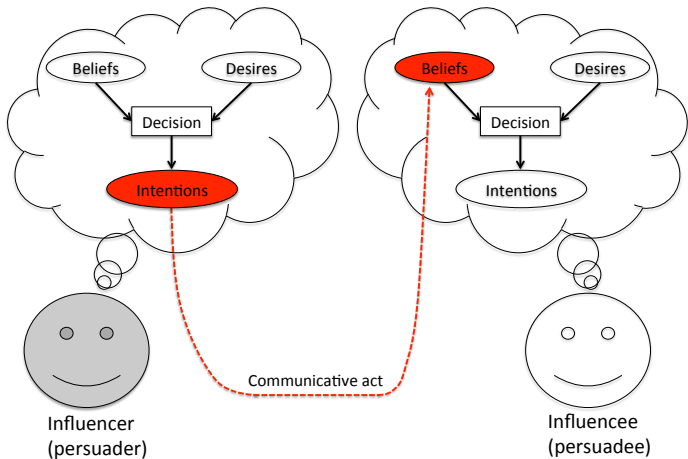




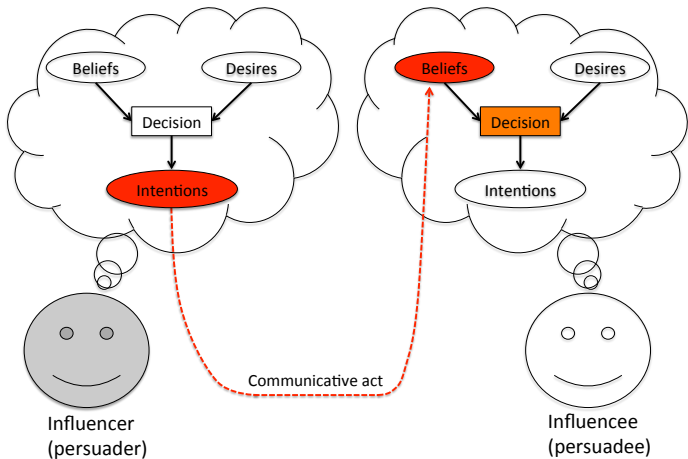
# Cognitive planning for persuasion and influence



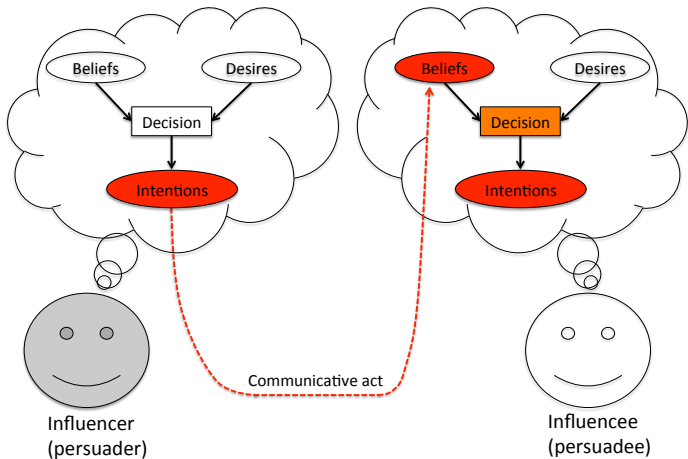
# Cognitive planning for persuasion and influence



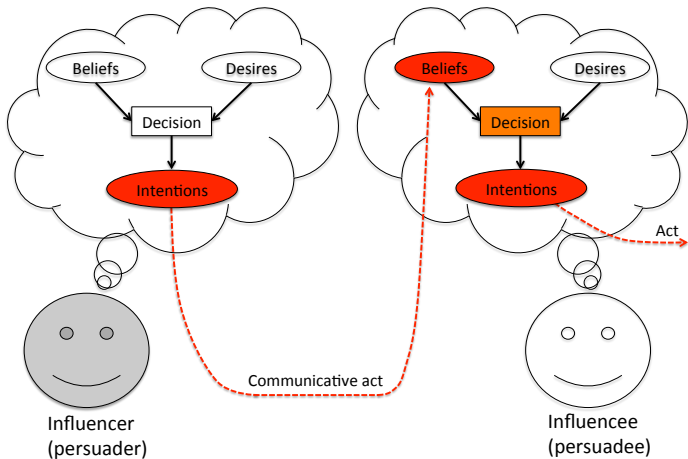
# Cognitive planning for persuasion and influence



# Cognitive planning for persuasion and influence



# Cognitive planning for persuasion and influence





# Plan of the tutorial

- **Part 1** Logical framework for cognitive planning
  - Epistemic logic with a semantics exploiting belief bases
  - NP-fragment and reduction to SAT
  - Formalization of cognitive planning problem and complexity results
- **Part 2** QBF encoding of cognitive planning and TouIST platform
- **Part 3** Implementation and applications
  - Recommender system for sport activities
  - Conversational agent for motivational interviewing
- **Part 4** Formal extension and implementation for collaborative gaming
  - Extended logical framework and encoding of the Yōkai game
  - General architecture and implementation

Language:

$$\begin{aligned}\mathcal{L}_0 : \alpha & ::= p \mid \neg\alpha \mid \alpha_1 \wedge \alpha_2 \mid \alpha_1 \vee \alpha_2 \mid \Delta_i\alpha, \\ \mathcal{L} : \varphi & ::= \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \Box_i\varphi \mid \Diamond_i\varphi,\end{aligned}$$

with  $p \in \text{Atm}$  and  $i \in \text{Agt}$

$\Delta_i\alpha$ : agent  $i$  **explicitly believes** that  $\alpha$

$\Box_i\varphi$ : agent  $i$  **implicitly believes** that  $\varphi$

- E. Lorini (2020). Rethinking Epistemic Logic with Belief Bases. *Artificial Intelligence*, 282.
- E. Lorini (2018). In Praise of Belief Bases: Doing Epistemic Logic without Possible Worlds. In *Proc. of AAIL-18*, AAAI Press, pp. 1915-1922.



## Definition (State)

A **state** is a tuple  $B = (B_1, \dots, B_n, V)$  where:

- for every  $i \in \text{Agt}$ ,  $B_i \subseteq \mathcal{L}_0$  is agent  $i$ 's belief base,
- $V \subseteq \text{Atm}$  is the actual environment.

The set of all states is denoted by  $S$ .



## Definition (Satisfaction relation)

Let  $B = (B_1, \dots, B_n, V) \in \mathbf{S}$ . Then:

$$B \models p \iff p \in V$$

$$B \models \neg\alpha \iff B \not\models \alpha$$

$$B \models \alpha_1 \wedge \alpha_2 \iff B \models \alpha_1 \text{ and } B \models \alpha_2$$

$$B \models \Delta_i\alpha \iff \alpha \in B_i$$

## Definition (Multi-agent belief model)

A **multi-agent belief model** (MAB) is a pair  $(B, Cxt)$ , where  $B \in \mathbf{S}$  and  $Cxt \subseteq \mathbf{S}$ . The class of MABs is denoted by  $\mathbf{M}$ .

## Definition (Epistemic alternatives)

Let  $B = (B_1, \dots, B_n, V), B' = (B'_1, \dots, B'_n, V') \in \mathbf{S}$ . Then,

$BR_i B'$  if and only if  $\forall \alpha \in B_i : B' \models \alpha$ .

## Definition (Satisfaction relation )

Let  $(B, Cxt) \in \mathbf{M}$ . Then:

$$(B, Cxt) \models \alpha \iff B \models \alpha$$

$$(B, Cxt) \models \Box_i \varphi \iff \forall B' \in Cxt : \text{if } BR_i B' \text{ then } (B', Cxt) \models \varphi$$

## Theorem

*Checking satisfiability of  $\mathcal{L}(Atm, Agt)$  formulas in the class  $\mathbf{M}$  is a PSPACE-hard problem.*

Single-reasoner fragment with  $\text{Agt} = \{m, h\}$ :

$$\mathcal{L}_{\text{Frag}} : \varphi ::= \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \Box_m \alpha \mid \Diamond_m \alpha$$

where  $\alpha$  ranges over  $\mathcal{L}_0$

$$\text{Recall: } \mathcal{L}_0 : \alpha ::= p \mid \neg\alpha \mid \alpha_1 \wedge \alpha_2 \mid \alpha_1 \vee \alpha_2 \mid \Delta_m \alpha \mid \Delta_h \alpha$$

Agent  $m$ : the 'machine'

Agent  $h$ : the 'human'



# Polynomial reduction to SAT

$$\mathcal{L}_{\text{Frag}} \xrightarrow{\text{nnf}} \mathcal{L}_{\text{Frag}}^{\text{NNF}} \xrightarrow{\text{tr}_1} \mathcal{L}_{\text{Mod}} \xrightarrow{\text{tr}_2} \mathcal{L}_{\text{Prop}}$$

Figure: Summary of reduction process

## Theorem

*Checking satisfiability of formulas in  $\mathcal{L}_{\text{Frag}}$  in the class  $\mathbb{M}$  is an NP-complete problem.*



# Dynamic extension

Dynamic language:

$$\mathcal{L}_{\text{Frag}}^+ : \varphi ::= \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \Box_m \alpha \mid \Diamond_m \alpha \mid [+_i \alpha] \varphi$$

$[+_i \alpha] \varphi$ :  $\varphi$  holds after agent  $i$  has privately expanded her belief base with  $\alpha$

## Definition (Satisfaction relation, cont.)

Let  $B = (B_1, \dots, B_n, V) \in \mathbf{S}$  and let  $(B, Cxt) \in \mathbf{M}$ . Then:

$$(B, Cxt) \models [+_i \alpha] \varphi \iff (B^{+_i \alpha}, Cxt) \models \varphi$$

with  $V^{+_i \alpha} = V$ ,  $B_i^{+_i \alpha} = B_i \cup \{\alpha\}$  and  $B_j^{+_i \alpha} = B_j$  for all  $j \neq i$ .



# Dynamic extension

The following equivalences are valid in the class M:

$$[+i\alpha]\alpha' \leftrightarrow \begin{cases} \top, & \text{if } \alpha' = \Delta_i\alpha, \\ \alpha', & \text{otherwise;} \end{cases}$$

$$[+i\alpha]\neg\varphi \leftrightarrow \neg[+i\alpha]\varphi;$$

$$[+i\alpha](\varphi_1 \wedge \varphi_2) \leftrightarrow [+i\alpha]\varphi_1 \wedge [+i\alpha]\varphi_2;$$

$$[+i\alpha](\varphi_1 \vee \varphi_2) \leftrightarrow [+i\alpha]\varphi_1 \vee [+i\alpha]\varphi_2;$$

$$[+i\alpha]\Box_m\alpha' \leftrightarrow \begin{cases} \Box_m(\alpha \rightarrow \alpha'), & \text{if } i = m, \\ \Box_m\alpha', & \text{otherwise;} \end{cases}$$

$$[+i\alpha]\Diamond_m\alpha' \leftrightarrow \begin{cases} \Diamond_m(\alpha \wedge \alpha'), & \text{if } i = m, \\ \Diamond_m\alpha', & \text{otherwise.} \end{cases}$$



# Dynamic extension

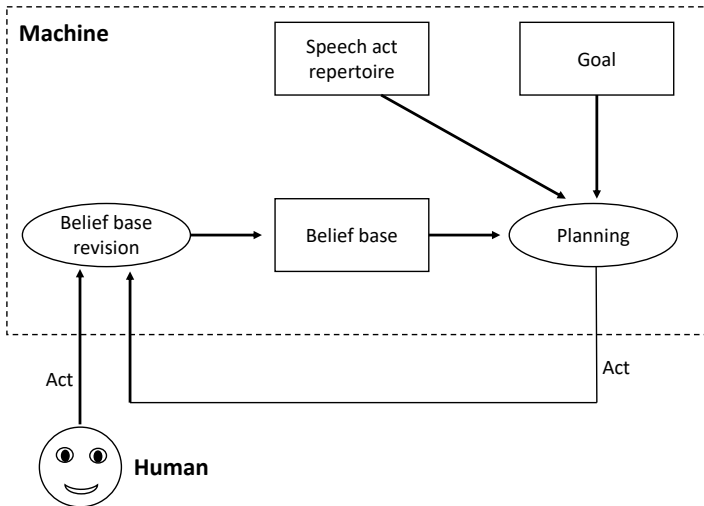
Polynomial reduction of satisfiability for  $\mathcal{L}_{\text{Frag}}^+$  to satisfiability for  $\mathcal{L}_{\text{Frag}}$  via the previous reduction axioms

## Theorem

*Checking satisfiability of formulas in  $\mathcal{L}_{\text{Frag}}^+$  in the class  $\mathbf{M}$  is an NP-complete problem.*



# General architecture





# General architecture

## ■ Interrogative phase

- Agent  $m$  gathers information about agent  $h$ 's cognitive state
- Sequence of questions by  $m$  to  $h$
- After  $h$ 's answer to  $m$ 's question,  $m$ 's expands/revises its beliefs

## ■ Informative phase

- Core of the persuasion/influence process
- Sequence of assertions by  $m$  aimed at modifying  $h$ 's cognitive state



# Cognitive planning problems

⇒ **Agent  $m$ 's set of informative actions:**

$$Act_m = \{+_m\alpha : \alpha \in \mathcal{L}_0\}$$

with elements  $\epsilon, \epsilon', \dots$

⇒ **Agent  $m$ 's set of yes-no questions:**

$$Que_m = \{?_{m,h}\alpha : \alpha \in \mathcal{L}_0\}$$

with elements  $\lambda, \lambda', \dots$



# Cognitive planning problems

⇒ **Answer function:**

$$\mathcal{A} : Que_m \longrightarrow 2^{Act_m}$$

with  $\mathcal{A}(?_{m,h}\alpha) = \{ +_m \Delta_h \alpha, +_m \neg \Delta_h \alpha \}$   
and default answer  $da(?_{m,h}\alpha) = +_m \Delta_h \alpha$

⇒ **Necessary consequence of a question:**

$$[\lambda]\varphi \stackrel{\text{def}}{=} \bigwedge_{\rho \in \mathcal{A}(\lambda)} [\rho]\varphi$$



# Cognitive planning problems

⇒ **Set of events:**  $Evt_m = Act_m \cup Que_m$  with elements  $\gamma, \gamma', \dots$

⇒ **Executability precondition function:**

$$\mathcal{P} : Evt_m \longrightarrow \mathcal{L}_{Frag}$$

⇒ **Necessary consequence of an event:**

$$\langle\langle \gamma \rangle\rangle \varphi \stackrel{\text{def}}{=} \mathcal{P}(\gamma) \wedge [\gamma] \varphi$$

$\langle\langle \gamma \rangle\rangle \varphi$  : event  $\gamma$  can take place and  $\varphi$  holds after its occurrence



# Cognitive planning problems

## Definition (Informative planning problem)

An interrogative planning problem is a tuple  $\langle \Sigma, Op_{\text{inf}}, \alpha_G \rangle$  where:

- $\Sigma \subset \mathcal{L}_0$  is a finite set of agent  $m$ 's available information,
- $Op_{\text{inf}} \subset Act_m$  is a finite set of agent  $m$ 's informative actions,
- $\alpha_G \in \mathcal{L}_0$  is agent  $m$ 's goal.

A **solution plan** to the informative planning problem is a sequence of informative actions  $\epsilon_1, \dots, \epsilon_k$  from  $Op_{\text{inf}}$  for some  $k$  such that

$$\Sigma \models_{\mathbf{M}} \langle\langle \epsilon_1 \rangle\rangle \dots \langle\langle \epsilon_k \rangle\rangle \Box_m \alpha_G$$



# Cognitive planning problems

## Definition (Interrogative planning problem)

An interrogative planning problem is a tuple

$\langle \Sigma, Op_{inf}, Op_{quest}, \alpha_G \rangle$  where:

- $\Sigma \subset \mathcal{L}_0$  is a finite set of agent  $m$ 's available information,
- $Op_{inf} \subset Act_m$  is a finite set of agent  $m$ 's informative actions,
- $Op_{quest} \subset Que_m$  is a finite set of agent  $m$ 's questions,
- $\alpha_G \in \mathcal{L}_0$  is agent  $m$ 's goal.



# Cognitive planning problems

A **strong solution plan** to the interrogative planning problem is a sequence of questions  $\lambda_1, \dots, \lambda_m$  from  $Op_{\text{quest}}$  such that

$$\Sigma \models_{\mathbf{M}} \langle\langle \lambda_1 \rangle\rangle \dots \langle\langle \lambda_m \rangle\rangle \top$$

and  $\forall \rho_1 \in \mathcal{A}(\lambda_1), \dots, \forall \rho_m \in \mathcal{A}(\lambda_m), \exists \tau_1, \dots, \tau_k \in Op_{\text{inf}}$  such that

$$\Sigma \models_{\mathbf{M}} [\rho_1] \dots [\rho_m] \langle\langle \tau_1 \rangle\rangle \dots \langle\langle \tau_k \rangle\rangle \Box_{\mathbf{m}} \alpha_G$$





# Cognitive planning problems

A **weak solution plan** to the interrogative planning problem is a sequence of questions  $\lambda_1, \dots, \lambda_m$  from  $Op_{\text{quest}}$  such that

$$\Sigma \models_{\mathbf{M}} \langle\langle \lambda_1 \rangle\rangle \dots \langle\langle \lambda_m \rangle\rangle \top$$

and  $\exists \tau_1, \dots, \tau_k \in Op_{\text{inf}}$  such that

$$\Sigma \models_{\mathbf{M}} [da(\lambda_1)] \dots [da(\lambda_m)] \langle\langle \tau_1 \rangle\rangle \dots \langle\langle \tau_k \rangle\rangle \Box_m \alpha_G$$



# Cognitive planning problems

Problems:

- ES-INF: checking existence of a solution for an informative planning problem
- EWS-INT: checking existence of a weak solution for an interrogative planning problem
- ESS-INT: checking existence of a strong solution for an interrogative planning problem

## Theorem

*The ES-INF problem and EWS-INT problem are  $NP^{NP} = \Sigma_2^P$ -complete.*



# Belief revision

- Propositional language  $\mathcal{L}_{\text{PROP}}$  built from the following set of atomic formulas:

$$\text{Atm}^+ = \text{Atm} \cup \{p_{\Delta_i\alpha} : \Delta_i\alpha \in \mathcal{L}_0\}$$

- Translation  $tr_{\text{PROP}} : \mathcal{L}_0 \rightarrow \mathcal{L}_{\text{PROP}}$ :

$$tr_{\text{PROP}}(p) = p$$

$$tr_{\text{PROP}}(\neg\alpha) = \neg tr_{\text{PROP}}(\alpha)$$

$$tr_{\text{PROP}}(\alpha_1 \wedge \alpha_2) = tr_{\text{PROP}}(\alpha_1) \wedge tr_{\text{PROP}}(\alpha_2)$$

$$tr_{\text{PROP}}(\Delta_i\alpha) = p_{\Delta_i\alpha}$$

- For  $X \subseteq \mathcal{L}_0$ ,  $tr_{\text{PROP}}(X) = \{tr_{\text{PROP}}(\alpha) : \alpha \in X\}$
- $X$  is propositionally consistent if and only if  $\perp \notin Cn(tr_{\text{PROP}}(X))$



# Belief revision

- Agent  $m$ 's core (or, immutable) information set  $\Sigma_{core} \subseteq \mathcal{L}_0$
- Agent  $m$ 's volatile (or, mutable) information set  $\Sigma_{mut} \subseteq \mathcal{L}_0$
- Agent  $m$ 's input set  $\Sigma_{input} \subseteq \mathcal{L}_0$



# Belief revision

Revision of  $(\Sigma_{core}, \Sigma_{mut})$  by  $\Sigma_{input}$ :

- 1 if  $\Sigma_{core} \cup \Sigma_{input}$  is not propositionally consistent then  
 $Rev(\Sigma_{core}, \Sigma_{mut}, \Sigma_{input}) = (\Sigma_{core}, \Sigma_{mut})$
- 2 otherwise,  $Rev(\Sigma_{core}, \Sigma_{mut}, \Sigma_{input}) = (\Sigma'_{core}, \Sigma'_{mut})$  with

$$\Sigma'_{core} = \Sigma_{core}$$

$$\Sigma'_{mut} = \bigcap_{X \in MCS(\Sigma_{core}, \Sigma_{mut}, \Sigma_{input})} X$$

where  $X \in MCS(\Sigma_{core}, \Sigma_{mut}, \Sigma_{input})$  if and only if:

- $X \subseteq \Sigma_{mut} \cup \Sigma_{input}$
- $\Sigma_{input} \subseteq X$
- $X \cup \Sigma_{core}$  is propositionally consistent
- there is no  $X' \subseteq \Sigma_{mut} \cup \Sigma_{input}$  such that  $X \subset X'$  and  $X' \cup \Sigma_{core}$  is propositionally consistent



# Plan of the tutorial

- 1 Logical framework for cognitive planning (Emiliano)
- 2 **QBF encoding of cognitive planning and TouIST platform (Frédéric)**  
**NOW!**
  - $\exists\forall$  optimal encoding of simple cognitive planning
  - TouIST: Toulouse Integrated Satisfiability Tool
- 3 Implementation and applications (Jorge)
- 4 Formal extension and implementation for collaborative gaming (Dominique & Jorge)





# Part 2 - QBF encoding of cognitive planning and TouIST platform

≡ optimal encoding



## ES-INF planning problem (recall)

A **solution plan** to an ES-INF planning problem  $\langle \Sigma, Op, \alpha_G \rangle$  is a sequence of operators  $\epsilon_1, \dots, \epsilon_k$  from  $Op$  such that  $\Sigma \models_{\mathbf{M}} \langle\langle \epsilon_1 \rangle\rangle \dots \langle\langle \epsilon_k \rangle\rangle \Box_m \alpha_G$

### Theorem

*An ES-INF planning problem  $\langle \Sigma, Op, \alpha_G \rangle$  has a solution plan if and only if it has a poly-size solution plan  $\epsilon_1, \dots, \epsilon_k$  with  $k \leq |Op|$  and  $\epsilon_i \neq \epsilon_j$  for all  $i < j$ .*

### Theorem

*Checking plan existence for an ES-INF planning problem is  $\Sigma_2^P$ -complete.*





# $\exists\forall$ encoding of ES-INF planning problem $\langle \Sigma, Op, \alpha_G \rangle$

## Theorem

Let  $\varphi \in \mathcal{L}_{\text{Frag}}^+$  and let  $\Sigma \subset \mathcal{L}_0$  be finite.

Then,  $\Sigma \models_{\text{M}} \varphi$  if and only if  $\models_{\text{M}} \bigwedge_{\alpha \in \Sigma} \Box_{\text{m}} \alpha \rightarrow \varphi$ .

A plan candidate  $\epsilon_1, \dots, \epsilon_k$  is a solution plan for  $\langle \Sigma, Op, \alpha_G \rangle$  if and only if the following formula is valid:

$$\left( \bigwedge_{\alpha \in \Sigma} \Box_{\text{m}} \alpha \right) \rightarrow \langle\langle \epsilon_1 \rangle\rangle \dots \langle\langle \epsilon_{k-1} \rangle\rangle \langle\langle \epsilon_k \rangle\rangle \Box_{\text{m}} \alpha_G$$

which can be also written as:

$$\left( \bigwedge_{\alpha \in \Sigma} \Box_{\text{m}} \alpha \right) \rightarrow \left( \bigwedge_{i \in \{1, \dots, k\}} [+_{\text{m}} \alpha_{\epsilon_1}] \dots [+_{\text{m}} \alpha_{\epsilon_{i-1}}] \mathcal{P}(\epsilon_i) \right) \wedge [+_{\text{m}} \alpha_{\epsilon_1}] \dots [+_{\text{m}} \alpha_{\epsilon_{k-1}}] [+_{\text{m}} \alpha_{\epsilon_k}] \Box_{\text{m}} \alpha_G$$



# $\exists\forall$ encoding of ES-INF planning problem $\langle \Sigma, Op, \alpha_G \rangle$

- Global selector variables  $V_s = \{s_{\epsilon \preceq \epsilon'} : \epsilon, \epsilon' \in Op\} \subseteq Atm$
- $\varphi_{V_s}$  denotes the conjunction of the following axioms:

$$\bigwedge_{\epsilon \in Op} \left( \neg s_{\epsilon \preceq \epsilon} \rightarrow \bigwedge_{\substack{\epsilon' \in Op \\ \epsilon \neq \epsilon'}} \neg s_{\epsilon \preceq \epsilon'} \wedge \neg s_{\epsilon' \preceq \epsilon} \right) \quad (S1)$$

$$\bigwedge_{\epsilon \in Op} \bigwedge_{\substack{\epsilon' \in Op \\ \epsilon \neq \epsilon'}} \left( \neg s_{\epsilon \preceq \epsilon'} \vee \neg s_{\epsilon' \preceq \epsilon} \right) \quad (S2)$$

$$\bigwedge_{\epsilon \in Op} \bigwedge_{\substack{\epsilon' \in Op \\ \epsilon \neq \epsilon'}} \bigwedge_{\substack{\epsilon'' \in Op \\ \epsilon \neq \epsilon'' \\ \epsilon' \neq \epsilon''}} \left( s_{\epsilon \preceq \epsilon'} \wedge s_{\epsilon' \preceq \epsilon''} \rightarrow s_{\epsilon \preceq \epsilon''} \right) \quad (S3)$$



# $\exists\forall$ encoding of ES-INF planning problem $\langle \Sigma, Op, \alpha_G \rangle$

- $\varphi_{V_s}$  denotes the conjunction of the following axioms:

$$\bigwedge_{\epsilon \in Op} \bigwedge_{\substack{\epsilon' \in Op \\ \epsilon \neq \epsilon'}} \left( s_{\epsilon \preceq \epsilon} \wedge s_{\epsilon' \preceq \epsilon'} \rightarrow s_{\epsilon \preceq \epsilon'} \vee s_{\epsilon' \preceq \epsilon} \right) \quad (S4)$$

$$\bigwedge_{\epsilon \in Op} \bigwedge_{\epsilon' \in Op} \left( s_{\epsilon \preceq \epsilon'} \leftrightarrow \square_m s_{\epsilon \preceq \epsilon'} \right) \quad (S5)$$





## Dynamic extension (recall)

The following equivalences are valid in the class  $\mathbf{M}$ :

$$[+i\alpha]\alpha' \leftrightarrow \begin{cases} \top, & \text{if } \alpha' = \Delta_i\alpha, \\ \alpha', & \text{otherwise;} \end{cases}$$

$$[+i\alpha]\neg\varphi \leftrightarrow \neg[+i\alpha]\varphi;$$

$$[+i\alpha](\varphi_1 \wedge \varphi_2) \leftrightarrow [+i\alpha]\varphi_1 \wedge [+i\alpha]\varphi_2;$$

$$[+i\alpha](\varphi_1 \vee \varphi_2) \leftrightarrow [+i\alpha]\varphi_1 \vee [+i\alpha]\varphi_2;$$

$$[+i\alpha]\Box_m \alpha' \leftrightarrow \begin{cases} \Box_m(\alpha \rightarrow \alpha'), & \text{if } i = m, \\ \Box_m \alpha', & \text{otherwise;} \end{cases}$$

$$[+i\alpha]\Diamond_m \alpha' \leftrightarrow \begin{cases} \Diamond_m(\alpha \wedge \alpha'), & \text{if } i = m, \\ \Diamond_m \alpha', & \text{otherwise.} \end{cases}$$



# $\exists \forall$ encoding of ES-INF planning problem

- For  $\epsilon \in Op$ , we define the function  $\Pi_{\preceq \epsilon} : \mathcal{L}_{\text{Frag}} \longrightarrow \mathcal{L}_{\text{Frag}}$ .

$$\Pi_{\preceq \epsilon}(p) = p,$$

$$\Pi_{\preceq \epsilon}(\alpha) = \begin{cases} s_{\epsilon' \preceq \epsilon} \vee \Delta_m \alpha_{\epsilon'}, & \text{if } \exists \epsilon' \neq \epsilon : \alpha = \Delta_m \alpha_{\epsilon'}, \\ \alpha, & \text{otherwise;} \end{cases}$$

$$\Pi_{\preceq \epsilon}(\neg \varphi) = \neg \Pi_{\preceq \epsilon}(\varphi),$$

$$\Pi_{\preceq \epsilon}(\varphi_1 \wedge \varphi_2) = \Pi_{\preceq \epsilon}(\varphi_1) \wedge \Pi_{\preceq \epsilon}(\varphi_2),$$

$$\Pi_{\preceq \epsilon}(\varphi_1 \vee \varphi_2) = \Pi_{\preceq \epsilon}(\varphi_1) \vee \Pi_{\preceq \epsilon}(\varphi_2),$$

$$\Pi_{\preceq \epsilon}(\Box_m \alpha) = \Box_m \left( \left( \bigvee_{\substack{\epsilon' \in Op \\ \epsilon \neq \epsilon'}} s_{\epsilon' \preceq \epsilon} \wedge \neg \alpha_{\epsilon'} \right) \vee \alpha \right)$$

$$\Pi_{\preceq \epsilon}(\Diamond_m \alpha) = \Diamond_m \left( \bigwedge_{\substack{\epsilon' \in Op \\ \epsilon \neq \epsilon'}} (\neg s_{\epsilon' \preceq \epsilon} \vee \alpha_{\epsilon'}) \wedge \alpha \right)$$



## $\exists\forall$ encoding of ES-INF planning problem

Let  $F_{\langle \Sigma, Op, \alpha_G \rangle}$  be the reduction in  $\mathcal{L}_{Prop}$  of the following formula of  $\mathcal{L}_{Frag}$ :

$$\varphi_{V_s} \wedge \left( \left( \bigwedge_{\alpha \in \Sigma} \square_m \alpha \right) \rightarrow \bigwedge_{\epsilon \in Op} (s_{\epsilon \preceq \epsilon} \rightarrow \Pi_{\preceq \epsilon} (\mathcal{P}(\epsilon))) \wedge \varphi_P \right)$$

### Theorem

The ES-INF planning problem  $\langle \Sigma, Op, \alpha_G \rangle$  has a solution plan iff the following quantified boolean formula is true:

$$Q = \exists_{s_{\epsilon \preceq \epsilon'} \in V_s} \quad \forall_{p \in \text{Vars}(F_{\langle \Sigma, Op, \alpha_G \rangle}) \setminus V_s} \quad F_{\langle \Sigma, Op, \alpha_G \rangle}$$



# Part 2 - QBF encoding of cognitive planning and TouIST platform

TouIST Platform



# Motivation for a new tool

## Remark 1: existing languages are not intuitive

- ⊕ SAT solvers are very efficient
- ⊖ but with a “low level” input language

Formula	Solver input (DIMACS)
$a \rightarrow$	<code>p cnf 5 4</code>
$b \vee \neg(a \rightarrow c)$	<code>-1 5 3 0</code>
$\vee (c \rightarrow \neg a)$	<code>-1 -5 4 1 0</code>
	<code>-1 -5 4 -2 0</code>
	<code>-1 -3 -2 -1 0</code>





# Motivation for a new tool

## Remark 2: existing languages are not compact

- ⊕ SMT-LIB is much more expressive than DIMACS
- ⊖ but with a lack of compactity

Formula

$$\bigwedge_{i \in [1..1000]} x_i$$

SMT-LIB example

```
(assert
  (and x1
    (and x2
      (and x3
        (and x4
          (and x5
            (and x6
              ... x1000))))))))
```



# Motivation for a new tool

## Technical goal:

- ⊕ an intuitive language for modeling
- ⊕ a user-friendly graphical interface

## Academic goal:

- ⊕ teaching of logic
- ⊕ research (comparison of solvers, encodings)





# Principle of use for reasoning on a little example

## Example of the politician

If growth increases, then unemployment decreases ;

Now, growth decreases.

Then unemployment increases !

## Problem

Is the conclusion justified ?

$\{\text{growth} \rightarrow \neg\text{unemployment}, \neg\text{growth}\} \models \text{unemployment}$





# Principle of use for reasoning on a little example

$$\{\text{growth} \rightarrow \neg\text{unemployment}, \neg\text{growth}\} \models \text{unemployment}$$

The screenshot shows the Touist (english) v3.5.2 interface. On the left, a 'Connectors' panel lists logical symbols:  $a \wedge b$ ,  $a \vee b$ ,  $\neg a$ ,  $a \oplus b$ ,  $a \Rightarrow b$ , and  $a \Leftrightarrow b$ . The main editor contains a list of four lines:

- 1 growth  $\Rightarrow$  not unemployment
- 2 not growth
- 3 not unemployment
- 4

The right pane displays the parsed formula in a more readable format:

$$\begin{aligned} & \text{growth} \Rightarrow \neg\text{unemployment} \\ & \neg\text{growth} \\ & \neg\text{unemployment} \end{aligned}$$

At the bottom, there is a status bar showing '4:1', a 'Solve' button with a gear icon, and a dropdown menu currently set to 'SAT'.



# Principle of use for reasoning on a little example

Touist (english) v3.5.1

Results  Tr...  False  Others

Name ▲	Value
growth	False
unemployment	False

A model is found, then the reasoning **is not valid !**

$\{\text{growth} \rightarrow \neg\text{unemployment}, \neg\text{growth}\} \not\models \text{unemployment}$



## Implementation aspects



TouIST (Toulouse Integrated Satisfiability Tool)

Graphical interface for writing logical formulas  
in a **compact language** and using different **external solvers**.

- under MIT license, sources : [github.com/touist/touist](https://github.com/touist/touist)
- graphical interface (**Java**)
- translation to solver languages by a compiler (**OCaml**)
- compatible with **SAT, SMT and QBF**





# Implementation aspects

Compatible solvers :

Solver	Formula	ToulST input language
SAT	$a \wedge b \rightarrow c$	a and b => c
SMT (QF-LIA)	$(x + y - z \geq 4) \vee b$	(x + y - z >= 4) or a
SMT (QF-LRA)	$(x + y - z > 4.0) \rightarrow c$	(x + y - z > 4.0) => c
SMT (QF-IDL)	$(x - y > 2) \wedge c$	(x - y > 2) and c
SMT (QF-RDL)	$(x - y > 1.0) \wedge c$	(x - y > 1.0) and c
QBF	$\forall b. a \wedge b$	forall b: a and b

QF-LIA = Quantifier-Free Linear Integer Arithmetic

QF-LRA = Quantifier-Free Linear Rational Arithmetic

QF-IDL = Quantifier-Free Integer Difference Logic

QF-RDL = Quantifier-Free Rational Difference Logic



# Implementation aspects

ToulST input language :

## ■ variables and sets

```
$N = 5  
$Rows = [1..$N]  
$Countries = [France, Italy, TheNetherlands]
```

## ■ generalized connectors

```
in(France) and in(Italy) and in(TheNetherlands)
```

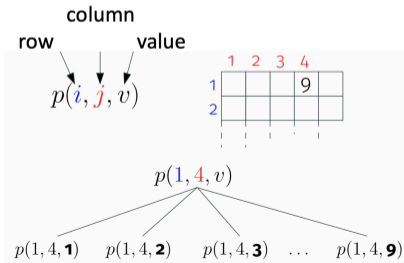
```
bigand $c in $Countries:  
  in($c)  
end
```





# Solving Puzzles with TouIST (SAT)

	2			4	8	
1			8	2	3	7
			9	3	2	1
	4	8				1
			6	1	8	
7					9	3
	1	5	9	4		
4	2	3		6		9
3	7					5



```
Touist (english) v3.5.2 - /Users/maris/ownCloud/touist/sudoku2019_9x9.touist
Co
22 ;; at least one value per cell
23 bigand $i in [1..$N]:
24   bigand $j in [1..$N]:
25     bigor $k in [1..$N]:
26       p($i, $j, $k)
27     end
28   end
29 end
30
31 ;; at most one value per cell
32 bigand $i in [1..$N]:
33   bigand $j in [1..$N]:
34     bigand $k1 in [1..$N]:
35       bigand $k2 in [1..$N] when $k1=$k2:
36         not p($i, $j, $k1) or not p($i, $j, $k2)
37       end
38     end
39   end
40 end
41
42 ;; at most once the same value per row
43 bigand $i in [1..$N]:
44   bigand $j1 in [1..$N]:
45     bigand $j2 in [1..$N] when $j1=$j2:
46       bigand $k in [1..$N]:
47         not p($i, $j1, $k) or not p($i, $j2, $k)
48       end
49     end
50   end
51 end
52
53 Co
54 ;; at most once the same value per column
55 bigand $i1 in [1..$N]:
56   bigand $i2 in [1..$N] when $i1=$i2:
57     bigand $j in [1..$N]:
58       p($j, $i1, $j) or not p($j, $i2, $j)
59     end
60   end
61 end

X0,6 ← []
X0,7 ← [3]
X0,8 ← [5]
X0,9 ← []

∧_{i∈[1..N]} ∧_{j∈[1..N]} ∧_{k∈x_{i,j}} p_{i,j,k}
∧_{i∈[1..N]} ∧_{j∈[1..N]} ∨_{k∈[1..N]} p_{i,j,k}
∧_{i∈[1..N]} ∧_{j∈[1..N]} ∧_{k1∈[1..N]} ∧_{k2∈[1..N]} (¬p_{i,j,k1} ∨ k1≠k2)
∧_{i∈[1..N]} ∧_{j1∈[1..N]} ∧_{j2∈[1..N]} ∧_{k∈[1..N]} (¬p_{i,j1,k} ∨ j1≠j2)
∧_{i1∈[1..N]} ∧_{i2∈[1..N]} ∧_{j∈[1..N]} ∧_{k∈[1..N]} (¬p_{i1,j,k} ∨ i1≠i2)
∧_{i1∈[0..R-1]} ∧_{i2∈[0..R-1]} ∧_{j∈[1..R]} ∧_{k∈[1..R]} (¬p_{i1,j,k} ∨ i1≠i2)
```



# Solving Puzzles with TouIST (SMT)

	1		0
		0	
	0		
1	1		0

0	1	1	0
1	0	0	1
0	0	1	1
1	1	0	0

A compact SMT encoding  
with atoms of QF\_LIA  
(linear arithmetic on integers)

```

26 ;; There is an equal number of 1s and 0s in each row
27 bigand $i in $N:
28   (x($i,1)+x($i,2)+x($i,3)+x($i,4)=2)
29   and (x(1,$i)+x(2,$i)+x(3,$i)+x(4,$i)=2)
30 end
31
32 ;; There is no more than two of either number adjacent
33 bigand $i,$j in [1..$NR-2],[1..$NC]:
34   x($i,$j)!=x($i+1,$j) or x($i+1,$j)!=x($i+2,$j)
35 end
36 bigand $i,$j in [1..$NR],[1..$NC-2]:
37   x($j,$i)!=x($j,$i+1) or x($j,$i+1)!=x($j,$i+2)
38 end
39
40 ;; There can be no identical rows or columns
41 bigand $i,$j in $N,$N when $i!=$j:
42   bigor $k in $N:
43     (x($i,$k)!=x($j,$k))
44   end
45   and
46   bigor $k in $N:
47     (x($k,$i)!=x($k,$j))
48   end
49 end
50
51

```

Generated formula:

$$\begin{aligned}
& \bigwedge_{i,j \in \{1..4\}, [1..4], x_{ij}} (x_{ij} = \text{value}) \\
& \bigwedge_{i,j \in \mathbb{N}, \mathbb{N}} ((x_{ij} = 0) \vee (x_{ij} = 1)) \\
& \bigwedge_{i,j \in \mathbb{N}, \mathbb{N}} ((x_{i,1} + x_{i,2} + x_{i,3} + x_{i,4} = 2)) \\
& \bigwedge_{i,j \in [1..NR-2], [1..NC]} (x_{ij} \neq x_{i+1,j} \vee x_{i+1,j} \neq x_{i+2,j}) \\
& \bigwedge_{i,j \in [1..NR], [1..NC-2]} (x_{j,i} \neq x_{j,i+1} \vee x_{j,i+1} \neq x_{j,i+2}) \\
& \bigwedge_{i,j \in \mathbb{N}, \mathbb{N}} \left( \bigvee_{k \in \mathbb{N}} (x_{i,k} \neq x_{j,k}) \wedge \bigvee_{k \in \mathbb{N}} (x_{k,i} \neq x_{k,j}) \right)
\end{aligned}$$



# Solving an ES-INF planning problem with TouIST

- **SAT solver:** to verify the validity of a plan candidate

$$\left( \bigwedge_{\alpha \in \Sigma} \Box_m \alpha \right) \rightarrow \langle\langle \epsilon_1 \rangle\rangle \dots \langle\langle \epsilon_{k-1} \rangle\rangle \langle\langle \epsilon_k \rangle\rangle \Box_m \alpha_G$$

$$\mathcal{L}_{\text{Frag}} \xrightarrow{\text{nnf}} \mathcal{L}_{\text{Frag}}^{\text{NNF}} \xrightarrow{\text{tr}_1} \mathcal{L}_{\text{Mod}} \xrightarrow{\text{tr}_2} \mathcal{L}_{\text{Prop}}$$

- **QBF solver:** to extract a solution plan from  $\exists \forall$  encoding

$$Q = \exists_{s_{\epsilon \preceq \epsilon'} \in V_s} s_{\epsilon \preceq \epsilon'} \quad \forall_{p \in \text{Vars}(F_{\langle \Sigma, Op, \alpha_G \rangle}) \setminus V_s} p \quad F_{\langle \Sigma, Op, \alpha_G \rangle}$$



# Plan of the tutorial

- 1 Logical framework for cognitive planning (Emiliano)
- 2 QBF encoding of cognitive planning and ToulST platform (Frédéric)
- 3 **Implementation and applications (Jorge) NOW!**
  - Recommender system for sport activities
  - Conversational agent for motivational interviewing
- 4 Formal extension and implementation for collaborative gaming (Dominique & Jorge)



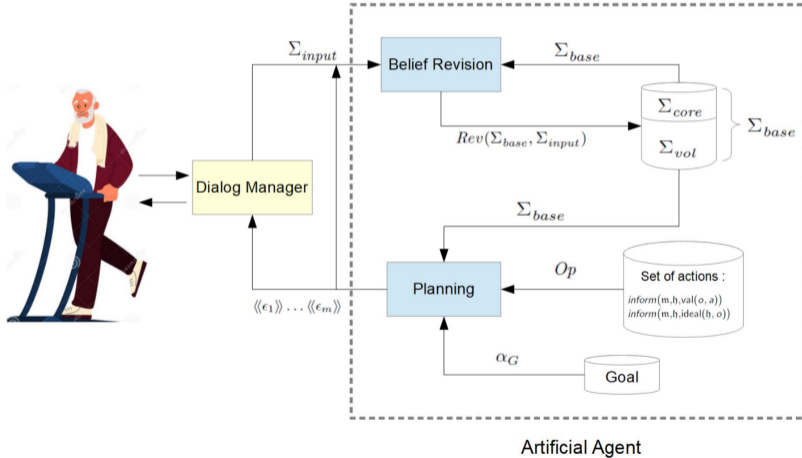


# Part 3 - Implementation and applications

Introduction



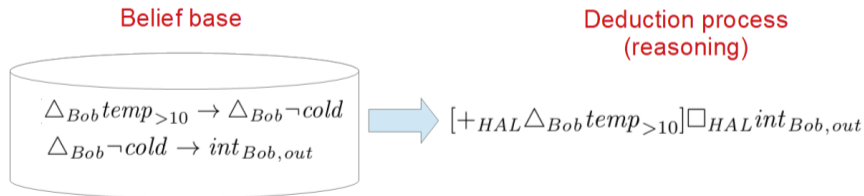
# System Architecture





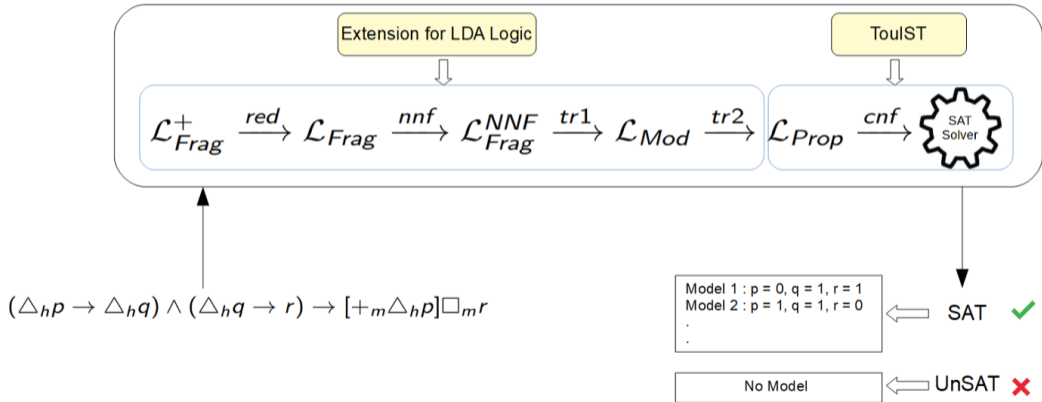
# Deduction process

We consider a conversational agent that has to interact with a human user in order to support her activity and to take care of her well-being. Specifically, HAL is an artificial companion which takes care of an elderly person called Bob and keeps him company. Bob has to do regular physical activity to be in good health.





# Checking satisfiability of formulas in $\mathcal{L}_{Frag}$







# Part 3 - Implementation and applications

Recommender system for sport activities



## Example 1: Artificial assistant

- Agent  $h$ : human user who has to choose a sport to practice
- Agent  $m$ : artificial assistant
- Agent  $m$ 's **goal**: agent  $h$  forms the intention to practice a sport
- **Solution plan**: sequence of speech acts by  $m$





## Example 1: Artificial assistant(cont.)

<i>Opt</i>	<i>Var</i>					
	<b>env</b>	<b>loc</b>	<b>soc</b>	<b>cost</b>	<b>dan</b>	<b>intens</b>
sw	<i>water</i>	<i>mixed</i>	<i>single</i>	<i>med</i>	<i>low</i>	<i>high</i>
ru	<i>land</i>	<i>outdoor</i>	<i>single</i>	<i>low</i>	<i>med</i>	<i>high</i>
hr	<i>land</i>	<i>outdoor</i>	<i>single</i>	<i>high</i>	<i>high</i>	<i>low</i>
te	<i>land</i>	<i>mixed</i>	<i>mixed</i>	<i>high</i>	<i>med</i>	<i>med</i>
so	<i>land</i>	<i>mixed</i>	<i>team</i>	<i>med</i>	<i>med</i>	<i>med</i>
yo	<i>land</i>	<i>mixed</i>	<i>single</i>	<i>med</i>	<i>low</i>	<i>low</i>
di	<i>water</i>	<i>mixed</i>	<i>single</i>	<i>high</i>	<i>high</i>	<i>low</i>
sq	<i>land</i>	<i>indoor</i>	<i>mixed</i>	<i>high</i>	<i>med</i>	<i>med</i>

**Table:** For every option  $o \in Opt$  and variable  $x \in Var$ , we denote by  $v_{o,x}$  the corresponding entry in the table.



# Example 1: Artificial assistant(cont.)

$$\alpha_1 \stackrel{\text{def}}{=} \bigwedge_{\substack{o \in \text{Opt} \\ x \in \text{Var} \\ v, v' \in \text{Val}_x: v \neq v'}} (\text{val}(o, x \mapsto v) \rightarrow \neg \text{val}(o, x \mapsto v')),$$

$$\alpha_2 \stackrel{\text{def}}{=} \bigwedge_{\substack{o \in \text{Opt} \\ x \in \text{Var} \\ v, v' \in \text{Val}_x: v \neq v'}} (\Delta_{\mathfrak{h}} \text{val}(o, x \mapsto v) \rightarrow \Delta_{\mathfrak{h}} \neg \text{val}(o, x \mapsto v')),$$

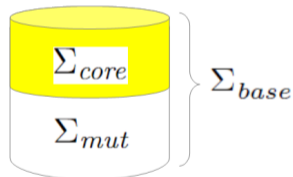
$$\alpha_3 \stackrel{\text{def}}{=} \bigwedge_{\Gamma, \Gamma' \in 2^{\text{Des}^*}: \Gamma \neq \Gamma'} (\text{des}(\mathfrak{h}, \Gamma) \rightarrow \neg \text{des}(\mathfrak{h}, \Gamma')),$$

$$\alpha_4 \stackrel{\text{def}}{=} \bigvee_{\Gamma \in 2^{\text{Des}^*}} \text{des}(\mathfrak{h}, \Gamma),$$

$$\alpha_5 \stackrel{\text{def}}{=} \bigwedge_{o \in \text{Opt}} (\text{ideal}(\mathfrak{h}, o) \leftrightarrow \bigvee_{\Gamma \in 2^{\text{Des}^*}} (\text{des}(\mathfrak{h}, \Gamma) \wedge \bigwedge_{\gamma \in \Gamma} f_{\text{comp}}(o, \gamma))),$$

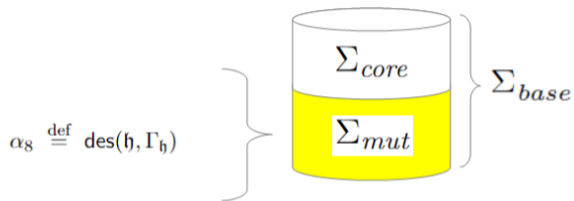
$$\alpha_6 \stackrel{\text{def}}{=} \bigwedge_{o \in \text{Opt}} (\text{justif}(\mathfrak{h}, o) \leftrightarrow \bigvee_{\Gamma \in 2^{\text{Des}^*}} (\text{des}(\mathfrak{h}, \Gamma) \wedge \bigwedge_{\gamma \in \Gamma} f_{\text{comp}}^{\mathfrak{h}}(o, \gamma))).$$

$$\alpha_7^{o,x} \stackrel{\text{def}}{=} \text{val}(o, x \mapsto v_{o,x}).$$





## Example 1: Artificial assistant(cont.)



$$\Gamma_h = \{ \text{env} \mapsto \text{land}, \text{intens} \mapsto \text{med}, \sim \text{loc} \mapsto \text{indoor}, \\ [\text{cost} \mapsto \text{high}] \rightsquigarrow \text{soc} \mapsto \text{mixed} \}.$$

**Potential intention:**  $\text{potIntend}(h, o) \stackrel{\text{def}}{=}} \Delta_h \text{ideal}(h, o) \wedge \text{justif}(h, o)$





## Example 1: Artificial assistant(cont.)

### Operators:

$$Op = \{ inform(m, h, val(o, a)) : o \in Opt \text{ and } a \in Assign \} \cup \\ \{ inform(m, h, ideal(h, o)) : o \in Opt \}$$

### Executability preconditions:

$$\mathcal{P}(inform(m, h, val(o, a))) = \Box_m \left( val(o, a) \wedge \bigwedge_{v \in Val_{dan}} (val(o, dan \mapsto v) \rightarrow \Delta_h val(o, dan \mapsto v)) \right)$$

if  $a \notin Assign_{dan}$ ,

$$\mathcal{P}(inform(m, h, val(o, a))) = \Box_m val(o, a)$$

if  $a \in Assign_{dan}$ ,

$$\mathcal{P}(inform(m, h, ideal(h, o))) = \Box_m (ideal(h, o) \wedge justif(h, o)).$$





## Example 1: Artificial assistant(cont.)

**Planning goal:**  $\alpha_G \stackrel{\text{def}}{=} \bigvee_{o \in Opt} \text{potIntend}(h, o)$

$\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6$  is a solution for the planning problem  $\langle \Sigma, Op, \alpha_G \rangle$  where:  
 $\Sigma = \{\alpha_1, \dots, \alpha_8\} \cup \{\alpha_7^{o,x} : o \in Opt \text{ and } x \in Var\}$  and

$\epsilon_1 \stackrel{\text{def}}{=} \text{inform}(m, h, \text{val}(te, \mathbf{dan} \mapsto med)),$

$\epsilon_2 \stackrel{\text{def}}{=} \text{inform}(m, h, \text{val}(te, \mathbf{env} \mapsto land)),$

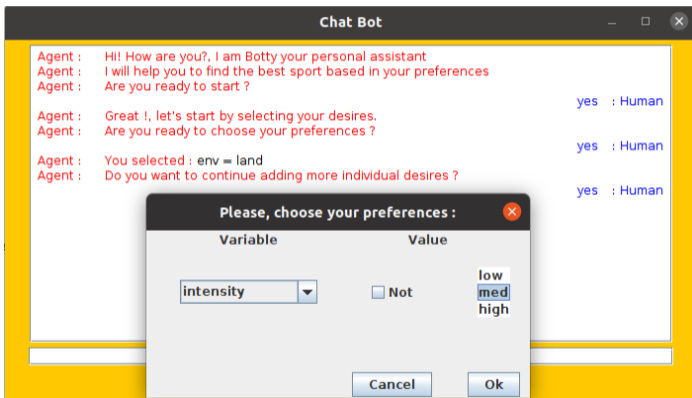
$\epsilon_3 \stackrel{\text{def}}{=} \text{inform}(m, h, \text{val}(te, \mathbf{intens} \mapsto med)),$

$\epsilon_4 \stackrel{\text{def}}{=} \text{inform}(m, h, \text{val}(te, \mathbf{loc} \mapsto mixed)),$

$\epsilon_5 \stackrel{\text{def}}{=} \text{inform}(m, h, \text{val}(te, \mathbf{soc} \mapsto mixed)),$

$\epsilon_6 \stackrel{\text{def}}{=} \text{inform}(m, h, \text{ideal}(h, te)).$





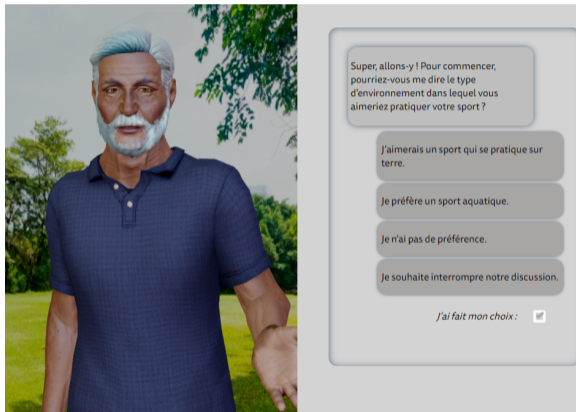
■ Source code:

<https://www.irit.fr/CoPains/software/>





# Web interface

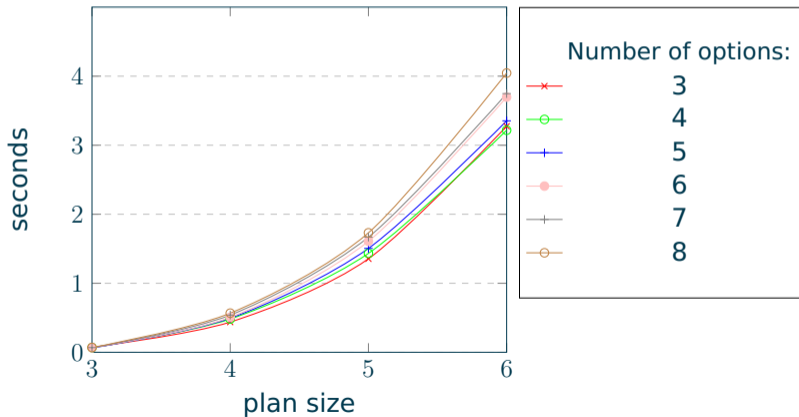


- Web avatar developed in collaboration with the company DAVI:

<https://cognitive-planning.schm.fr/>



Processing time used by the brute force approach:





## Experiments(cont.)

Processing time (in seconds) used by the brute force approach:

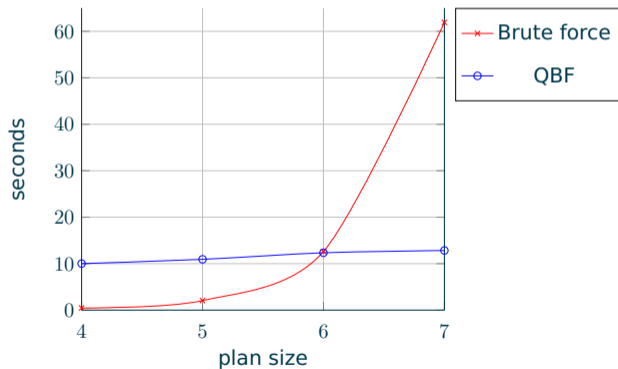
Plan size	Number of options (sports)					
	3	4	5	6	7	8
3	0.059	0.067	0.063	0.066	0.068	0.070
4	0.438	0.482	0.494	0.506	0.539	0.567
5	1.355	1.433	1.505	1.608	1.668	1.731
6	3.274	3.217	3.353	3.696	3.747	4.045





# Experiments(cont.)

Comparison in performance between the brute force and the QBF approaches:





# Experiments(cont.)

Comparison in performance between the brute force and the QBF approaches:

Size of the plan	Planning module processing time (seconds)	
	Brute force	QBF
4	0.464	10.015
5	2.059	10.936
6	12.610	12.343
7	61.920	12.837



## Conclusions

- Our implementation demonstrates that our NP-complete fragment and the cognitive planning problem formulated in this logic are suitable for real-world applications in the domain of HMI.
- We intend to include a setting parameter in the artificial agent in order to let the system select the most convenient approach (SAT or QBF) depending on the scenario.
- We plan to extend the implemented system by speech acts of type question to capture both sides of interaction, from agent  $m$  to agent  $h$  (handled by the actual implementation) and from agent  $h$  to agent  $m$ .



# Part 3 - Implementation and applications

Conversational agent for motivational interviewing



## Example 2: Virtual coaching

Agent  $m$  is a virtual coaching agent which has to motivate agent  $h$  to practice a physical activity.

⇒ Agent  $h$  has to become aware of the inconsistency between her current behavior and her desires

Desires : <i>DesAtm</i>	Conditions : <i>CondAtm</i>	Actions : <i>ActAtm</i>
<i>dr</i> : "h has dietary restrictions" <i>pw</i> : "h puts on weight" <i>lw</i> : "h loses weight" <i>at</i> : "h is attractive" <i>gh</i> : "h is in good health" <i>st</i> : "h is stressed"	<i>ow</i> : "h has an office work" <i>sl</i> : "h has a sedentary life style" <i>co</i> : "h is a commuter/time in traffic"	does( $h, ps^*$ )

\**ps* : "h practices sport"







## Example 2: Virtual coaching(cont.)

Agent  $m$ 's model of agent  $h$ 's cognitive state:

$$\alpha_1 \stackrel{\text{def}}{=} \bigwedge_{l \in Lit} \Delta_h \text{nec}(\{l\}, l),$$

$$\alpha_2 \stackrel{\text{def}}{=} \bigwedge_{l \in Lit} (\Delta_h \text{nec}(\emptyset, l) \leftrightarrow \Delta_h l),$$

$$\alpha_3 \stackrel{\text{def}}{=} \bigwedge_{l \in Lit, X, X' \in LitSet: X' \subseteq X} \left( \Delta_h \text{nec}(X, l) \rightarrow \left( \bigwedge_{l' \in X'} \Delta_h l' \rightarrow \Delta_h \text{nec}(X \setminus X', l) \right) \right),$$

$$\alpha_4 \stackrel{\text{def}}{=} \bigwedge_{l \in Lit, X, X' \in LitSet: X \subseteq X'} (\Delta_h \text{nec}(X, l) \rightarrow \Delta_h \text{nec}(X', l)),$$

$$\alpha_5 \stackrel{\text{def}}{=} \bigwedge_{l \in Lit} \left( (\text{des}(h, l) \leftrightarrow \Delta_h \text{des}(h, l)) \wedge (\neg \text{des}(h, l) \leftrightarrow \Delta_h \neg \text{des}(h, l)) \right),$$

$$\alpha_6 \stackrel{\text{def}}{=} \bigwedge_{a \in Act} \left( (\text{does}(h, a) \leftrightarrow \Delta_h \text{does}(h, a)) \wedge (\neg \text{does}(h, a) \leftrightarrow \Delta_h \neg \text{does}(h, a)) \right),$$



## Example 2: Virtual coaching(cont.)

$$\alpha_7 \stackrel{\text{def}}{=} \text{nec}(\{\neg dr, \neg pw, sl\}, \text{does}(h, ps)) \wedge \\ \text{nec}(\{at, \neg dr\}, \text{does}(h, ps)) \wedge \\ \text{nec}(\{sl, gh\}, \text{does}(h, ps)) \wedge \\ \text{nec}(\{gh\}, \neg st) \wedge \\ \text{nec}(\{co, ow\}, sl).$$

$$\alpha_G \stackrel{\text{def}}{=} \neg \text{does}(h, ps) \rightarrow \text{AwareIncon}(h, \neg \text{does}(h, ps)).$$

Plan computed by agent  $m$  :

Step	Informative planning problem	Interrogative planning problem
0		$?_{m,h} \text{does}(h, ps)$
1		$?_{m,h} \text{des}(h, gh), ?_{m,h} co, ?_{m,h} ow$
2	$!_{m,h}(\emptyset, \{co, ow\}, sl), !_{m,h}(\{gh\}, \{sl\}, \text{does}(h, ps))$	



## Example 2: Virtual coaching(cont.)

Speaker	Utterance	Speech act
m	Do you practice a sport regularly?	? <sub>m,h</sub> does(h,ps)
h	I don't	+ <sub>m</sub> ¬Δ <sub>h</sub> does(h,ps)
m	Do you wish to be in good health?	? <sub>m,h</sub> des(h,gh)
h	Yes	+ <sub>m</sub> Δ <sub>h</sub> des(h,gh)
m	Do you spend quite some time in the traffic everyday as a commuter?	? <sub>m,h</sub> co
h	Yes	+ <sub>m</sub> Δ <sub>h</sub> co
m	Do you have an office work?	? <sub>m,h</sub> ow
h	Yes	+ <sub>m</sub> Δ <sub>h</sub> ow
m	You spend quite some time in the traffic everyday as a commuter and you have an office work. Therefore, your life style is sedentary!	! <sub>m,h</sub> (∅, {co, ow}, sl)
m	Your life style is sedentary. Therefore, you will not satisfy your desire to be in good health unless you practice a sport regularly!	! <sub>m,h</sub> ({gh}, {sl}, does(h,ps))



# Plan of the tutorial

- 1 Logical framework for cognitive planning (Emiliano)
- 2 QBF encoding of cognitive planning and ToulST platform (Frédéric)
- 3 Implementation and applications (Jorge)
- 4 **Formal extension and implementation for collaborative gaming (Dominique & Jorge) NOW!**
  - Extended logical framework and encoding of the Yōkai game
  - General architecture and implementation



# Part 4 - Application to the Yōkai game

Introduction



# Why Yōkai game?

Because this game includes:

- reasoning about mental states (theory of mind),
- temporal reasoning,
- spatial reasoning.

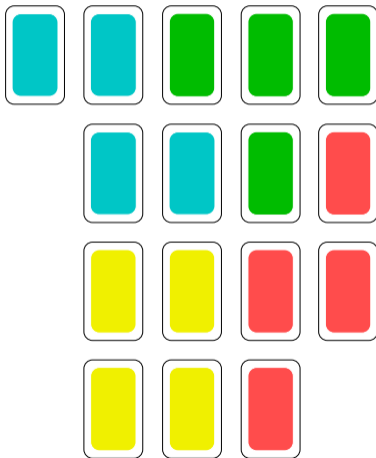


## Rules of Yōkai game [Griffon, 2019]

- 4 card colors and 4 cards of each color,
- 7 hints,
- initial state: shuffled and grouped cards face down,
- target end state (goal): group cards of the same color,
- 4 kinds of action:
  - to observe (the color of) a card,
  - to move a card (face down),
  - to active a hint,
  - to mark a card with a hint,
- constraint: communication outside of actions is not allowed.



## Example of a winning final configuration







## A round of play

At each round of play, each player execute 4 actions (no order constraint) :

- (to observe 1 card)  $\times 2$  ;
- to move 1 card (without creating a *separation*) ;
- to active 1 hint (x) or to mark 1 card with a hint about its color ( $\Rightarrow$  marked cards cannot be moved anymore).





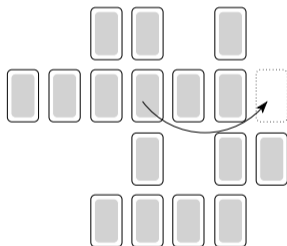
# What is a *separation* (between groups of cards)?

## Definition (separation)

There are at least two cards that cannot be linked by a “path”

## Definition (path)

A set of positions occupied by cards which are accessed only from their sides (NOT diagonally).

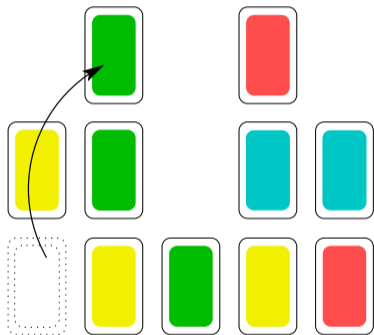




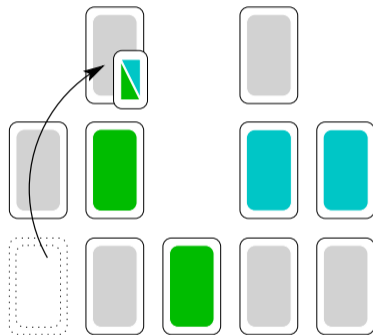
- an hint is a card containing 1 to 3 colors,
- “to mark a card” is to place a hint on this card,
- a card can only be marked with a hint including the color of the card,
- once marked, a card can no longer be observed, moved or marked.



# Example of game in progress ( $m$ plays against $h$ )



Cards of which  $m$  knows  
the color



Cards that  $m$  believes  $h$   
knows the color of



## End of game

- 2 ways to complete the game:
  - either because a player declares the end of the game,
  - or because there is no more hint to activate or to mark a card.

### Reminder

- Each round, a hint must be activated or played (to mark a card),
- there are 7 hints,
- there are a maximum of 14 rounds of play (7 for each player).



# Part 4 - Application to the Yōkai game

Formal framework extension



# Temporal extension of explicit beliefs

We first define the language  $\mathcal{L}_0(Atm)$  by:

$$\alpha ::= \underbrace{p^t \mid \Delta_h^t \alpha \mid now^{\geq t}}_{\mathcal{L}_0^T(Atm)} \mid \neg \alpha \mid \alpha_1 \wedge \alpha_2 \mid \Delta_m \alpha$$

where :

- $p$  ranges over  $Atm$ ,
- $p^t$  is read “atomic proposition  $p$  is true at time  $t$ ”,
- $now^{\geq t}$  is read “the actual time of the game play is at least  $t$ ”.





## Implicit beliefs (no change)

Language  $\mathcal{L}(Atm)$  is defined by:

$$\varphi ::= \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \Box_m \alpha$$

Note: only formulas from  $\mathcal{L}_0$  can be in the scope of an  $\Box_m$  operator.







# Semantics (reminder & new things)

- State:  $S = (B, V)$  with
  - $B \subseteq \mathcal{L}_0$  is agent  $m$ 's belief base ,
  - $V \subseteq \mathcal{L}_0^T$  is the actual situation,
- for every  $t, t' \in \mathbb{N}$ :
  - $now^{\geq 0} \in V$ ,
  - if  $now^{\geq t} \in V$  and  $t' \leq t$  then  $now^{\geq t'} \in V$ ,
  - $now^{\geq t} \in V$  iff  $now^{\geq t} \in B$ .





## Semantics (new things)

Let  $S = (B, V) \in \mathcal{S}$ . Then, for every  $x \in \mathcal{L}_0^T(Atm)$ :

$$S \models x \iff x \in V,$$

$$S \models \neg\alpha \iff S \not\models \alpha,$$

$$S \models \alpha_1 \wedge \alpha_2 \iff S \models \alpha_1 \text{ and } S \models \alpha_2,$$

$$S \models \Delta_m \alpha \iff \alpha \in B.$$





# Dynamic extension

- Language  $\mathcal{L}^+(Atm)$  is defined as follows:

$$\varphi ::= \alpha \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \Box_m \alpha \mid [+^t_m \alpha]\varphi,$$

- $[+^t_m \alpha]\varphi$  is read “ $\varphi$  holds after agent  $m$  has privately learned that  $\alpha$  and that the current time is at least  $t$ ”.
- $(S, Cxt) \models [+^t_m \alpha]\varphi \iff (S^{+^t_m \alpha}, Cxt) \models \varphi$  with

$$S^{+^t_m \alpha} = (B^{+^t_m \alpha}, V^{+^t_m \alpha}),$$

$$V^{+^t_m \alpha} = V \cup \{now^{\geq t'} : t' \leq t\},$$

$$B^{+^t_m \alpha} = B \cup \{\alpha\} \cup \{now^{\geq t'} : t' \leq t\}.$$



# Theorems

- Checking satisfiability of  $\mathcal{L}(Atm)$  formulas in the class  $\mathbf{M}$  is an NP-complete problem.
- Checking satisfiability of  $\mathcal{L}^+(Atm)$  formulas in the class  $\mathbf{M}$  is an NP-complete problem.
- Let  $\varphi \in \mathcal{L}^+(Atm)$  and let  $\Sigma \subset \mathcal{L}_0(Atm)$  be finite. Then
 
$$\Sigma \models_{\mathbf{M}} \varphi \text{ if and only if } \models_{\mathbf{M}} \bigwedge_{\alpha \in \Sigma} \Box_{\mathbf{m}} \alpha \rightarrow \varphi.$$

(Logical consequence problem with a finite set of premises can be reduced to the satisfiability problem.)





## Action selection (rappel)

- **Belief expansion events:**  $Act_m = \{+^t_m \alpha : \alpha \in \mathcal{L}_0, t \in \mathbb{N}\}$
- **Executability preconditions:**  $\mathcal{P}(\epsilon)$  for  $\epsilon \in Act_m$
- **Successful occurrence of an event:**  $\langle\langle +^t_m \alpha \rangle\rangle \varphi \stackrel{\text{def}}{=} \mathcal{P}(+^t_m \alpha) \wedge [+^t_m \alpha] \varphi$
- **Action selection problem (ASP):**  $\langle \Sigma, Op, \alpha_G \rangle$  where:
  - $\Sigma \subset \mathcal{L}_0(Atm)$  (m's belief base),
  - $Op \subset Act_m$  (m's action repertoire),
  - $\alpha_G \in \mathcal{L}_0(Atm)$  (m's goal).
- **Solution to ASP:**  $\epsilon \in Op$  such that  $\Sigma \models_M \langle\langle \epsilon \rangle\rangle \Box_m \alpha_G$ .





## Update, then revision

- Belief update :  $m$ 's explicit beliefs evolve from a time  $t$  to  $t + 1$ .
- Belief revision :  $m$  learns a new fact and adds it to its belief base.





# Beliefs update

- $Upd(\Sigma_c, \Sigma_h, \Sigma_f) = (\Sigma'_c, \Sigma'_h, \Sigma'_f)$  if and only if:
  - $\Sigma'_c = \Sigma_c$ ,
  - $\Sigma'_h = incrt(\Sigma_h)$ ,
  - $\Sigma'_f = incrt(\Sigma_f) \cup \{now^{\geq 0}\}$ .
  
- where for each finite  $X \subseteq \mathcal{L}_0$ ,  $incrt(X) = \{incrt(\alpha) : \alpha \in X\}$  such that:
  - $incrt(p^t) = p^{t+1}$
  - $incrt(\Delta_h^t \alpha) = \Delta_h^{t+1} incrt(\alpha)$
  - $incrt(\Delta_m \alpha) = \Delta_m incrt(\alpha)$
  - $incrt(now^{\geq t}) = now^{\geq t+1}$
  - $incrt(\neg \alpha) = \neg incrt(\alpha)$
  - $incrt(\alpha_1 \wedge \alpha_2) = incrt(\alpha_1) \wedge incrt(\alpha_2)$ .



## Belief revision (reminder)

- $Rev(\Sigma_c, \Sigma_h, \Sigma_f, \Sigma_{input}) =$ 
  - $(\Sigma_c, \Sigma_h, \Sigma_f)$  if  $\Sigma_c \cup \Sigma_{input}$  is not propositionally consistent ;
  - $(\Sigma'_c, \Sigma'_h, \Sigma'_f)$  otherwise, with  $\Sigma'_c = \Sigma_c$ ,  $\Sigma'_f = \bigcap_{X \in MCS(\Sigma_c, \Sigma_f, \Sigma_{input})} X$ ,  
 $\Sigma'_h = \bigcap_{X \in MCS(\Sigma_c \cup \Sigma'_f, \Sigma_h, \emptyset)} X$ .
  
- So:
  - $\Sigma_c$  is not modified,
  - $\Sigma_{input}$  is added to  $\Sigma_f$  (iff  $\Sigma_{input}$  and  $\Sigma_c$  are consistent),
  - belief revision satisfies minimal change for factual information.







# Part 4 - Application to the Yōkai game

Logical modeling of Yōkai



Let be the following sets:

$$TIME = \{0, 1, \dots, end\} \text{ where } end = 56$$

$$TIME^* = TIME \setminus \{0\}$$

$$GRID = \{1, \dots, 32\} \times \{1, \dots, 32\},$$

$$IPOS = \{(l, c) \in GRID : l, c \in \{15, \dots, 18\}\},$$

$$COLORS = \{r, g, b, y\},$$

$$HINTS = 2^{COLORS} \setminus \{\{\}, \{r, g, b, y\}\},$$

$$CARDS = \{1, \dots, 16\}$$

$$CARDS^n = \{X \in 2^{CARDS} : |X| = n\} \text{ with } n \in \mathbb{N}$$





# Atomic propositions

- $col_{x,c}^t$ : card  $x$  is of color  $c$  at time  $t$ ,
- $pos_{x,p}^t$ : card  $x$  is at position  $p$  at time  $t$ ,
- $active_h^t$ : hint  $h$  is enabled at time  $t$ ,
- $mark_{x,h}^t$ : card  $x$  is marked with hint  $h$  at time  $t$ ,
- $legMov_{x,p}^t$ : to move card  $x$  to position  $p$  at time  $t$  is legal.





# Initial factual belief base

$$\Sigma_f^W = \{now^{\geq 0}\} \cup \bigcup_{x \in CARDS} \{pos_{x,\sigma(x)}^0\} \cup \bigcup_{\substack{x \in CARDS \\ c \in COLORS}} \{\neg \Delta_m col_{x,c}^0\} \cup$$

$$\bigcup_{h \in HINTS} \{\neg active_h^0\} \cup \bigcup_{\substack{x \in CARDS \\ h \in HINTS}} \{\neg mark_{x,h}^0\}$$

$$\Sigma_f^h = \bigcup_{\alpha \in \Sigma_f^W} \{\Delta_h^0 \alpha\} \cup \bigcup_{\substack{x \in CARDS \\ c \in COLORS}} \{\neg \Delta_h^0 col_{x,c}^0\}$$

$$\Sigma_f = \Sigma_f^W \cup \Sigma_f^h$$





# Initial hypothetical belief base

$$\Sigma_h = \bigcup_{\substack{x \in \text{CARDS} \\ h \in \text{HINTS} \\ c \in \text{COLORS} \setminus h}} \{ \text{mark}_{x,h}^0 \rightarrow \neg \text{col}_{x,c}^0, \Delta_h^0 \text{mark}_{x,h}^0 \rightarrow \Delta_h^0 \neg \text{col}_{x,c}^0 \}$$

## Example

1)  $\Sigma_h \supseteq \{ \text{mark}_{1,\{g,r\}}^0 \rightarrow \neg \text{col}_{1,b}^0, \text{mark}_{1,\{g,r\}}^0 \rightarrow \neg \text{col}_{1,y}^0 \}$ .

2)  $\text{mark}_{1,\{g,r\}}^0 \in \Sigma_f$ .

So, we can deduce that  $\Box_m ((\text{col}_{1,g}^0 \vee \text{col}_{1,r}^0) \wedge \neg \text{col}_{1,b}^0 \wedge \neg \text{col}_{1,y}^0)$  holds.



# Core belief base (integrity constraints)

## Rules of the game:

$$\Sigma_c^{ic} = \bigcup_{t \in TIME} \left\{ \bigwedge_{x \in CARDS} \bigvee_{p \in GRID} pos_{x,p}^t, \right. \quad (ICP1)$$

$$\bigwedge_{\substack{x \in CARDS \\ p, p' \in GRID: p \neq p'}} \neg(pos_{x,p}^t \wedge pos_{x,p'}^t), \quad (ICP2)$$

$$\bigwedge_{\substack{p \in GRID \\ x, x' \in CARDS: x \neq x'}} \neg(pos_{x,p}^t \wedge pos_{x',p}^t), \quad (ICP3)$$

$$\dots \left. \right\}$$



# Core belief base (frame axioms)

- Description of what does not change:

$$\text{posFA}_X^t \stackrel{\text{def}}{=} \bigwedge_{\substack{x \in \text{CARDS} \setminus X \\ p \in \text{GRID}}} (\text{pos}_{x,p}^t \leftrightarrow \text{pos}_{x,p}^{t-1})$$

$$\text{colFA}_X^t \stackrel{\text{def}}{=} \bigwedge_{\substack{x \in \text{CARDS} \setminus X \\ c \in \text{COLORS}}} \left( (\text{col}_{x,c}^t \leftrightarrow \text{col}_{x,c}^{t-1}) \wedge (\neg \Delta_m \text{col}_{x,c}^t \leftrightarrow \neg \Delta_m \text{col}_{x,c}^{t-1}) \right)$$

$$\text{hintFA}_H^t \stackrel{\text{def}}{=} \bigwedge_{h \in \text{HINTS} \setminus H} (\text{active}_h^t \leftrightarrow \text{active}_h^{t-1}) \wedge \bigwedge_{\substack{x \in \text{CARDS} \\ h \in \text{HINTS} \setminus H}} (\text{mark}_{x,h}^t \leftrightarrow \text{mark}_{x,h}^{t-1})$$



# Core belief base (successor state axioms)

## ■ Computation of a future state:

$$\Sigma_c^{ssa} = \bigcup_{t \in \text{TIME}^*} \left\{ \bigwedge_{\substack{x \in \text{CARDS} \\ p \in \text{GRID}}} (pos_{x,p}^t \wedge \neg pos_{x,p}^{t-1} \rightarrow posFA_{\{x\}}^t \wedge colFA_{\emptyset}^t \wedge hintFA_{\emptyset}^t), \right. \quad (\text{SSA4})$$

$$\bigwedge_{\substack{x \in \text{CARDS} \\ c \in \text{COLORS}}} (col_{x,c}^t \wedge \neg \Delta_m col_{x,c}^{t-1} \rightarrow posFA_{\emptyset}^t \wedge colFA_{\{x\}}^t \wedge hintFA_{\emptyset}^t), \quad (\text{SSA5})$$

$$\bigwedge_{h \in \text{HINTS}} (active_h^t \wedge \neg active_h^{t-1} \rightarrow posFA_{\emptyset}^t \wedge colFA_{\emptyset}^t \wedge hintFA_{\{h\}}^t), \quad (\text{SSA6})$$

$$\left. \bigwedge_{\substack{x \in \text{CARDS} \\ h \in \text{HINTS}}} (mark_{x,h}^t \wedge \neg mark_{x,h}^{t-1} \rightarrow posFA_{\emptyset}^t \wedge colFA_{\emptyset}^t \wedge hintFA_{\{h\}}^t) \right\} \quad (\text{SSA7})$$



 **Core belief base**

## ■ Finally:

$$\Sigma_c = \Sigma_c^{ic} \cup \Sigma_c^{ssa}$$



# Action repertoire

- $ACT \subseteq EVT$  of agent  $m$  is defined as follows:

$$+_m^{col_{x,c}^t} \stackrel{\text{def}}{=} +_m^t \left( col_{x,c}^t \wedge \Delta_m col_{x,c}^t \wedge \left( \Delta_h^t \bigvee_{c' \in COLORS} \Delta_m col_{x,c'}^t \right) \wedge \Delta_h^t now^{\geq t} \right)$$

$$+_m^{pos_{x,p}^t} \stackrel{\text{def}}{=} +_m^t \left( pos_{x,p}^t \wedge \Delta_m pos_{x,p}^t \wedge \Delta_h^t pos_{x,p}^t \wedge \Delta_h^t now^{\geq t} \right)$$

$$+_m^{actHint_h^t} \stackrel{\text{def}}{=} +_m^t \left( active_h^t \wedge \Delta_m active_h^t \wedge \Delta_h^t active_h^t \wedge \Delta_h^t now^{\geq t} \right)$$

$$+_m^{markHint_{x,h}^t} \stackrel{\text{def}}{=} +_m^t \left( mark_{x,h}^t \wedge \Delta_m mark_{x,h}^t \wedge \Delta_h^t mark_{x,h}^t \wedge \Delta_h^t now^{\geq t} \right)$$



# Actions preconditions

$$\mathcal{P}(+_m^{\text{col}}{}_{x,c}^{t+1}) \stackrel{\text{def}}{=} \text{now}^=t \wedge \square_m \bigwedge_{h \in \text{HINTS}} \neg \text{mark}_{x,h}^t$$

$$\mathcal{P}(+_m^{\text{pos}}{}_{x,p}^{t+1}) \stackrel{\text{def}}{=} \text{now}^=t \wedge \square_m \bigwedge_{h \in \text{HINTS}} \neg \text{mark}_{x,h}^t \wedge \square_m \text{legMov}_{x,p}^t$$

$$\mathcal{P}(+_m^{\text{actHint}}{}_h^{t+1}) \stackrel{\text{def}}{=} \text{now}^=t$$

$$\mathcal{P}(+_m^{\text{markHint}}{}_{x,h}^{t+1}) \stackrel{\text{def}}{=} \text{now}^=t \wedge \square_m \text{active}_h^t \wedge \bigvee_{c \in h} \square_m \text{col}_{x,c}^t \wedge \square_m \bigwedge_{\substack{h' \in \text{HINTS} \\ h' \neq h}} \neg \text{mark}_{x,h'}^t$$



## Goals modeling (observe a card around $X$ )

- goal = motivational attitude for action,
- $X \subseteq CARDS : 1 \leq |X| \leq 3$ , a set of cards of the same color,

$$\alpha_{obsArroundCards(X)}^t \stackrel{\text{def}}{=} \bigvee_{\substack{x' \in CARDS \\ x' \notin X}} \left( \left( \bigwedge_{c \in COLORS} \neg \Delta_m col_{x',c}^t \right) \wedge \right. \\ \left. nbgCards_{X \cup \{x'\}}^t \wedge \left( \bigvee_{c \in COLORS} \Delta_m col_{x',c}^{t+1} \right) \right)$$

$$\mathcal{P}_g(\alpha_{obsArroundCards(X)}^t) \stackrel{\text{def}}{=} nbgCards_X^t$$





# Goals modeling (observe a card whose color is unknown)

Let  $x \in CARDS$ :

$$\alpha_{obsUnknownColorCard}^t(x) \stackrel{\text{def}}{=} \bigvee_{c \in COLORS} \Delta_m col_{x,c}^{t+1}$$

$$\mathcal{P}_g(\alpha_{obsUnknownColorCard}^t(x)) \stackrel{\text{def}}{=} \bigwedge_{c \in COLORS} \neg \Delta_m col_{x,c}^t$$





# Goals modeling (observe a card randomly)

- Default goal for moving ( $x \in CARDS$ ):

$$\alpha_{obsRandomly(x)}^t \stackrel{\text{def}}{=} \bigvee_{c \in COLORS} \Delta_m col_{x,c}^{t+1}$$

$$\mathcal{P}_g(\alpha_{obsRandomly(x)}^t) \stackrel{\text{def}}{=} \top$$





## How to manage the different goal?

- With meta algorithms generating an ordered list of goals
- $\text{getSubsetsGtLeq}(X, \tau_m, \tau_M) = \{X' \in 2^X : \tau_m < |X'| \leq \tau_M\}$

```

1: function tryToObserve( $S, \alpha_g^t$ )
2:    $success \leftarrow \text{false}$ 
3:   while  $S \neq \emptyset$  && ! $success$  do
4:      $x \leftarrow \text{pop}(S)$ 
5:     if  $\mathcal{P}_g(\alpha_{g(x)}^t)$  then
6:        $success \leftarrow \text{checkPlan}(\alpha_{g(x)}^t)$ 
7:     end if
8:   end while
9:   return  $success$ 

```

```

1: ▷ Parameters computation for  $\alpha_{obsArroundCards(X)}^t$ 
2:  $S \leftarrow \{\}$ 
3: for all  $c \in COLORS$  do
4:    $X_c \leftarrow \{x \in CARDS : \Delta_m col_{x,c}^t \in \Sigma_f\}$ 
5:    $S \leftarrow S \cup \text{getSubsetsGtLeq}(X_c, 0, |X_c|)$ 
6: end for
7:  $S' \leftarrow \text{sortByDecreasingSize}(S)$ 
8:   ▷  $\forall X_i, X_{i+1} \in S', |X_i| \geq |X_{i+1}|$ 

```



## How to manage the different goal?

- 1: **if** !tryToObserve( $S'$ ,  $\alpha_{obsArroundCards}^t$ ) **then**
- 2:      $X \leftarrow \{x \in CARDS : \exists c \in COLORS, \Delta_m col_{x,c}^t \in \Sigma_f\}$
- 3:      $X' \leftarrow CARDS \setminus X$                      ▷ The set of cards whose color is unknown
- 4:     **if** !tryToObserve( $X'$ ,  $\alpha_{obsUnknownColorCard}^t$ ) **then**
- 5:         tryToObserve( $CARDS$ ,  $\alpha_{obsRandomly}^t$ )
- 6:     **end if**
- 7: **end if**





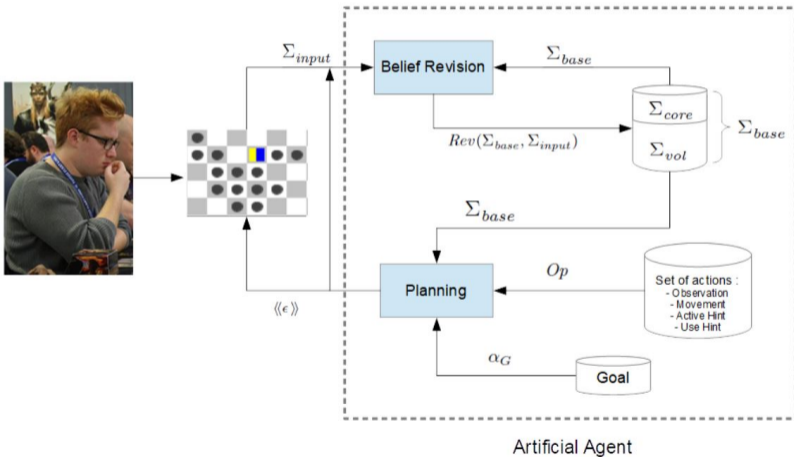


# Part 4 - Application to the Yōkai game

Implementation

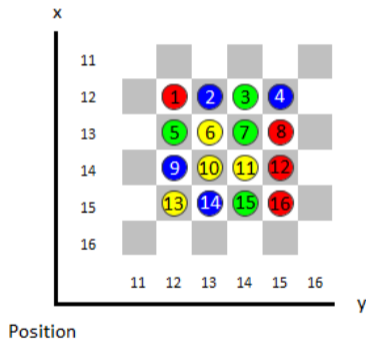


# System Architecture





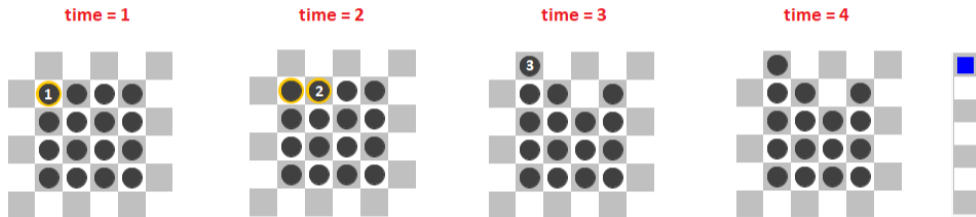
# Cards distribution



- Source code:  
<https://github.com/iritlab/yokai>



# Agent $m$ 's first round



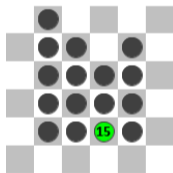
- 1:  $col\_1\_R\_t4$  and  $tm\_col\_1\_R\_t4$  and ( $th\_tm\_col\_1\_G\_t4$  or  $th\_tm\_col\_1\_Y\_t4$  or  $th\_tm\_col\_1\_B\_t4$  or  $th\_tm\_col\_1\_R\_t4$ )
- 2:  $col\_2\_B\_t4$  and  $tm\_col\_2\_B\_t4$  and ( $th\_tm\_col\_2\_G\_t4$  or  $th\_tm\_col\_2\_Y\_t4$  or  $th\_tm\_col\_2\_B\_t4$  or  $th\_tm\_col\_2\_R\_t4$ )
- 3:  $pos\_3\_p\_11\_12\_t4$  and  $tm\_pos\_3\_p\_11\_12\_t4$  and  $th\_pos\_3\_p\_11\_12\_t4$
- 4:  $act\_1\_t4$  and  $tm\_act\_1\_t4$  and  $th\_act\_1\_t4$

$\Sigma_m$  at time point 4

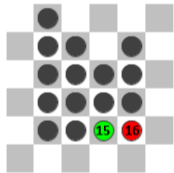


# Agent $h$ 's first round

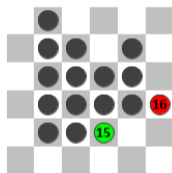
time = 5



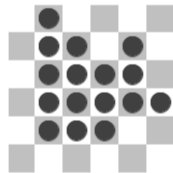
time = 6



time = 7



time = 8

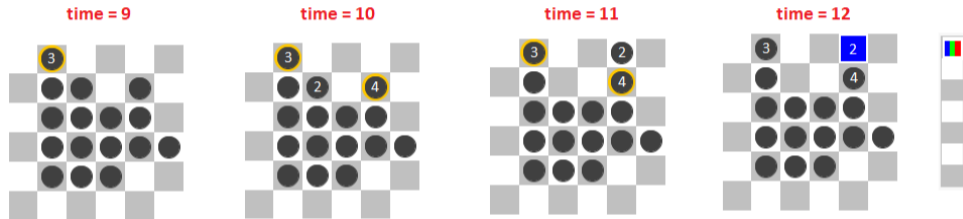


- 5: (th\_col\_15\_R\_t8 or th\_col\_15\_G\_t8 or th\_col\_15\_Y\_t8 or th\_col\_15\_B\_t8)  
 6: (th\_col\_16\_R\_t8 or th\_col\_16\_G\_t8 or th\_col\_16\_Y\_t8 or th\_col\_16\_B\_t8)  
 7: pos\_16\_p\_14\_16\_t8 and tm\_pos\_16\_p\_14\_16\_t8 and th\_pos\_16\_p\_14\_16\_t8  
 8: act\_6\_t8 and tm\_act\_6\_t8 and th\_act\_6\_t8

$$\Sigma_m \text{ at time point 8}$$



# Grouping cards



2: `col_2_B_t10` and `tm_col_2_B_t10` and (`th_tm_col_2_G_t10` or `th_tm_col_2_Y_t10` or `th_tm_col_2_B_t10` or `th_tm_col_2_R_t10`)

9: `col_3_G_t10` and `tm_col_3_G_t10` and (`th_tm_col_3_G_t10` or `th_tm_col_3_Y_t10` or `th_tm_col_3_B_t10` or `th_tm_col_3_R_t10`)

10: `col_4_B_t10` and `tm_col_4_B_t10` and (`th_tm_col_4_G_t10` or `th_tm_col_4_Y_t10` or `th_tm_col_4_B_t10` or `th_tm_col_4_R_t10`)



# Action selection

```
(not ([m](
not tm_col_5_G_t10 and
not tm_col_6_Y_t10 and
.
col_1_R_t10 and tm_col_1_R_t10 and (th_tm_col_1_G_t10 or th_tm_col_1_Y_t10 or th_tm_col_1_B_t10 or th_tm_col_1_R_t10) and
col_2_B_t10 and tm_col_2_B_t10 and (th_tm_col_2_G_t10 or th_tm_col_2_Y_t10 or th_tm_col_2_B_t10 or th_tm_col_2_R_t10) and
act_1_t10 and tm_act_1_t10 and th_act_1_t10 and
.
;;SSA11
((pos_2_p_11_15_t11 and not pos_2_p_11_15_t10) =>
((pos_1_p_12_12_t11 <=> pos_1_p_12_12_t10) and ((pos_3_p_11_12_t11 <=> pos_3_p_11_12_t10) and
((pos_4_p_12_15_t11 <=> pos_4_p_12_15_t10) and ((pos_5_p_13_12_t11 <=> pos_5_p_13_12_t10) and
((pos_6_p_13_13_t11 <=> pos_6_p_13_13_t10))))))
.
=> ([m] now_t10 and [m] (not mark_2_1_t10 and not mark_2_2_t10 and
not mark_2_3_t10 and not mark_2_4_t10 and not mark_2_5_t10 and
not mark_2_6_t10 and not mark_2_7_t10) and
plus(pos_2_p_11_15_t11 and tm_pos_2_p_11_15_t11 and th_pos_2_p_11_15_t11,
[m]((pos_2_p_11_15_t11 and pos_4_p_12_15_t11) or (pos_2_p_12_14_t11 and pos_4_p_12_15_t11) or
(pos_2_p_12_16_t11 and pos_4_p_12_15_t11) or (pos_4_p_11_13_t11 and pos_2_p_12_13_t11) or
(pos_4_p_12_14_t11 and pos_2_p_12_13_t11))))))
```

Planning formula for grouping blue cards 2 and 4



# Belief revision

## Integrity constraint ICP2

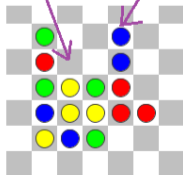
```

.
.
(not (pos_2_p_12_13_t11 and pos_2_p_12_11_t11) and
(not (pos_2_p_12_13_t11 and pos_2_p_10_12_t11) and
(not (pos_2_p_12_13_t11 and pos_2_p_11_11_t11) and
(not (pos_2_p_12_13_t11 and pos_2_p_11_13_t11) and
.
.
(not (pos_2_p_12_13_t11 and pos_2_p_11_15_t11) and
(not (pos_2_p_12_13_t11 and pos_2_p_12_14_t11) and
(not (pos_2_p_12_13_t11 and pos_2_p_12_16_t11) and
(not (pos_2_p_12_13_t11 and pos_2_p_13_11_t11) and

```

Initial position

Final position



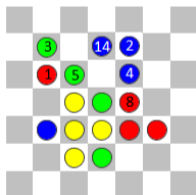




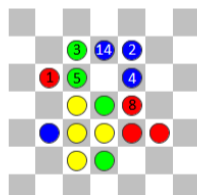
# Hierarchy of goals

*Sets of cards known by the machine :*

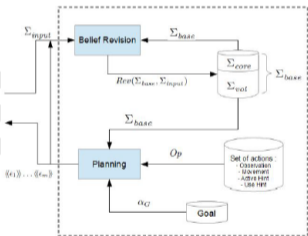
1	B		2	4	14
2	R		1	8	
3	G		3	5	
4	B		4	14	
5	B		2	4	
6	B		2	14	
7	Y		6		



*Human round at time t*



*Machine round at time t+3*



Artificial Agent

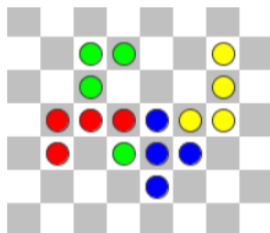
h - m



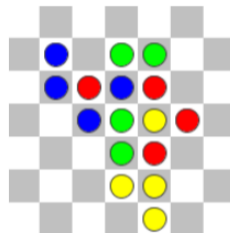
h - h



# Experiments



Case 01 - Failed game



Case 02 - Failed game

**Figure: Two different scenarios when players lost the game**





## Experiments(cont.)

Experiment set	Game	h-m		h-h	
		# 4-y-groups	Score	# 4-y-groups	Score
1	1	1	-	1	-
	2	1	-	2	-
	3	4	8	2	-
	4	1	-	2	-
	5	3	-	1	-
	6	4	8	4	9
	7	4	7	3	-
	8	2	-	2	-
	9	1	-	1	-
	10	1	-	0	-
Total:		22		18	

**Table: Total # 4-y-groups in experiment set 1**



## Experiments(cont.)

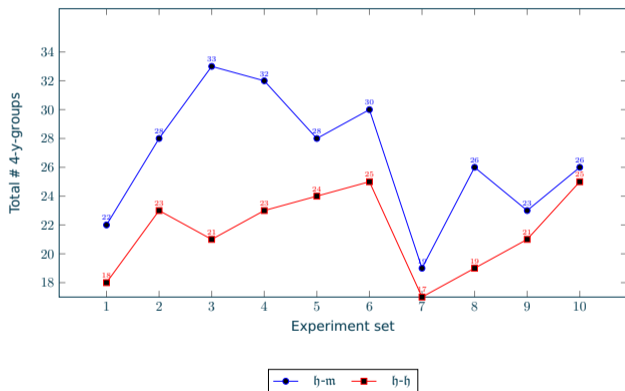
Config. type	Experiment set									
	1	2	3	4	5	6	7	8	9	10
h-m	22	28	33	32	28	30	19	26	23	26
h-h	18	23	21	23	24	25	17	19	21	25

**Table: Total # 4-y-groups by experiment set**





# Experiments(cont.)



**Figure: Total # 4-y-groups by experiment set**



Griffon, J. (2019).

Règles du jeu de Yōkai.

[https://www.play-in.com/pdf/rules\\_games/yokai\\_regles\\_fr.pdf](https://www.play-in.com/pdf/rules_games/yokai_regles_fr.pdf).