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A New PWM Control based on an Optimized Zero Sequence Component injection: Application in a Two-Level Inverter

Bourgeade Adrien¹, Malek Ghanes¹, Fadel Maurice², Bouarfa Abdelkader³, Barbot Jean-Pierre⁴

Abstract—In this paper, a new optimization problem is proposed for improvement of PWM control of an inverter that supplies a generic RLE load. Two objectives functions are studied separately: Weighted Total Harmonic Distortion (WTHD) of load voltages and total losses. The gap contribution is related to optimize the zero sequence component waveform by defining special decision variables. By doing so, the proposed PWM strategy allows to improve the WTHD and efficiency of converters by keeping a classical PWM implementation scheme. Moreover this approach has the same way of thinking than the two main different classes of PWM strategies that are Space-Vector-based PWM (SVM) methods and classic carrier-based PWM methods (CBPWM). It is well known that there is an equivalence between SVM and CBPWM through the zero sequence component injection. So, optimized zero sequences are proposed for the two objectives functions considered, and the simulation results show a significant improvement compared to classical CBPWM strategies. Moreover the proposed strategy allows an implementation on existing hardware by modifying only the modulation component software.

Index Terms—Optimized PWM, optimal SVM, PWM, SVM, Zero Sequence Component

I. INTRODUCTION

Carrier-based PWM (CBPWM) exists since almost one century [1] to control power electronic devices such as inverters. Many works tried to improve the PWM by injecting harmonics in the modulated signal [2]–[6], or on searching vector modulation methods [7], [8].

More recently it also appeared Optimal Pulse Patterns (OPP) strategies as [9]–[15]. Most of the papers are concerned about minimisation of Total Harmonic Distortion (THD) or Weighted Total Harmonic Distortion (WTHD). Different algorithms uses WTHD because it is a good way to evaluate current behaviour in an electric motor. Indeed motors behaves as a low pass filter, and WTHD allows to take into account this dynamic without considering a specific electrical motor.

Moreover, the control loop is considered as a black box, control loop is supposed to give a correct information about sinusoidal signal frequency (f_1), phase reference and desired

voltage ($m.E_{DC}$, here m is called modulation index and E_{DC} the bus voltage).

According to the literature, this paper is dedicated to improve the performances of existing strategies (SVM, THIPWM $\frac{1}{6}$,...). In order to achieve this goal, a new PWM control consisting on using an optimized Zero Sequence Component (ZSC) that generalizes the CBPWM is proposed. By doing so, the following issues are improved in terms of two objectives functions: the WTHD and efficiency. Furthermore with the proposed strategy only the modulation component software is changed without modifying the existing hardwares. The well known robustness of classical CBPWM and the phase symmetry are preserved. The strategy is applied to a two-level inverter and developed within an EV manufacturer to feed a synchronous motor.

In simulation the proposed strategy is tested and compared to SVM and THIPWM $\frac{1}{6}$ by using a two level inverter feeding an RLE load. The comparison is realised by taking in account two objectives functions, WTHD and the efficiency RLE load, and proposed strategy is done in simulation according to WTHD and efficiency on an RLE motor. SVM and THIPWM are selected to be compared to the proposed strategy because SVM is a gold standard PWM and THIPWM one is the gold standard of CBPWM strategies [3], [4]. SVM is a good starting point because it has been demonstrated that it is equivalent to CBPWM with a specific zero sequence injection [8], [16], [17].

Next, section II describes the objective functions for the considered case. Then, a description of the proposed method is developed in section III. Section IV provides a resolution way. Simulation results and discussion are given in section V. The paper ends with a conclusion in section VI.

II. OBJECTIVE FUNCTION DESCRIPTION

In order to test and compare strategies, an ideal inverter is considered. It is represented by eq. (1) where S_x , is the command of the leg x . E_{DC} The bus voltage and finally \mathbf{V} , is the vector of the simple voltages.

$$\mathbf{V} = \frac{E_{DC}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} \quad (1)$$

To achieve the comparison, two main objectives functions (WTHD and efficiency of RLE load) are used. Even if these objective functions are approximating, they afford a good indication on solution quality. The commutation and conduction losses of RLE model (Fig. 1) are computed. The

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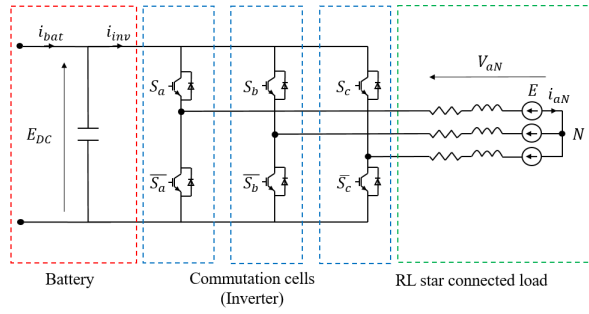


Fig. 1. Scheme of the RLE model

minimisation of harmonic losses and switch losses are then performed.

A. WTHD

The WTHD does not depend on a load and allows to evaluate solution quality quite faster and easier. The WTHD equation is represented in eq. (2).

$$V_{\text{WTHD},\%} = \frac{100}{V_1} \sqrt{\sum_{n \geq 2} \left(\frac{V_n}{n}\right)^2} \quad (2)$$

Where V_1 is the desired voltage (expressed as mE_{DC}), which is also the fundamental voltage and V_n is the voltage of the harmonic number n [4].

In order to implement it in a solver, a modified WTHD is considered in eq. (3). This equation does not have any physic meaning but is useful to analyse the solutions.

$$V_{\text{WTHDm}} = \sqrt{\sum_{n \geq 2} \frac{a_{n,V}^2 + b_{n,V}^2}{n^2}} \quad (3)$$

Where $a_{n,V}$ and $b_{n,V}$ are the Fourier coefficients of the single phase of the load.

B. Efficiency of the RLE load

The considered case, is the RLE load connected to a to level inverter, see Fig. 1.

The efficiency is defined as follows

$$\eta_{RLE} = \frac{\mathcal{P}_u}{\mathcal{P}_{tot} + \mathcal{P}_u} \quad (4)$$

Where \mathcal{P}_u represents the useful power and \mathcal{P}_{tot} the total losses.

To compute efficiency, evaluation of the losses in the inverter is given in the next.

1) *Losses in the inverter:* The main losses in the inverter are due to IGBT and Diode losses. For such components, there are two kind of losses, the conduction losses and the switching losses. The first one is due to the internal resistance of the semiconductor. The second one, is due to the on/off transition delay.

$$\mathcal{P}_{cond} = \frac{1}{2\pi} \int_0^{2\pi} S(\alpha) V_{ce} |i(\alpha)| d\alpha \quad (5)$$

$$\mathcal{P}_{sw} = \frac{1}{2} f_1 \sum_{i=1}^{N_{dec}} (E_{on}(\alpha_i) + E_{off}(\alpha_i)) + \frac{1}{2} f_1 \sum_{i=1}^{N_{dec}} [(-1)^i (E_{on}(\alpha_i) - E_{off}(\alpha_i))] \quad (6)$$

Equations (5) and eq. (6) [6] are quite simple. Indeed they depend on few parameters and on the angle position. The functions V_{ce} , E_{on} and E_{off} are given by the data sheets of the chosen semiconductors under a polynomial form. Moreover, S is equal to one when the switch is on and zero otherwise. This switching function S depends of the comparison between the carrier and the modulating signal, the current i depends of S and switching losses E_{on} and E_{off} depend on i . With this knowledge a strong link between inverter losses and command can be done.

$$\mathcal{P}_{tot} = 3\mathcal{P}_{cond} + 3\mathcal{P}_{sw} + 3 \sum_{n \geq 2} (V_n I_n \cos(\phi_n)) \quad (7)$$

In eq. (7) V_n, I_n designate voltage and current for the harmonic n respectively for the load phase voltage and load phase current. $\mathcal{P}_{cond}, \mathcal{P}_{sw}$ are the conduction and switching losses, and ϕ_n is the angle between harmonic voltage and harmonic current.

For this evaluation, the Fourier decomposition of the current is given here after.

2) *Fourier decomposition of load currents:* From Fig. 1, the analytical solution of the phase load current is given by:

$$\begin{aligned} i(\alpha) = & \left(i_0 - \sum_{n \geq 1} \frac{Ra_{n,V} - nL\omega b_{n,V}}{(nL\omega)^2 + R^2} \right) e^{-\frac{R\alpha}{L\omega}} \\ & + \sum_{n \geq 1} \frac{(Ra_{n,V} - nL\omega b_{n,V}) \cos(n\alpha)}{(nL\omega)^2 + R^2} \\ & + \sum_{n \geq 1} \frac{(Rb_{n,V} + nL\omega a_{n,V}) \sin(n\alpha)}{(nL\omega)^2 + R^2} \\ & + \frac{Ra_{1,E} - L\omega b_{1,E}}{R^2 + (L\omega)^2} e^{-\frac{R\alpha}{L\omega}} \\ & - \frac{(Ra_{1,V} - L\omega b_{1,V})}{(L\omega)^2 + R^2} \cos(\alpha) \\ & - \frac{(Rb_{1,E} + L\omega a_{1,E})}{(L\omega)^2 + R^2} \sin(\alpha) \end{aligned} \quad (8)$$

where R represents a resistor, L an inductance and $\omega = 2\pi f_1$. Moreover $a_{n,V}$ and $b_{n,V}$ are the Fourier coefficients of the considered single phase voltage of the load, and $E(\omega) = E \sin(\omega t + \phi)$, with ϕ depending on the considered phase. It can be then decomposed as Fourier coefficients, $a_{1,E}$ and $b_{1,E}$. This equation conduce to $E(\omega) = a_{1,E} \cos(\omega t) + b_{1,E} \sin(\omega t)$

The steady state solutions of eq. (8) is defined as follow:

$$\begin{aligned}
i(\alpha) &= \sum_{n \geq 1} \frac{(Ra_{n,V} - nL\omega b_{n,V})}{(nL\omega)^2 + R^2} \cos(n\alpha) \\
&+ \sum_{n \geq 1} \frac{(Rb_{n,V} + nL\omega a_{n,V})}{(nL\omega)^2 + R^2} \sin(n\alpha) \\
&- \frac{(Ra_{1,V} - L\omega b_{1,V})}{(L\omega)^2 + R^2} \cos(\alpha) \\
&- \frac{(Rb_{1,E} + L\omega a_{1,E})}{(L\omega)^2 + R^2} \sin(\alpha)
\end{aligned} \quad (9)$$

From eq. (9), a current Fourier decomposition appears, and the fundamental of the current is:

$$\begin{cases} a_{1,I} = \frac{R(a_{1,V} - a_{1,E}) - L\omega(b_{1,V} - b_{1,E})}{(L\omega)^2 + R^2} \\ b_{1,I} = \frac{R(b_{1,V} - b_{1,E}) + L\omega(a_{1,V} - a_{1,E})}{(L\omega)^2 + R^2} \end{cases} \quad (10)$$

In the same way from eq. (9), harmonics are obtained with:

$$\begin{cases} a_{n,I} = \frac{Ra_{n,V} - nL\omega b_{n,V}}{(nL\omega)^2 + R^2} \quad \forall n \in \mathbb{N}^* \setminus \{1\} \\ b_{n,I} = \frac{Rb_{n,V} + nL\omega a_{n,V}}{(nL\omega)^2 + R^2} \quad \forall n \in \mathbb{N}^* \setminus \{1\} \end{cases} \quad (11)$$

In order to decompose the losses are decomposed in two parts

$$\mathcal{P}_K = 3\mathcal{P}_{cond} + 3\mathcal{P}_{sw} \quad (12)$$

and

$$\mathcal{P}_H = 3 \sum_{n \geq 2} (V_n I_n \cos(\phi_n)) \quad (13)$$

\mathcal{P}_H is similar to WTHD, consequently the WTHD is indirectly included in (7). The \mathcal{P}_K losses is useful to determine the impact of harmonic minimisation on the optimal solutions.

III. MULTIPLE HARMONIC INJECTION DESCRIPTION

A. Multiple harmonic injection

The zero sequence component modulation consist to find the optimal harmonics to inject in the modulating signal. Because it is impossible to compute a complete Fourier decomposition, it is necessary to set the maximal harmonic considered in computation, it has been fixed arbitrary to $n_h = 20$, corresponding to 60 harmonics over the electrical frequency. This number is sufficiently large to generate precisely classical zero sequence components, to inject in the modulating signal. Consequently, this technique can reproduce SVM, THIPWM, DPWMmin and DPWMmax. Nevertheless this harmonic regeneration does not work with some DPWM strategies, due to discontinuities which generate a strong Gibbs effect. This Gibbs effect generate unexpected extra switching. Then in the proposed zero sequence harmonic the generation of strong discontinuities (extra switching) is avoided.

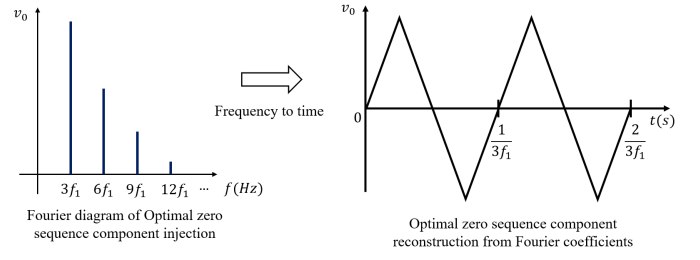


Fig. 2. Principle of ZSCM, a zero sequence component is generated from the computed optimal Fourier specter (here is represented as an example the SVM one).

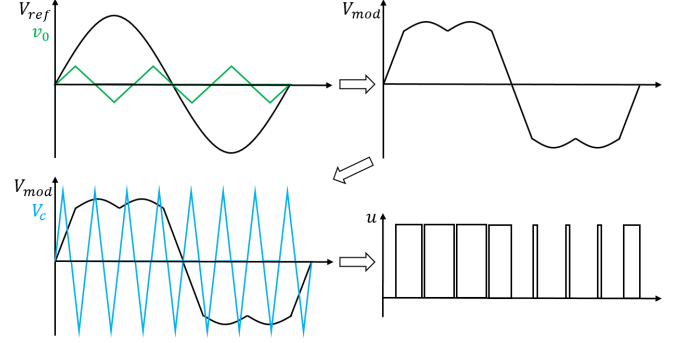


Fig. 3. Following the arrows, is represented the zero sequence component and the reference. On the second picture it is the modulated signal (sum of the two last ones). The comparison between the modulated signal and the carrier signal. Finally here is the command generated.

$$\begin{cases} v_0(t) = a_0 + \sum_{n=1}^{n_h} a_{n,v_0} \cos(3n\omega t) + b_{n,v_0} \sin(3n\omega t) \\ v_0(t) \in [-\frac{1}{2}; \frac{1}{2}] \end{cases} \quad (14)$$

Fig. 2 shows the principle scheme of the proposed method. The idea is to compute Fourier coefficients $(a_{0,v_0}, a_{n,v_0}, b_{n,v_0})$ with an optimisation algorithm in order to generate the ideal zero sequence component. This optimisation problem is computed for a specific modulation index (m), frequency (f_1) and number of switches per period ($4N$) with respect of the Fourier description of v_0 found in eq. (14). More precisely this equation is used to compute the modulated signal depending on time and not on frequency as before.

Intersection with a triangular function leads to generate commutation angles Fig. 2. The number of switching angles by period is equal to $4N$, where N is the number of switching per quarter period. The switches found are then enumerated as $0 < \alpha_1 < \alpha_2 \dots < \alpha_{4N} < 2\pi$.

With this angles determination, eq. (15), eq. (16) and eq. (17) [14] allow to compute switching sequence (S), and then phase voltage of the inverter is accessible thanks to eq. (1).

$$a_{0,S} = \frac{1}{2\pi} \left(2\pi - \sum_{j=1}^{4N} (-1)^j \alpha_j \right) \quad (15)$$

$$a_{n,S} = -\frac{1}{n\pi} \left(\sum_{j=1}^{4N} (-1)^j \sin(n\alpha_j) \right) \quad (16)$$

$$b_{n,S} = \frac{1}{n\pi} \left(\sum_{j=1}^{4N} (-1)^j \cos(n\alpha_j) \right) \quad (17)$$

Therefore, optimisation of eq. (3), eq. (7) and eq. (12) according to some constraints can be done in next section.

IV. OPTIMISATION PROBLEM SETTING

The **fmincon** function of the optimisation toolbox of MatLab is used to perform the optimisation problem. As fmincon propose only local minimum solutions. It is necessary to randomly chose many v_0 Fourier description to chose sufficiently large number of v_0 . Then a quality evaluation for each Fourier description is performed with eq. (18). This equation consider the chosen objective function, f , its description is given by eq. (3), eq. (7) or eq. (12). In the objective function f (WTHD, \mathcal{P}_{tot} or \mathcal{P}_K), no constraints are considered. In order to find solutions with constraints respect, two penalty terms is added in eq. (18).

$$f_v = f(\mathbf{x}) + K_1 \sum_{i=1}^9 (\max(c_i, 0)) + K_2 \sum_{i=1}^9 g(c_i) \quad (18)$$

Where $\mathbf{x} = (a_{0,v_0}, a_{n,v_0}, b_{n,v_0})$ is the decision vector, K_1 and K_2 are two penalty coefficients (In this paper setting equal to $K_1 = K_2 = 10^6$). Moreover $\mathbf{c} = (c_1, c_2 \dots, c_9)$ is the vector of constraints, these constraints are the same then for the problem eq. (20). In eq. (18), g is defined as follow:

$$g(c_i) = \begin{cases} 1 & \text{if } c_i > \epsilon \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

Remark, eq. (18), was inspired by the work of Sierra et al. [18].

The first penalty of eq. (18) indicates how far away the current solution is from the feasible domain and the second penalty evaluates how many constraints are violated.

After selection of the best first v_0 (starting point) according to eq. (18). Fmincon find a local minimum around the starting point with respect to the problem defined by (20). This problem try to minimise objective function (eq. (3), eq. (7) or eq. (12)), which means decreasing WTHD and increasing efficiency. Those objectives are constrained by nine constraints, those constraints force the solutions to respect the desired voltage.

$$\begin{cases} f.o & \min(f(\mathbf{x})) \\ s.c & a_{0,V_a} = 0, a_{1,V_a} = 0, b_{1,V_a} = mE_{DC} \\ & a_{0,V_b} = 0, a_{1,V_b} = -\frac{\sqrt{3}}{2}mE_{DC}, b_{1,V_b} = -\frac{1}{2}mE_{DC} \\ & a_{0,V_c} = 0, a_{1,V_c} = +\frac{\sqrt{3}}{2}mE_{DC}, b_{1,V_c} = -\frac{1}{2}mE_{DC} \end{cases} \quad (20)$$

In order to preserve at least this local minimum, the solution from the previous modulation index is chosen as a reference starting point with respect to the new random starting points. For example, the solutions found for $m =$

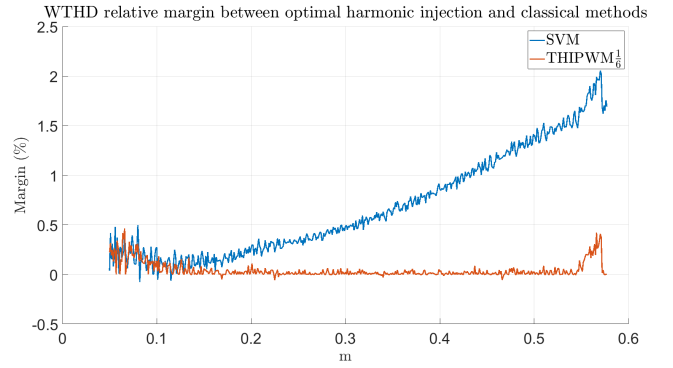


Fig. 4. Relative margin, (ϵ_r eq.(21)) for optimal harmonic injection compared with SVM and THIPWM $\frac{1}{6}$ applied to WTHD (eq.(2)) for N=5

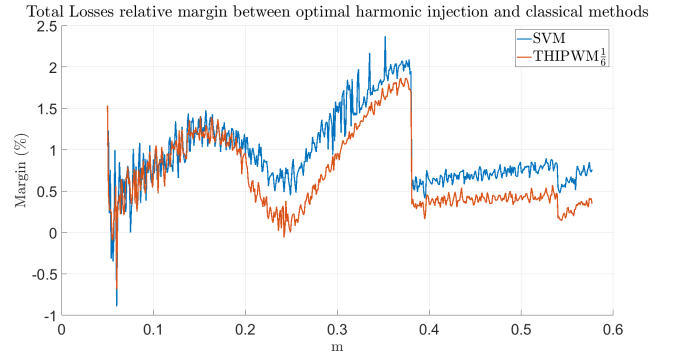


Fig. 5. Relative margin, (ϵ_r eq.(21)) for optimal harmonic injection compared with SVM and THIPWM $\frac{1}{6}$ applied to Total losses (\mathcal{P}_{tot} eq.(7)) for N=5

0.5, will be one of the starting point of the computation for the next discretisation step $m = 0.501$, this step is equal to 0.001. In addition, and to ensure that harmonic injection increase the solution quality, the solutions obtained (v_0) from Fourier decomposition of SVM, DPWMMin, DPWMMax, THIPWM1/6 and THIPWM1/4 are also considered as starting points for (20).

V. RESULTS AND DISCUSSION

A. Results

The Fig.4, represent a WTHD comparison between SVM, THIPWM $\frac{1}{6}$ and optimal harmonic injection. The idea here is to represent the relative margin (defined in eq. (21)) between our strategy and two classical ones. Fig.5 does the same comparison for the RLE load total losses and Fig.6 for the switching losses only.

$$\epsilon_r = 100 \frac{y_c - y_o}{y_c} \quad (21)$$

In eq. (21) y_c represent the objective function cost of the classical symmetry (SVM or THIPWM $\frac{1}{6}$ in this paper) and y_o the proposed objective function value of harmonic injection one.

For the proposed method the WTHD is improved in front of SVM for any modulation index and reach almost 2% for high modulation index (Fig. 4). On another hand, our optimal

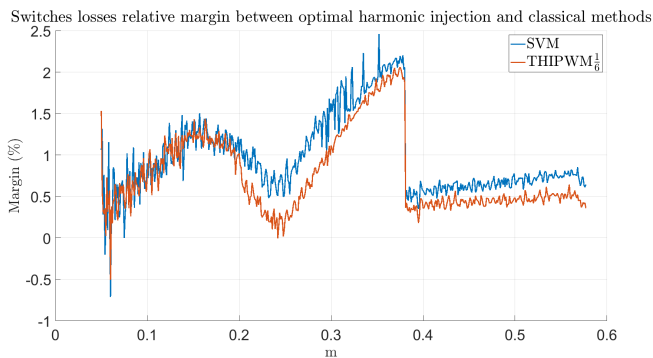


Fig. 6. Relative margin, (ϵ_r , eq.(21)) for optimal harmonic injection compared with SVM and THIPWM $\frac{1}{6}$ applied to switches losses only (\mathcal{P}_K , eq.(12)) for $N = 5$

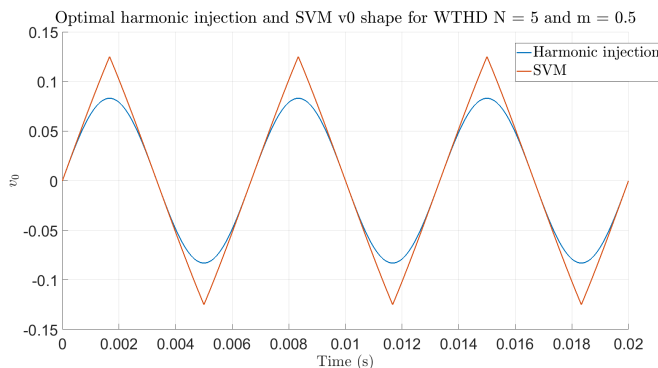


Fig. 7. Normalised and centred v_0 comparison between harmonic injection and SVM for WTHD (eq.(2)) considering $N = 5$ and $m = 0.5$

computation afford a small gain in comparison to THIPWM $\frac{1}{6}$ (according to WTHD objective function), the improvement never overcome 0.5% of improvement. Furthermore, for THIPWM $\frac{1}{6}$ the WTHD improvement is only for low and high modulation index. This observation is a confirmation of the literature, where THIPWM $\frac{1}{6}$ is considered as the best manner to reduce current harmonics.

Considering total losses objective function (Fig.5, eq. (7)) and switching losses objective function (Fig.6, eq. (12)), the harmonics losses (eq. (13)), seems to have a negligible impact on relative margin improvement. Furthermore, improvement appears to fit the same curve for both objective functions. But for these figures the improvement is more quantifiable than for WTHD one (Fig. 4), and is around 1% of increasing for every modulation index.

In Fig.7 and Fig.8 is represented the zero sequence component for WTHD objective function and total losses. For the WTHD (Figs. 7 and 4), as expected the optimal harmonic signal is near THIPWM $\frac{1}{6}$ wave form for $m = 0.5$, which is not surprising according to the corresponding improvement of harmonic injection seen on Fig. 4.

About total losses and switching losses (Figs.8, 5 and 6), the optimal curve for low modulation index ($m = 0.1$) is quite surprising because it does not look like any classical carrier based strategy. The curve looks like a sawtooth function, and a small Gibbs effect appears near the

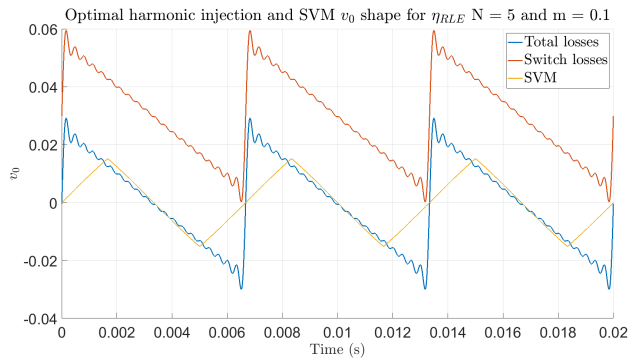


Fig. 8. Normalised and centred v_0 comparison between harmonic injection and SVM for Total losses (eq.(7)) and considering switches losses (eq.(12)) for $N = 5$ and $m = 0.1$

discontinuities.

Furthermore the optimal injection for total losses and for switching losses only, seems to be the same, except a different mean value (a_0 in eq. (14)). Consequently the optimal curve for switch losses is similar to the one obtained by a DPWM strategy. This observation leads to consider eq. (13) imposes that v_0 signal to have a mean value equal to zero in an optimal case.

Let's notice on Figs. 4, 5 and 6, SVM and THIPWM $\frac{1}{6}$ solutions has a negative relative margins. This implies that classical solutions are better than optimal zero sequence component. Nevertheless, for these modulation index, SVM and THIPWM $\frac{1}{6}$ do not respect the specified constraints. So, both classical strategies are unfeasible according to constraints definition (see eq. (20)) and especially inside tolerance.

B. Discussion

The carrier based property of the proposed strategy will ensure a fast implementation. The main problem seems to be the synchronism, because one of our main assumptions is to set invariant the number of commutations per period.

In this paper, the idea beyond this method is really simple, because it is just a particular case of harmonic injection. Because of the carrier based property of the method, this strategy will be easier to implement in comparison to OPPs. Off-lines solutions affording better solutions but are way more complicated to implement on a real engine.

Furthermore, ElectroMagnetic Compatibility (EMC) [19], or noise and harshness [20]–[22] problems are ignored but should be considered in future works. Moreover, results has been obtained in simulations. And in the next months an implementation will be done.

VI. CONCLUSION

The results in this paper highlight the benefits of the proposed method with respect to SVM and THIPWM $\frac{1}{6}$ (more generally in front of every classical carrier based zero sequence component). About WTHD, results support the well known observation that THIPWM $\frac{1}{6}$ is the better injection to increase current harmonic quality. For the RLE

load 1% of improvement could seem negligible, nevertheless considering, for example, the particular case of electric vehicles the range is one of the most important criterion to consider. Consequently each percent of power saved is a real improvement.

As the optimisation strategy can be easily generalized to more voltage levels, in a future work the proposed method will be applied to a multilevel converter.

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