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MTPV for Continuous Flux-Weakening Strategy Control Law for IPMSM

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Abstract- Synchronous motors with Interior Permanent Magnets (IPMSM) are particularly effective for operation at high speed due to their high power density. However the speeds generate large electromagnetic forces and it is necessary to set up a flux-weakening. Classical control algorithms do flux-weakening by injecting a negative Id current in respect of current norm. The value of this current is usually defined on the LUTs (Look Up Tables) depending on the speed and the torque required reflecting open-loop operation. This article presents continuous control law in closed loop ensuring the operation from the speed zero up to the maximum speed. The generation of the current reference I_d is based on strategy to Minimum Current per Torque (MinCPT) followed a Maximum Torque per Volt (MTPV) strategy. The goal is to maximize the speed for a given power. The effectiveness of this unified algorithm is tested on an experimental platform.

Keywords— Flux-Weakening, Maximum Torque per Volt, Torque Speed Characteristic, High Speed IPMSM Introduction

I. INTRODUCTION

Electric motors have better efficiency than thermal engines, and electric power is cheaper than fossil fuel energy. In particular, electric traction requires that the motors have high torque at low speed and a wide range of operation at constant power. The interior permanent magnet synchronous motor (IPMSM) guarantees the best power density and efficiency [1], and is also suitable for embedded systems. The volume and mass of an IPMSM are proportional to its torque. For embedded system applications this type of engine will be privileged and one will seek to make it operate at high speed to reduce its size. Certainly its torque will be reduced but it can be associated with a reducer to adapt it if necessary. In this context, the motor is fed by an electric battery through a threephase voltage source inverter (VSI). The control is generally carried out in a d-q reference frame and the torque reference is used to calculate the d-q current component references which are then regulated by two current loops. A Pulse Width Modulation (PWM) strategy is used to generate duty cycle for the phases of the machine. As the speed increases, the electromotive force of the machine grows, and due to the battery voltage limit, from a certain speed, it is necessary to introduce a flux-weakening of the magnet perceived by the windings of the machine by injecting a negative current on the axis d. This flux attenuation operation is fundamental for high speed operation.

The literature presents several strategies for flux-weakening [2 to 24] with [2, 3, 4, 5] presenting the state of the art of flux attenuation techniques. Most methods use off-line LUTs to calculate the value of the current to be injected as a function of the desired speed and torque [8, 9, 10, and 11]. Other solutions integrate analytical calculations [6, 7] or combining variants of several solutions. In any case, the implementation is often based on an algorithm switching triggered according to the operating point. The maximum power that the battery can deliver is generally not taken into account explicitly.

The work presented in this article proposes a continuous control law ensuring the control of the machine from the zero speed to the maximum speed taking into account the limitation of the battery power. The recommended path takes into account the current limitation (MinCPT) and then the voltage limitation (MTPV) in order to maximize the speed while taking care to reduce the electrical losses in the machine. The performance obtained with this algorithm is evaluated on a small IPMSM experimental bench.

II. OPERATION IN THE TORQUE-SPEED PLANE

A. PMSM modelling and limits

The PMSM electrical model equations with d-q Park Transform operation are presented by the electric part in (1) and (2) and the electromagnetic torque is expressed in (3).

$$V_d = R.I_d + L_d \frac{dI_d}{dt} - \omega.L_q.I_q$$
(1)

$$V_q = R.I_q + L_q \frac{dI_q}{dt} + \omega.L_d.I_d + \omega.\varphi_f$$
(2)

$$T = \frac{3}{2} \cdot p \cdot \left(\phi_f \cdot I_q + (L_d - L_q) I_d I_q \right)$$
(3)

 $V_d,\,V_q\!:\!d,\,q \text{ voltage components};$

 I_d , I_q : d, q current components

 L_d , L_q : d and q inductances;

p : number of pairs of poles

ω: Electrical rotor angular velocity;

R: Resistance by phase winding

 $\Phi_{\rm f}$: Flux linkage of PMSM;

T: Electromagnetic torque.

We define a maximum current I_{max} (4) that integrates the limiting possibilities of the electrical machine, those of the inverter and also those of the battery.

$$\sqrt{I_d^2 + I_q^2} \le I_{\max} \tag{4}$$

We also define a maximum available voltage V_{max} (5) that integrates the DC battery voltage V_{DC} and the modulation method used, here a space vector modulation (6).

$$\sqrt{V_d^2 + V_q^2} \le V_{\max} \tag{5}$$

$$V_{\max} = \frac{V_{DC}}{\sqrt{3}} \tag{6}$$

At high speed the resistance is usually negligible in (1) and (2). So, the voltage limit can be written in steady state as given in (7).

$$\sqrt{\left(\omega L_q I_q\right)^2 + \left(\omega L_d I_d + \omega \phi_f\right)^2} \le V_{\max} \tag{7}$$

This equation determines an ellipse in the plane I_q - I_d having for center, height and width respectively:



Depending on the parameters of the machine and the power source, the center of the ellipse may be within the maximum circle of current or outside. In the first case, it is theoretically possible to completely weaken the magnet flux and reach very high speeds.

B. Torque-Speed and Power Limit Plan.

For a buried magnet machine (IPMSM), the direct and quadrature inductances are different and an additional torque contribution appears due to the salient torque [3]. The

components I_d and I_q must be defined in order to obtain a given torque. If one wishes to minimize the copper losses the two components are defined according to Maximum Torque by Ampere law (MTPA), equations (9) and (10). The parameters of the machine studied are given in Table 1 below. In the torque-speed plane, it is easy to show the limitation of power according to a hyperbolic curve, Fig. 2.

The typical operation of this machine results in high torque at low speed and a wide speed range at constant power. Fig. 2 shows the maximum torque achievable as a function of speed and nominal operation for different control strategies for a power of 1kW. The cyan colour curve shows the batteryinduced limitation, for example 800 W. The maximum torque at low speed given by the maximum torque per ampere (MTPA) is plotted in blue-solid line, which is also the minimum of Current per unit torque (MinCPT), see below equations (14) and (15). When the voltage limit is reached (12), it is necessary to attenuate the flux to attain the operating points at a higher speed. The dotted blue curve shows the evolution of the speed without flux-weakening strategy. The dotted red curve shows the evolution at maximum current and then when the power limit is reached a maximum voltage evolution which leads to a drop in the useful power.



TABLE I. IPMSM PARAMETERS

Reduced size salient pole PMSM parameters				
R	0.97	Ω		
L _d	4.73	mH		
Lq	5.77	mH		
$\phi_{\rm f}$	0.0345	Wb		
Number of Poles	10			
I _{max}	8	А		
V _{DC}	200	V		
Battery power	800	W or 1 kW		

The red-solid curve finally shows the evolution at maximum torque per unit of volt (MTPV) which allows to

operating at maximum power. This last part constitutes the main objective of this article.

III. MTPA STRATEGY FOR ID AND IQ DEFINITION.

When the device has a rotation speed loop the controller provides a current reference I_{norm} i.e. the total current magnitude that can be injected into the machine. For minimizing the copper losses, the contributions on the d-axis and on the q-axis should be judiciously distributed in order to obtain a maximum torque for a given current, there is the MTPA strategy. So we have to solve the following problem:

$$I_d, I_q \Rightarrow \max T \quad \text{with } \sqrt{I_d^2 + I_q^2} = I_{norm}$$
(9)

$$\frac{dT}{dI_d} = 0 \quad \text{with} \quad I_q = \sqrt{I_{norm}^2 - I_d^2} \tag{10}$$

This calculation gives the following results, Fig. 3:

$$I_d = \frac{\phi_f - \sqrt{\phi_f^2 + 8(L_d - L_q)^2 I_{norm}^2}}{-4(L_d - L_q)}$$
(11)

$$I_q = \sqrt{I_{norm}^2 - I_d^2} \tag{12}$$



Fig.3. Evolution of Id and Iq in MTPA strategy

IV. MINCPT STRATEGY FOR ID AND IQ DEFINITION.

When the reference is a desired torque Tref, as in the case of the electric traction, it is necessary to correctly distribute the contributions of the d,q axes in order to minimize the current norm, in relation to this desired torque. Consequently, there is a Minimum current strategy for a given torque (MinCPT). The problem is then formulated as follows:

$$I_d, I_q \Rightarrow \min I_{norm} = \sqrt{I_d^2 + I_q^2} \text{ with } T = T_{ref}$$
 (13)

By using Lagrange multiplier [31], we can define this equation (14) which has two complex solutions, a positive and a negative solution that we retain. Note that we can establish an analytical solution using a Ferrari-type method [30]. The value of I_q is obtained using (15) and the relation (14).

$$I_{d}^{4}(L_{d} - L_{q})^{3} + I_{d}^{3}(L_{d} - L_{q})^{2}3\phi_{f} + I_{d}^{2}(L_{d} - L_{q})3\phi_{f}^{2} + I_{d}\phi_{f}^{3} - \frac{4(L_{d} - L_{q})T_{ref}}{9p^{2}}^{2} = 0$$
(14)

$$I_{q} = \frac{T_{ref}}{3/2.p.(\phi_{f} + (L_{d} - L_{q})I_{d})}$$
(15)



In fact this law is equivalent to the previous one as shown in Fig. 4, which changes the input variables.

V. MTPV STRATEGY FOR ID AND IQ DEFINITION.

The objective is to maximize the torque obtained from the available voltage. We therefore impose the norm of the voltage at the maximum voltage Vmax. This strategy is called MTPV (Maximum Torque Per Volt). We assume the negligible resistance at high speed with respect to the terms in L. ω . The electrical model of the IPMSM is thus expressed in steady state by (16) and (17).

$$V_d = -L_q \cdot \omega \cdot I_q \tag{16}$$

$$V_a = \omega L_d I_d + \omega \phi_f \tag{17}$$

The problem is then stated as follows:

$$V_d, V_q \Rightarrow \max T = \frac{3}{2} p \left(\phi_f + \left(L_d - L_q \right) \left(\frac{V_q}{\omega L_d} - \frac{\phi_f}{L_d} \right) \right) \left(-\frac{V_d}{\omega L_q} \right)$$
(18)
with $V_d^2 + V_q^2 = V_{max}^2$

By using the Lagrange multiplier method, it is easy to extract the values of V_d and V_q which maximize the torque produced by the machine.

$$V_{q} = \frac{\phi_{f} - \sqrt{\phi_{f}^{2} + V_{\max}^{2} 8 \left(\frac{L_{d} - L_{q}}{L_{q}\omega}\right)^{2}}}{-4 \left(\frac{L_{d} - L_{q}}{L_{q}\omega}\right)}$$
(19)

$$V_d = -\sqrt{V_{\text{max}}^2 - V_q^2} \tag{20}$$

The currents are then deducted by (16), (17), (19) and (20):



Fig.5. Evolution of Id, Iq and Torque in function of speed

Fig. 5 shows the evolution of the current components (I_d and I_q) as well as the torque as a function of the speed when the voltage module is maximum. This mode shows a decrease in the current norm as a function of the speed, which limits

copper losses. The curves in Fig. 5 have been plotted without taking into account the limitation of the battery; therefore the total usable power is $V_{DC}*I_{max}=1,6kW$..

VI. CONTINOUS FLUX WEAKENING STRATEGY

There are numerous flux weakening strategies in the literature, they are based on different principles. We can cite the following approaches:

- Direct open loop algorithm by direct analytical calculation (see previous section III, IV, V). [23], [24])

- With a table approach, called Look Up Tables and Torque and Flux Control (TFC) considerations [6], [7], [8]).

- With a single regulator for the current loop: SCR (Single Current Regulator) [9])

- With vector control of current and regulation on the voltage standard [17]:

Here, we seek to develop a control law operating to adapt continuously the references of I_d and I_q as a function of the speed while taking into account the voltage saturation and the power available for the source. For this strategy we propose the diagram of figure 6. It is composed of 4 essential blocks:

The block 1 generates an allowable torque reference T from a Tref torque reference considering the limits of current, voltage and power of the battery. This procedure takes into account the performance achievable with the MTPV approach.

The block 2 gives the current I_d out of the flux-weakening to be applied in order to obtain the torque by minimizing the current norm. For this purpose, the MinCPT strategy is used. The current I_d is thus calculated from (14) with T_{ref} =T.



Fig. 6 Scheme of the PMSM control with continous flux weakening, (where AW is an anti-windup structure).

The block3 represents the voltage regulator. The output of this regulator is saturated in such a way as to adapt the current to be injected. This contribution is always negative. If the current I_d calculated for the MTPV strategy is more constraining then the saturation of the output current of the flux weakening strategy control takes account of this current limit. The output current of the saturation is then added to the current IdMinCPT to obtain the total reference current I_{dref} .

In block 4 the reference current I_{qref} is calculated using the inversion of the equation of the torque (3). Then the current I_{qref} is saturated in such a way that the final reference current standard respects the current limit I_{max} .

VII. EXPERIMENTAL RESULTS

To evaluate the proposed control laws an experimental setup is realized with 2 PMSM mounted on the same axis (Fig. 7). The first driven machine is powered to act the second one working as a load. The assembly is controlled by a dSPACE system (DS1104) by using standard 3-phase inverter operating at 8 kHz switching frequency in SVM mode.



Fig.7 Experimental setup

Three control algorithms are tested and compared in this part:

- The control algorithm without flux weakening procedure with the current loop alone. The current set points are directly defined by the MTPA strategy.

- The control algorithm with standard flux weakening strategy.

- The control algorithm that we propose with the flux weakening procedure including the MTPV strategy in a unified way.

The three algorithms in this section include the prediction of the variation of the angle (due to the computation delay and the discontinuous nature of the inverter) for inverse Park transformation. The tests are carried out with a DC bus voltage of 50V. With this voltage and maximum torque set point the flux weakening starts from 869 rpm. The current limit is 6.2A. This limit is set voluntarily, a little beyond the critical current point define by (8). The measurements are carried out in steady state by scanning the speed approximately every 500 rpm. There is no maximum speed limit included in the system control algorithms. The set point is a torque reference. The maximum speed reached by the motor with each algorithm results from the torque created by the motor and mechanical viscous friction. Without a flux weakening strategy, we observe in Fig. 8 that the torque generated decreases greatly as soon as the voltage limit is reached. The maximum speed achieved in this case is 1551 rpm. From these experiments with a flux weakening strategy including the MTPV we observe that we have the same results as by experiment with the standard flux weakening algorithm up to 5000 rpm. We remark that starting from 5000 rpm the conventional strategy gives us less torque than the algorithm with MTPV.



Fig.8. Torque-Speed plane for 3 flux weakening startegy

This additional torque makes it possible to achieve a higher speed thanks to this new algorithm. The conventional flux weakening strategy ensures a maximum speed of 7227 rpm and the MTPV strategy 8023 rpm. Table II summarizes the results obtained. We validate by experience that this new algorithm allows to increase the torque and thus the power transmitted at high speed compared to the usual control strategies.

TABLE II

Flux Weakening Strategy	Maximal Speed		Stator Current frequency	
Without FW	1551	rpm	129,25	Hz
Classical FW	7227	rpm	602,25	Hz
FW + MTPV	8023	rpm	668,58	Hz

VIII. CONCLUSION

The high-speed operation of IPMSM requires the use of a flux weakening procedure. This technic relies on the injection of a negative current on the d-axis in order to reduce the flux in the air gap. To maximize the torque for a given speed it is important to take into account the limitation of the current norm and the maximum voltage. The conventional methods are based on maximum current and maximum voltage operation. The MTPV approach allows operation with a maximum voltage but with a current standard that is not necessarily maximum. The use of this principle in a continuously evolving control scheme allows a significant gain in maximal speed (> 10% with 8023 rpm). This result is interesting for embedded systems because for a maximum speed desired the size of the machine will be reduced. This is the case for electric traction for electric cars, for example.

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