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Globally Converging Observers for SPMSM Sensorless Control

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Abstract-This paper proposes a kind of globally converging observer for sensorless control of surface mount permanent magnet synchronous motors (SPMSM). The observers are based on rotor flux-oriented synchronous coordinates, and can converge to the real rotor position from any unknown initial positions. The observers can be performed by extended Kalman filter (EKF), state observer or model reference adaptive system (MRAS). By some compensation in q-axe equation, the unexpected equilibrium points can be eliminated and then observers can converge to the unique expected equilibrium point. Then the motors can start up from any unknown initial positions. Simulation and experimentation results have verified the effects of the method.

Keywords: surface mount permanent magnet synchronous motor, initial position, extended Kalman filter, model reference adaptive system, state observer

I. INTRODUCTION

Permanent magnet synchronous motors (PMSM) are more and more used because of its high power density, large torque to inertia ratio and high efficiency. High performance control of PMSM needs the information of rotor position which is generally measured by a mechanical sensor. But the mechanical sensor is expensive and increases the complexity and volume of the system. So mechanical sensorless control is becoming a research focus now.

Some sensorless control methods have been proposed before. Generally there are the methods based on back electromotive force (EMF), model reference adaptive system (MRAS)^[1], state observer and Kalman filter. The EMF and MRAS methods are simple but are greatly depended on the accuracy of the parameters. Methods based on state observer methods are more robust because of the feedback correction. The problem with state observer is how to design the feeback matrix and how to linearize the nonlinear equations of PMSM. Extended Kalman filter (EKF) is an extension in the nonlinear system of linear Kalman filter (LKF). Since the equations of the PMSM are nonlinear, extended Kalman filter is more suitable for sensorless control of PMSM than linear state observer.

In PMSM sensorless control, initial position is needed for start up. One method is to fix the initial position by some DC current or DC voltage before start up. By effects of the DC current, the rotor will stop at an expected initial position. The problem of this method is the value and duration of the voltage and current are hard to be known before. They are determined by load torque and inertia and should be tried by experiments. If the voltage amplitude and duration are not well selected, the DC current will damage the motor. Another method is to estimate or measure the rotor position with some test voltage. This method need the motor have some salient or stator has some saturation. In non-salient or just little salient motors, for example, the surface mount permanent magnet synchronous motor (SPMSM), this method cannot be used very well.^[1]

In this paper, the equilibrium points of the proposed observer are analyzed and then some measures are taken to move off the unexpected points. Since only one expected equilibrium point exist, the observer can converge to the expected points in spite of the large initial position errors. This method can be used for the MRAS, state observer and extended Kalman filter as long as they are built based on rotor flux oriented synchronous coordinate. Simulation and experiments have been performed to verify the efficiency of the method.

II. MODEL OF THE SYSTEM

In rotor flux oriented synchronous coordinate (D-Q axes), the voltage equations of the SPMSM are:

$$U_D = RI_D + L\frac{dI_D}{dt} - \omega LI_Q \quad ; \tag{1a}$$

$$U_{\varrho} = RI_{\varrho} + L\frac{dI_{\varrho}}{dt} + \omega LI_{D} + \omega K_{e} .$$
^(1b)

Here U_D and U_Q are stator voltages in D-Q axes. I_D and I_Q are stator currents. *L* is stator inductances, *R* is stator resistance. ω is rotor electrical angle speed. K_e is rotor flux amplitude on D axes.

Use currents as variables, the equations are:

$$\frac{dI_D}{dt} = \frac{U_D}{L} - \frac{R \cdot I_D}{L} + \omega \cdot I_Q; \qquad (2a)$$

$$\frac{dI_{Q}}{dt} = \frac{U_{Q}}{L} - \frac{R \cdot I_{Q}}{L} - \omega \cdot I_{D} - \frac{K_{e}}{L} \cdot \omega \quad .$$
(2b)

Since the real rotor position is not known exactly in sensorless control system, the coordinate is oriented by the estimated rotor flux position $\hat{\theta}$. The axes become d and q as shown in Fig. 1.

Rotor position estimation error is defined by:

$$\gamma = \theta - \hat{\theta} \quad . \tag{3}$$



Fig.1. Real and estimated axes

The voltages and currents in D-Q axes and d-q axes have the relations as (4).

$$\begin{cases} U_d = U_D \cos \gamma - U_Q \sin \gamma \\ U_q = U_Q \cos \gamma + U_D \sin \gamma \\ \int I_d = I_D \cos \gamma - I_Q \sin \gamma \\ I_q = I_Q \cos \gamma + I_D \sin \gamma \end{cases}$$
(4)

The equations in d-q axes are defined as follows:

$$\begin{cases} \frac{dI_d}{dt} = \frac{U_d}{L} - \frac{I_d}{\tau} + \omega \cdot I_q + \frac{K_e}{L} \cdot \omega \sin \gamma \\ \frac{dI_q}{dt} = \frac{U_q}{L} - \frac{I_q}{\tau} - \omega \cdot I_d - \frac{K_e}{L} \cdot \omega \cos \gamma \end{cases};$$
(5)

The movement equation of the motor is:

$$J\frac{d\Omega}{dt} = T_{em} - T_L(\Omega) \,. \tag{6}$$

Here J is mechanical inertia; Ω is rotor mechanical angle speed. T_{em} is electromagnetic torque of SPMSM.

$$T_{em} = p \cdot K_e \cdot I_Q = p \cdot K_e (I_q \cos \gamma - I_d \sin \gamma)$$
(7)

 $T_L(\Omega)$ is load torque which is the function of rotor speed. Generally there are three kinds of load torque:

- 1) First is friction torque which is determined by rotor speed and has an opposite direction with rotor speed. Mark the torque as T_f
- 2) Second is block torque which has a fix value when rotor speed is not zero and has an opposite direction with rotor speed. When rotor speed is zero, it equals the electromagnetic torque of SPMSM and has an opposite direction. Mark the torque as T_{B} .
- 3) The third is electromotive torque which has a fix direction. If the electromagnetic torque of SPMSM is smaller than the electromotive torque, the system will reverse. Generally this load torque is supplied by electric motor. Mark the torque as T_M

Most of load torques is the first two kinds.

If rotor electrical angle speed ω is used instead of Ω ,

$$\Omega = \omega / p \tag{8}$$

Then (6) becomes:

$$\frac{d\omega}{dt} = \frac{p^2 \cdot K_e}{J} \cdot \left(I_q \cos \gamma - I_d \sin \gamma \right) - T_L(\omega) .$$
(9a)

With
$$\frac{d\theta}{dt} = \omega$$
 (9b)

If we ignore the effects of inverter and suppose the voltages used in observer are precise, suppose the observer equations are:

$$\frac{d\hat{I}_d}{dt} = \frac{U_d}{L} - \frac{\hat{I}_d}{\tau} + \hat{\omega} \cdot \hat{I}_q \quad ; \tag{10a}$$

$$\frac{d\hat{I}_q}{dt} = \frac{U_q}{L} - \frac{\hat{I}_q}{\tau} - \hat{\omega} \cdot \hat{I}_d - \frac{K_e}{L} \cdot \hat{\omega}; \qquad (10b)$$

$$\frac{d\hat{\theta}}{dt} = \hat{\omega}.$$
 (10c)

Then the errors between real and estimated state variables are written by:

 $\varepsilon_d = I_d - \hat{I}_d$ and $\varepsilon_q = I_q - \hat{I}_q$

$$\frac{d}{dt}\varepsilon_d = -\frac{\varepsilon_d}{\tau} + \left(\omega \cdot I_q - \hat{\omega} \cdot \hat{I}_q\right) + \frac{K_e}{L} \cdot \omega \sin\gamma \quad (11a)$$

$$\frac{d}{dt}\varepsilon_q = -\frac{\varepsilon_q}{\tau} - (\omega \cdot I_d - \hat{\omega} \cdot \hat{I}_d) - \frac{K_e}{L} (\omega \cos \gamma - \hat{\omega}).$$
(11b)

$$\frac{d}{dt}\gamma = \frac{d}{dt}\theta - \frac{d}{dt}\hat{\theta} = \omega - \hat{\omega}$$
(11c)

If there are equilibrium points, they should satisfy:

$$\begin{cases} \frac{d}{dt} \varepsilon_d = 0 \\ \frac{d}{dt} \varepsilon_q = 0 \\ \frac{d}{dt} \gamma = \omega - \hat{\omega} = 0 \end{cases}$$
(12)

Since the current I_d and I_q are obtained by the measured currents I_a , I_b and I_c , Some feedback or adaptive algorithms will be used to make estimated currents to be the same as measured currents. Suppose the algorithms are effective and then in steady states there are:

$$\varepsilon_d = I_d - \hat{I}_d \to 0 \text{ and } \varepsilon_q = I_q - \hat{I}_q \to 0$$
 (13b)
Then the equations become:

Then the equations become:

$$\frac{d}{dt}\varepsilon_d = \left(\omega \cdot I_q - \hat{\omega} \cdot \hat{I}_q\right) + \frac{K_e}{L} \cdot \omega \sin\gamma ; \qquad (14a)$$

$$\frac{d}{dt}\varepsilon_{q} = -\left(\omega \cdot I_{d} - \hat{\omega} \cdot \hat{I}_{d}\right) - \frac{K_{e}}{L} \cdot \left(\omega \cos \gamma - \hat{\omega}\right).$$
(14b)

If we just consider the first two equations of (11), there are three kinds of equilibrium points:^[2]

$$\begin{cases} \hat{\omega} = \omega \neq 0\\ \gamma = 0 \end{cases} ; \tag{15a}$$

$$\begin{cases} \hat{\omega} = \omega = 0 \\ T_{em} = p \cdot K_e (I_a \cos \gamma - I_d \sin \gamma) = T_L(\Omega); \end{cases}$$
(15b)

$$\begin{cases} \hat{\omega} = \frac{L^2 I_q^2 - K_e^2}{L^2 I_q^2 + K_e^2} \omega \\ \cos \gamma = \frac{L^2 I_q^2 - K_e^2}{L^2 I_q^2 + K_e^2}; \sin \gamma = \frac{-2K_e I_q \cdot L}{L^2 I_q^2 + K_e^2} \end{cases}$$
(15c)

Only the first one is the expected point.

In some kinds of observers, the third equilibrium point does exist. Normally $|LI_q| < K_e$, then it is a situation of reverse. But it doesn't matter the third equation of (14), so in fact this wrong situation is made by the disadvantage of the observer. If the observer is very well designed and the parameters are well adjusted, the third equation of (14) affects well, the third situation will not exist.^[3]

For the second solution, they are determined by the style of load torque and speed regulator. Because the observed speed is zero, so the output of speed regulator (torque reference) will increase until the maximum value. In most of the control systems of SPMSM, d-axe currents are controlled to be zero.

$$I_d = \hat{I}_d = 0 \tag{16}$$

If the load torque is consist only by friction and block torque, the situation has two kinds:

$$\hat{\omega} = \omega = 0; \gamma = \pm 0.5\pi . \tag{17}$$

The electromagnetic torque will always be zero:

$$T_{em} = p \cdot K_e \cdot I_Q = p \cdot K_e (I_q \cos \gamma - I_d \sin \gamma) = 0$$
(18)

If the load is electromotive torque or d-axe current is not zero, position error γ will be other values which will satisfy:

$$T_{em} = p \cdot K_e (I_q \cos \gamma - I_d \sin \gamma) = T_L.$$
⁽¹⁹⁾

The second kinds of equilibrium points are mainly determined by the second equation, so we can add some compensation in the second equation to break out this balance. We change the equation to:

$$\frac{d\hat{I}_q}{dt} = \frac{\hat{U}_q}{L} - \frac{\hat{I}_q}{\tau} - \hat{\omega} \cdot \hat{I}_d - \frac{K_e}{L} \cdot \hat{\omega} + \frac{k \cdot RI_q}{L} .$$
(20)
Here k is a coefficient and $k \ge 0$

With the compensation, the unexpected equilibrium points can be avoided. In steady states, the compensation will be considered as a little error in stator resistance parameter. Its effects will be eliminated by the robustness of the system. Normally the observer can be realized by extended Kalman filter, state observer or MRAS method, simulations and experiments based on these three methods have been proposed to verify the effects of the system.

III. EXTENDED KALMAN FILTER

Based on the analyses former, an observer based extended Kalman filter (EKF) is built.^{[3][4][5][7]} The parameters of the motor are as shown in TABEL I.

Equations of EKF with q-axe compensation are:

$$\frac{dI_{d}}{dt} = \frac{U_{d}}{L} - \frac{I_{d}}{\tau} + \hat{\omega} \cdot \hat{I}_{q}$$

$$\frac{d\hat{I}_{q}}{dt} = \frac{\hat{U}_{q}}{L} - \frac{\hat{I}_{q}}{\tau} - \hat{\omega} \cdot \hat{I}_{d} - \frac{K_{e}}{L} \cdot \hat{\omega} + \frac{k \cdot RI_{q}}{L}$$

$$\frac{d\hat{\omega}}{dt} = 0$$

$$\frac{d\hat{\theta}}{dt} = \hat{\omega}$$
(21)

Without compensation in q-axe equation (k=0), simulation results during start up with little initial position errors is shown in Fig. 2.

By simulation results, the converging areas of the three equilibrium points are shown in Fig. 3a when torque current is positive. It is shown that when the initial position error $-\pi/2 < \gamma_0 < \gamma_a$, the motor can start up successfully. Otherwise, system will converge to the other two equilibrium points. If the initial position error γ_0 located in area A, γ will converge to zero directly. If γ_0 located in area B $(0 < \gamma_0 < \gamma_b)$, γ will increase and then decrease to zero. If γ_0 is in area C ($\gamma_b < \gamma_0 < \gamma_a$), γ will increase and pass the un-converging area, finally converge to zero. The values of γ_a and γ_b cannot be easily obtained by theory analyses. The converging area of equilibrium point $\gamma = -0.5\pi$ is determined by the block toque and generally the area is very little as shown in (22). We call it block-torque-dependent equilibrium points.

$$p \cdot K_e I_{q \max} \cos(-0.5\pi + \Delta \gamma_c) = T_B \qquad (22)$$

TABLE I.PARAMETERS OF THE MOTOR

Parameters	Values
Stator inductance $L_d = L_q = L$	0.00125 H
Stator phase resistance R	0.155 Ω
Rotor magnet flux K_e	0.153 Web
Number of pole pairs <i>p</i>	4



Fig. 2. Start-up with little initial position error (simulation) Converging areas when torque current is negative are shown in Fig. 3b.

Some kinds of observer can also be designed to have only one equilibrium points as in (15a) and other two blocktorque-dependent equilibrium points as shown in (15b). Then the observer is almost globally converging when block torque is little. ^[6]



Fig.3a. Converging areas of the different equilibrium points when Iq>0



Fig.3b. Converging areas of the different equilibrium points when Iq<0



Fig.4. Start up with large initial position error

With compensation, simulation results when there is a large initial position error $(4\pi/3)$ is shown in Fig.5. The motor can start up successfully under the effects of compensation. Simulations also show that with the same load torque and mechanical inertia, the coefficient of compensation *k* has no relations with the initial position error. With the same value of *k*, result of start up without initial position errors is shown in Fig. 6.

To test the start up ability with load torque, simulation result with an electromotive torque of 5 Nm is shown in Fig. 7. Simulations also show that the system can start up when there is only a block torque.

Simulations also show that in steady state, the compensation has very little effects in rotor speed and position estimation.

Some experiments have been done on a platform with DSP C6711 as controller. Experiment system used in this paper is shown in Fig.8 .In experiments, the initial value of estimated rotor position is always zero, and the real rotor position is random. Experiment results in Fig. 9 and Fig. 10 show that the motor can start up from any unknown initial position under the effects of the compensation and are well accorded with the simulation results. In spite of the short reverse course during start up when the initial position error is larger, the observer can be used in most of systems which have a not too high start-up performance requirement.

In practice experiments, if there are disturbance and DC offset in current measurement, some reverse state will exist after start up when there is very little friction and block torque, as shown in Fig.11. But it isn't the situation described in (15c). In this situation, estimated speed is still the same with real speed, and position estimation error is about $\pi/2$. So this situation is a practical variation of (15b).



Fig. 6. Start-up with compensation and no initial position error (simulation)





Fig. 8. Block diagram of the sensorless drive system



Fig. 9. Start-up with large initial position error (experiment)



Fig. 10. Start-up with little initial position error (experiment)



In [6], it is said that the start-up performance is produced by Kalman filter which has a better robustness than ordinary state observers, following analyses will show that the global convergence is produced by the compensation, but not a particular advantage of Kalman filter.

IV. STATE OBSERVER

The same equations are used as (21). The difference between EKF and state observer is that the feedback matrix of EKF is updated in every calculation period by Kalman iteration algorithm while the feedback matrix of state observer is fixed or just be changed artificially. The iteration equation of state observer is:

$$X(k+1) = X(k) + X \cdot T_e + L \cdot (y - C \cdot X(k)) .$$
(22)

In state observer, the value of the feedback matrix will affect the stability of the system and it is difficult to know the ideal values of matrix L. Here we use the steady state values of matrix K of extended Kalman filter. The values of matrix are as follows:

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \\ L_{31} & L_{32} \\ L_{41} & L_{42} \end{bmatrix} = \begin{bmatrix} 0.99 & -0.00001 \\ -0.00001 & 0.99 \\ 0.168 & -0.97 \\ 0.00136 & -0.00784 \end{bmatrix}.$$
 (23)

The same compensation as extended Kalman filter is used in q-axe equation. Simulation results with different initial position errors are shown in Fig. 12 and Fig. 13 while experiment results are shown in Fig. 14 and Fig. 15.

Simulations and experiments also show the same results with extended Kalman filter when there is no compensation with large initial position errors. The converging areas are also the same as shown in Fig. 3a and Fig. 3b.



Fig. 12. Start-up with compensation (simulation)



Fig. 13. Start-up with compensation (simulation)



Fig. 14. Start-up with compensation and a large initial position error (experiment)



Fig. 15. Start-up with compensation and a little initial position error (experiment)

V. MRAS METHOD

Another familiar observer used for SPMSM is MRAS method. We take one kinds of MRAS methods as example.^[1] SPMSM model (reference model) is built on d-q axes:

$$\begin{cases}
\frac{dI_d}{dt} = \frac{U_d}{L} - \frac{I_d}{\tau} + \omega \cdot I_q \\
\frac{dI_q}{dt} = \frac{U_q}{L} - \frac{I_q}{\tau} - \omega \cdot I_d - \frac{K_e}{L} \cdot \omega
\end{cases}$$
(24)

Adaptive model is used to estimate d-q currents:

$$\frac{dI_d}{dt} = \frac{U_d}{L} - \frac{I_d}{\tau} + \hat{\omega} \cdot \hat{I}_q \\
\frac{d\hat{I}_q}{dt} = \frac{U_q}{L} - \frac{\hat{I}_q}{\tau} - \hat{\omega} \cdot \hat{I}_d - \frac{K_e}{L} \cdot \hat{\omega}$$
(25)

Equations for state estimation errors can be obtained by the subtraction of (24) and (25).

$$\begin{cases} \frac{d\varepsilon_d}{dt} = -\frac{\varepsilon_d}{\tau} + \omega \cdot I_q - \hat{\omega} \cdot \hat{I}_q \\ \frac{d\varepsilon_q}{dt} = -\frac{\varepsilon_q}{\tau} - \omega \cdot I_d + \hat{\omega} \cdot \hat{I}_d - \frac{K_e}{L} \cdot (\omega - \hat{\omega}) \end{cases}$$
(26)
Use: $V = \varepsilon^T \varepsilon = \varepsilon_q^2 + \varepsilon^2 > 0$ (27)

Based on Liapunov and Popov's stability criterion, rotor speed identification algorithm can be set as:

$$\varepsilon = I_d \hat{I}_q - I_q \hat{I}_d - \frac{K_e}{L} (I_q - \hat{I}_q); \qquad (28a)$$

$$\hat{\omega} = \left(K_1 + \frac{K_2}{s}\right) \cdot \varepsilon \quad . \tag{28b}$$

Rotor position is obtained by the integration of rotor speed. The system is shown in Fig.16. In simulation, if there is no compensation in q-axe equation of the adaptive model, the motor cannot start up when the initial position error is too large. With compensation as former, simulation and experiment results shown that the motor can start up from any unknown initial positions (Fig.17, Fig. 18, Fig. 19 and Fig. 20).



Fig. 16. Block diagram of MRAS method



Fig. 19. Start-up with a large initial position error (experiment)



Fig. 20. Start-up with a large initial position error (experiment) It shows that the model reference adaptive system based on d-q axes equations also has a global converging ability under the effects of q-axe compensation.

Simulation results show that the full order observer and MRAS method with the same motor parameters have the same converging areas as shown in Fig. 3.

CONCLUSION

The compensation in q-axe equations can move off the unexpected equilibrium points of the observer. With the compensation, observer can converge to the real state of the motor from any unknown initial rotor position. In steady state, the compensation just has little effect in the precision of rotor speed and position estimation. The method is verified by extended Kalman filter, state observer and model reference adaptive systems. It shows that the method can be used in almost all the observers based on rotor flux oriented synchronous axes. The globally converging observer is a useful method to solve the problem of SPMSM start up. The remaining problem is that the compensation coefficient k is not easily to be obtained by theory analyses or calculation, today the only way is to adjust the coefficient by practice.

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