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# Sensorless Control of PMSM Based on Extended Kalman Filter

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## Keywords

«Permanent magnet motor», «Synchronous motor », «Vector control», « Sensorless control ».

## Abstract

This paper proposes a sensorless control system based on extended Kalman filter (EKF) for permanent magnet synchronous motors (PMSM). The EKF equations are built in rotor flux oriented synchronous coordinate, so it can easily be used for either non-salient or salient pole motors. Inertia and other mechanic parameters are not needed in this observer. Rotor speed and position can be estimated exactly and then a sensorless control system is built. The initial rotor position and the mechanic parameters are not needed in this system. By some compensation in observer equations, the observer can be always stable and has only one expected equilibrium point. So the motor can start up from any unknown initial positions.

## Introduction

Permanent magnet synchronous motors (PMSM) are more and more used because of its high power density, large torque to inertia ratio and high efficiency. The rotor flux is generated by the permanent magnet on the rotor. So the rotor flux position is the same as the rotor electrical position. And the precious rotor position is needed for the high performance control. Since mechanic position sensor is usually too expensive, increases the cost and decrease the stability of the system, mechanic sensorless control is becoming a research focus now.

Some sensorless control methods have been proposed before. Generally there are the methods based on back electromotive force (EMF), model reference adaptive system (MRAS) (example in [1]) and state observer method. The methods based on back EMF are simple but doesn't work well in low speed region because the back EMF is too small compared with the noise. The method based on MRAS also cannot get a satisfying performance in low speed region and is greatly depended on the accuracy of the reference model. The state observer method is not suitable for the nonlinear model and is hard to know the feedback matrix. Also some methods based on Kalman filter have been proposed, most of which are founded in the static two-phase coordinate, since the stator inductance of salient pole motor is a variable of rotor position in static two-phase coordinate, these observers can hardly be used for salient pole motors [2][3][4].

In this paper, a new state observer based on extended Kalman filter is used to observe the rotor position and speed. The observer model is set up in the rotor flux oriented synchronous coordinate, so it can be used easily in either salient or non-salient pole motor because the stator inductances in synchronous coordinate are always constant. Extended Kalman filter can solve nonlinear equation directly by numeric iteration. Kalman filter also considers the errors of the parameters and the noises in the measurement, so it is very robust with the parameters' errors and measurement noises. Also the initial rotor position is not necessary for the start-up. By a proper compensation in the observer equation, the other unexpected equilibrium points of the observer are moved off. The motor can start-up successfully from any unknown initial position[5][6][7].

## Observer based on extended Kalman filter

In rotor flux oriented synchronous coordinate (d,q axes), PMSM model is shown in (1).

$$\begin{cases} \frac{dI_d}{dt} = \frac{U_d}{L_d} - \frac{R \cdot I_d}{L_d} + \omega \cdot \frac{L_q}{L_d} I_q \\ \frac{dI_q}{dt} = \frac{U_q}{L_q} - \frac{R \cdot I_q}{L_q} - \omega \cdot \frac{L_d}{L_q} I_d - \frac{\psi_r}{L_q} \cdot \omega \\ \frac{d\omega}{dt} = 0 \\ \frac{d\theta}{dt} = \omega \end{cases} \quad (1)$$

Here  $I_d$  and  $I_q$  are the currents in d and q axes.  $R$  is stator resistance while  $L_d$  and  $L_q$  are the stator phase inductances in d and q axes. For non-salient motors,  $L_d$  is the same as  $L_q$ .  $U_d$  and  $U_q$  are stator voltages and  $R$  is stator resistance.  $\omega$  is the rotor electrical angle speed and  $\theta$  is rotor electrical angle (rotor flux angle).  $\psi_r$  is rotor flux amplitude. Rotor speed is considered to change more slowly compared with other variables.

State equations for PMSM can be written as (2).

$$\begin{aligned} \dot{x} &= g(x,u) + w \\ y &= C \cdot x + v \end{aligned} \quad (2)$$

$$\text{Here } x = [I_d \quad I_q \quad \omega \quad \theta]^T \quad (3a)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (3b)$$

$$y = \begin{bmatrix} I_d \\ I_q \end{bmatrix} \quad (3c)$$

Here  $w$  and  $v$  are random disturbances. In fact  $w$  is the process noise which stands for the errors of the parameters;  $v$  is the measurement noise which stands for the errors in the measurement and sample.

The noise covariance matrixes are defined as follows:

$$Q = \text{cov}(w) = E\{ww^T\} \quad (4a)$$

$$R = \text{cov}(v) = E\{vv^T\} \quad (4b)$$

Extended Kalman filter can be built by the derivation below:

$$x(k+1) = f = x(k) + \dot{x} \cdot T_e = \begin{cases} I_d(k) + \left( \frac{U_d}{L_d} - \frac{R \cdot I_d}{L_d} + \omega \cdot \frac{L_q}{L_d} I_q \right) \cdot T_e \\ I_q(k) + \left( \frac{U_q}{L_q} - \frac{R \cdot I_q}{L_q} - \omega \cdot \frac{L_d}{L_q} I_d - \frac{\psi_r}{L_q} \cdot \omega \right) \cdot T_e \\ \omega(k) \\ \theta(k) + \omega \cdot T_e \end{cases} \quad (5)$$

Define matrix  $F$ :

$$F = \frac{\partial f}{\partial x} = \begin{bmatrix} 1 - \frac{T_e}{\tau_d} & T_e \omega \frac{L_q}{L_d} & T_e \frac{L_q}{L_d} I_q & 0 \\ -\frac{L_d}{L_q} T_e \omega & 1 - \frac{T_e}{\tau_q} & T_e \left( -\frac{L_d}{L_q} I_d - \frac{K_e}{L_q} \right) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & T_e & 1 \end{bmatrix} \quad (6)$$

$\tau_d = L_d / R$ ,  $\tau_q = L_q / R$  are stator constants.

Define matrix  $P$  as the error covariance of observation

$$P_k = E\{e_k^T \cdot e_k\} = \sum_{i=1}^n E\{[x_i - \hat{x}_i][x_i - \hat{x}_i]^T\} \quad (7)$$

$E\{\cdot\}$  is the computation of expectation value.

Extended Kalman filter can be realized by iteration as follows:

1. Compute the state ahead and the error covariance ahead.

$$\hat{x}_{k|k-1} = \hat{x}_{k-1|k-1} + \dot{x} \cdot T_e \quad (8a)$$

$$P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + Q_{k-1} \quad (8b)$$

2. Compute the Kalman gain.

$$K_k = P_{k|k-1} \cdot C^T \cdot (C \cdot P_{k|k-1} \cdot C^T + R_{k-1})^{-1} \quad (8c)$$

3. Update estimation with measurement.

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - C \cdot \hat{x}_{k|k-1}) \quad (8d)$$

4. Update the error covariance matrix.

$$P_{k|k} = [I - K_k \cdot C] \cdot P_{k|k-1} \quad (8e)$$

Based on extended Kalman filter, the sensorless control system is shown in Fig. 1. In this system, rotor flux oriented vector control is adopted. The d-axis current is controlled to be zero which can get the largest torque with the smallest phase currents. Since the terminal voltages of motor are hard to measure, the reference voltages are used in extended Kalman filter instead of the real voltages.

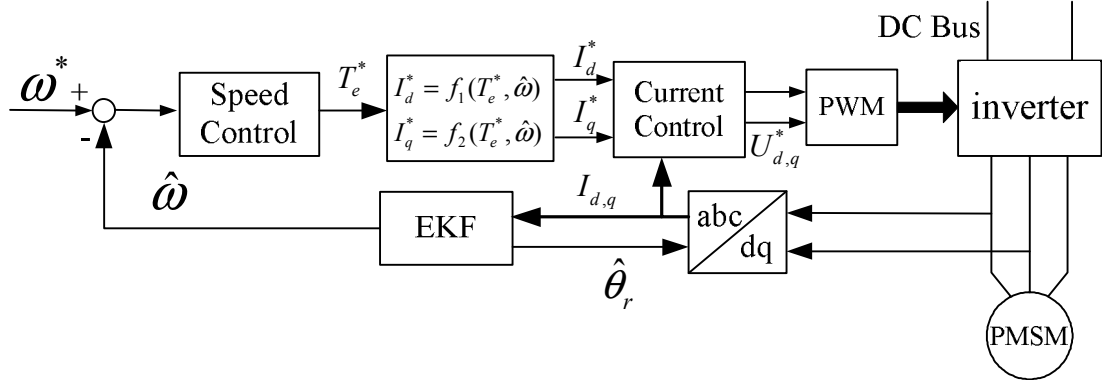


Fig. 1. Block diagram of the sensorless system

## Start up ability analyse

Take non-salient PMSM motor as example, in real rotor flux oriented coordinate ( $\gamma - \delta$  axes), PMSM model is:

$$\frac{dI_\gamma}{dt} = \frac{U_\gamma}{L} - \frac{R \cdot I_\gamma}{L} + \omega \cdot I_\delta; \quad (9a)$$

$$\frac{dI_\delta}{dt} = \frac{U_\delta}{L} - \frac{R \cdot I_\delta}{L} - \omega \cdot I_\gamma - \frac{K_e}{L} \cdot \omega. \quad (9b)$$

$U_\gamma$  and  $U_\delta$  are stator voltages in  $\gamma - \delta$  axes.  $I_\gamma$  and  $I_\delta$  are stator currents in  $\gamma - \delta$  axes.

Since the real rotor flux is not known in sensorless control, in the coordinate oriented by estimated rotor position (d-q axes), there are:

$$\begin{cases} U_d = U_\gamma \cos \gamma - U_\delta \sin \gamma \\ U_q = U_\delta \cos \gamma + U_\gamma \sin \gamma \end{cases} \quad (10a)$$

$$\begin{cases} I_d = I_\gamma \cos \gamma - I_\delta \sin \gamma \\ I_q = I_\delta \cos \gamma + I_\gamma \sin \gamma \end{cases} \quad (10b)$$

PMSM equations become:

$$\begin{cases} \frac{dI_d}{dt} = \frac{U_d}{L} - \frac{I_d}{\tau} + \omega \cdot I_q + \frac{K_e}{L} \cdot \omega \sin \gamma \\ \frac{dI_q}{dt} = \frac{U_q}{L} - \frac{I_q}{\tau} - \omega \cdot I_d - \frac{K_e}{L} \cdot \omega \cos \gamma \end{cases} \quad (11)$$

Here:

$\gamma = \theta - \hat{\theta}$  is rotor position error.

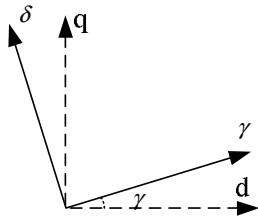


Fig.2. Real and estimated axes

In both coordinates, there always are:

$$\frac{d\theta}{dt} = \omega \quad (12)$$

Electromagnetic torque is:

$$T_{em} = p \cdot \psi_r \cdot I_\delta = p \cdot \psi_r (I_q \cos \gamma - I_d \sin \gamma) \quad (13)$$

Mechanical movement equation of PMSM is:

$$J \frac{d\Omega}{dt} = T_{em} - T_L(\Omega) \quad (14)$$

$T_L(\Omega)$  is load torque and it is a function of rotor speed.

If rotor electrical angle speed is used instead:

$$\frac{d\omega}{dt} = \frac{p^2 \cdot K_e}{J} \cdot (I_q \cos \gamma - I_d \sin \gamma) - T_L(\omega) \quad (15)$$

In the observer proposed in (1), the third equation for rotor speed just relies on state feedback. If we just consider the other three equations, (superscript ^ stands for estimated variables)

$$\frac{d\hat{I}_d}{dt} = \frac{\hat{U}_d}{L} - \frac{\hat{I}_d}{\tau} + \hat{\omega} \cdot \hat{I}_q \quad (16a)$$

$$\frac{d\hat{I}_q}{dt} = \frac{\hat{U}_q}{L} - \frac{\hat{I}_q}{\tau} - \hat{\omega} \cdot \hat{I}_d - \frac{K_e}{L} \cdot \hat{\omega} \quad (16b)$$

$$\frac{d\hat{\theta}}{dt} = \hat{\omega} \quad (16c)$$

State observation errors are defined as:

$$\varepsilon = \begin{bmatrix} \varepsilon_d \\ \varepsilon_q \\ \gamma \end{bmatrix} = \begin{bmatrix} I_d - \hat{I}_d \\ I_q - \hat{I}_q \\ \theta - \hat{\theta} \end{bmatrix} \quad (17)$$

Using (10) (11) and (16), there are:

$$\frac{d}{dt}\varepsilon_d = (\omega \cdot I_q - \hat{\omega} \cdot \hat{I}_q) + \frac{K_e}{L} \cdot \omega \sin \gamma ; \quad (18a)$$

$$\frac{d}{dt}\varepsilon_q = -(\omega \cdot I_d - \hat{\omega} \cdot \hat{I}_d) - \frac{K_e}{L} \cdot (\omega \cos \gamma - \hat{\omega}). \quad (18b)$$

$$\frac{d}{dt}\gamma = \frac{d}{dt}\theta - \frac{d}{dt}\hat{\theta} = \omega - \hat{\omega} \quad (18c)$$

The equilibrium points of the system former will satisfy:

$$\frac{d}{dt}\varepsilon = 0 \quad (19)$$

In the observer based on the equations former, besides the expected equilibrium point  $\gamma = 0$ , there is another equilibrium point:

$$\begin{cases} \hat{\omega} = \omega = 0 \\ T_{em} = p \cdot \psi_r (I_q \cos \gamma - I_d \sin \gamma) = T_L(\Omega) \end{cases} \quad (20)$$

Since the observed speed is zero, difference between reference speed and feedback speed exists, the output of speed regulator (reference torque) will arrive at the maximum limitation. But on this point, the electromagnetic torque equals load torque while current  $I_q$  equals desired torque current. So the motor cannot accelerate any more and will stay in this wrong situation.

Particularly, when load torque is just friction or block torque which are most familiar, the equilibrium point is  $\gamma = \pm\pi/2$  where the actual electromagnetic torque is zero although current  $I_q$  equals the desired value. On this point, since the rotor speed and electromagnetic torque are all zero, load torque is also null. Under the effects of the maximum limitation in speed regulator, the motor will stay in this situation.

In practice, when initial position error satisfy  $\cos \gamma > 0$ , the real electromagnetic torque has the same direction with desired torque, then PMSM can start up towards the desired speed direction, and the motor will start up successfully. Otherwise, PMSM reverses and the observer will get a wrong speed direction and will converge to the unexpected equilibrium points. This is verified by simulation in next section.

In [2], another equilibrium points are proposed which doesn't satisfy (11). It's the converging problem of some observers and these points don't exist in our observer.

The second kind of equilibrium points are mainly determined by the q-axis voltage equation, so we can add some compensation in the second equation to break out this balance. We change the equation to:

$$\frac{d\hat{I}_q}{dt} = \frac{\hat{U}_q}{L} - \frac{\hat{I}_q}{\tau} - \hat{\omega} \cdot \hat{I}_d - \frac{K_e}{L} \cdot \hat{\omega} + \frac{k \cdot RI_q}{L} \quad (21)$$

Here  $k$  is a coefficient positive.

With the compensation, the unexpected equilibrium points can be avoided. In steady states, the compensation will be considered as a little error in stator resistance parameter. Its effects will be eliminated by the robustness of the system. Also when motor is started up successfully, the coefficient  $k$  can be decreased artificially and the compensation can be moved off finally.

## Simulation results

Simulations have been done in MATLAB Simulink to verify the performance of the extended Kalman filter. Motor parameters are shown in TABLE I.

**Table I Motor parameters**

Stator resistance $R$	0.155 $\Omega$
Stator inductance $L_d = L_q$	0.00125 H
Number of pole pairs $p$	4
Rotor magnet flux $\psi_r$	0.153 Web

In extended Kalman filter, matrixes Q and R in (4a) and (4b) are difficult to be known exactly because the disturbances  $w$  and  $v$  are not known. The only possible method is to adjust the values of Q and R by practical simulations or experiments. In simulation, we use the values as follows:

$$P = \begin{bmatrix} 0.1 & & & \\ & 0.1 & & \\ & & 0.1 & \\ & & & 0.1 \end{bmatrix} \quad Q = \begin{bmatrix} 10 & & & \\ & 10 & & \\ & & 10 & \\ & & & 10 \end{bmatrix} \quad R = \begin{bmatrix} 1.0 & \\ & 1.0 \end{bmatrix}$$

Rotor speed and position estimation results are shown in Fig. 3 and Fig. 4. It shows that extended Kalman filter can observe rotor speed and position exactly.

If there is some initial rotor position error, when this error is too large, the motor cannot start up and will converge to the unexpected equilibrium point. As shown in Fig. 5.

With compensation as shown in (17), simulation results when there is a large initial position error ( $2\pi/3$ ) is shown in Fig.6. The motor can start up successfully under the effects of compensation.

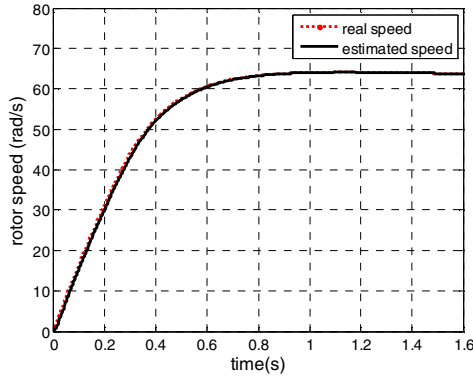


Fig. 3. Estimated and real rotor speed

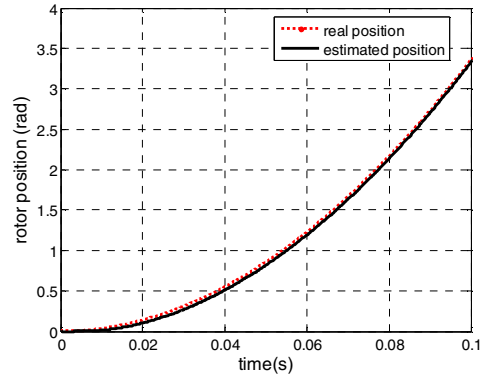


Fig. 4. Estimated and real rotor position

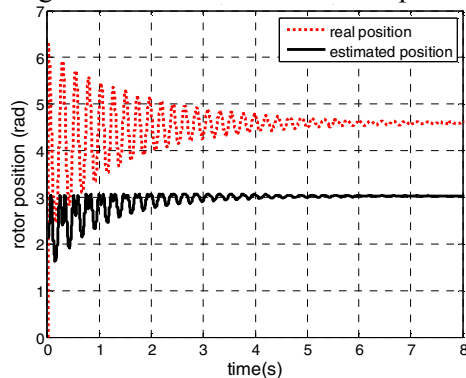


Fig. 5. Failure start up when initial position error is too large (simulation)

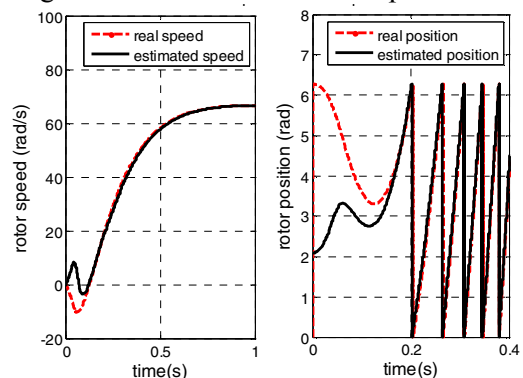


Fig. 6. Success start-up with compensation (simulation)

Simulations also show that with the same load torque and mechanic inertia, the coefficient of compensation has no relations with the initial position error. With the same value of  $k$ , result of start up without initial position errors is shown in Fig. 7. To test the start up ability with load torque, simulation result with an electromotive torque of 5Nm is shown in Fig. 8.

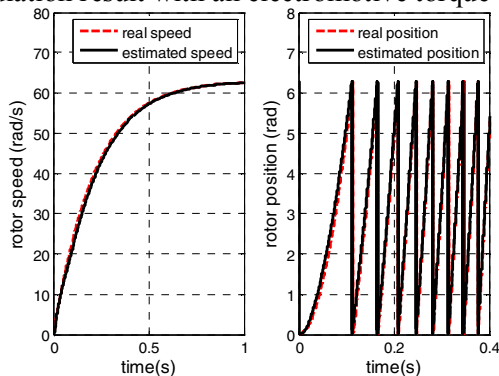


Fig. 7. Start-up with compensation and no initial position error

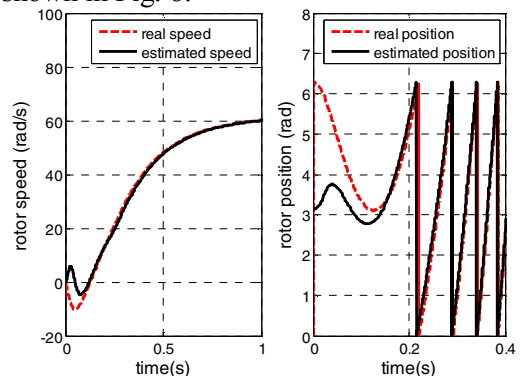


Fig. 8. Start-up with compensation and load torque

Simulations also show that the system can start up when there is a block torque and in steady state, the compensation has very little effects in rotor speed and position estimation.

## Experiment results

Experiments have been done on a platform with the DSP C6711 as the controller. The parameters of the motor are the same as Table 1. In fact, the parameters can be varied in a large field.

$$P = \begin{bmatrix} 0.1 & & & \\ & 0.1 & & \\ & & 0.1 & \\ & & & 0.1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.1 & & & \\ & 0.1 & & \\ & & 0.1 & \\ & & & 0.1 \end{bmatrix} \quad R = \begin{bmatrix} 0.1 & \\ & 0.1 \end{bmatrix}$$

The estimated and real speeds when the motor rotor mechanical angle speed is accelerated from  $2\pi$  rad/s to  $2\pi$  rad/s are shown in Fig.9. If a load torque impact is used in the rotor, rotor speed during dynamic state is shown in Fig. 10. It can be seen that in both steady and dynamic states, the estimated speed by EKF observer can also track the real rotor speed very well.

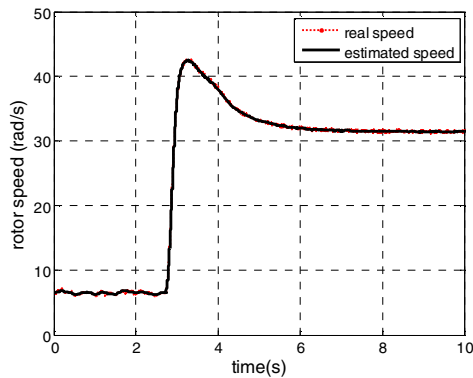


Fig. 9. The estimated and real speeds during acceleration

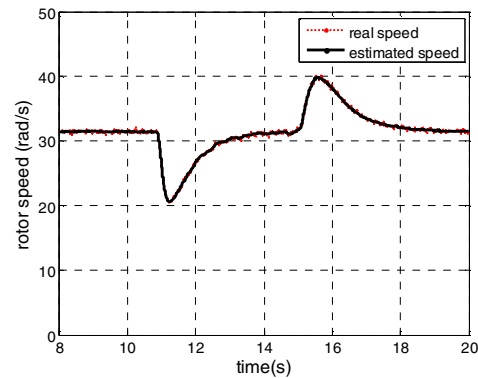


Fig.10. The estimated and real speeds during load impact

When there are some errors in the initial position, the estimated and real positions during start up periods are shown in Fig. 11 and Fig. 12. The initial value of the estimated position is always zero while the real position is random.

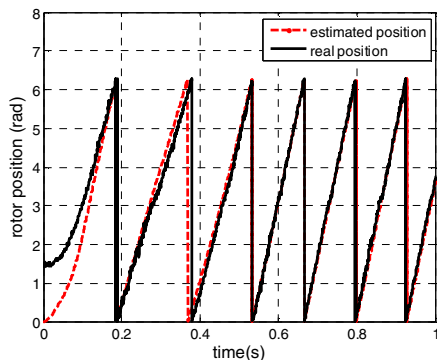


Fig. 11. Start-up with little initial position error (experiment)

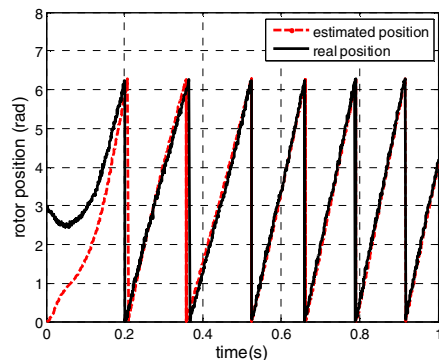


Fig. 12. Start-up with large initial position error (experiment)

It can be seen that the motor can start up from any unknown initial position. The extended Kalman filter can track the real rotor position quickly during start up periods. And there is few reverses or vibrations. It is obvious that the initial position measure or estimation in our system is not needed. In



steady state, there are some steady-state errors between the real and estimated rotor positions. That is because we use the reference voltages instead of the terminal voltages, the voltage errors caused the position estimation error.

## Conclusion

This paper proposed a sensorless control system based on extended Kalman filter for the PMSM. The Kalman filter can estimate the exact rotor speed and rotor position while the initial position and mechanic parameters are not needed. By proper compensation in q-axis equation, only expected equilibrium point is kept. Then the motor can start up at any unknown initial positions and it is not necessary to estimate the initial position before start up. The extended Kalman filter is set up at rotor flux oriented synchronous axes, so it can be easily used in either non-salient or salient motors. The problem is that the covariance matrixes of noises can only be determined by experiment since the noises and disturbances are not known in practice.

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