Sliding Mode controls for a two-cell chopper

Dominique Pinon, Maurice Fadel, Thierry Meynard

Sliding Mode controls for a two-cell chopper

D. Pinon, M. Fadel, T. Meynard
Laboratoire d'Electrotechnique et d'Electronique Industrielle
Unité Mixte de Recherche I.N.P.T.-E.N.S.E.E.I.H.T. / C.N.R.S.
B.P. 7122 - 2, rue Camichel, 31071 Toulouse Cedex 7 - France
Tel. : 05-61-58-82-08, Fax. : 05-61-63-88-75
email: pinon@leei.enseeiht.fr, fadel@leei.enseeiht.fr, meynard@leei.enseeiht.fr
URL : http://www.leei.enseeiht.fr/

Abstract

In this paper, the authors present three different sliding mode controls for a two-cell chopper. First, the synthesis of the control is treated. By the study of the limit cycle, the authors have developed a new control in steady state that imposes a phase shift of π between the control signals of the switches. They also present a sliding mode control at fixed frequency.

Keywords

Converter control, multilevel converter, sliding mode control, variable structure, limit cycle.

Introduction

The multi-cell converters have been developed to increase the converted power. A p-cell converter with an optimal phase of $2\pi/p$ between the interleaved control signal (frequency f_d) can create improved output signals : the fundamentals of those signals have a frequency of pf_d.
With an appropriate control, the performance in transient could be exceptional [1]. The fixed frequency controls for this converter are in general based on a decoupling of the input [2], [3] (linear or non-linear). With those methods, a too dynamical control may saturate the duty cycles in transient, which would cause one or several of the state variables to remain constant during the time where the duty cycles are saturated.
We will present here three different sliding mode controls. The first control is directly derived from the synthesis of the switching functions. By studying the limit cycle, we found a second control giving a phase of π between the control signal of the switches [4]. In a third part, we will present a new fixed frequency control based on the switching functions that we have defined. We will show that the performances are close to those obtained with a variable frequency control.

Model of the converter

We will study a two-cell chopper of which structure is presented on the figure 1.
In open loop, the phase between signals u_1 and u_2 is π. It produce an optimal waveform of the output current : the frequency if doubled (figure 1).
The instantaneous model of the chopper is non-linear :

$$\dot{x} = Ax + g_1(x)u_1 + g_2(x)u_2$$

With :

$$x = \begin{bmatrix} V_1 \\ i \end{bmatrix} A = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{R}{L} \end{bmatrix} g_1(x) = \begin{bmatrix} -\frac{i}{C} \\ \frac{V_1}{L} \end{bmatrix} g_2(x) = \begin{bmatrix} \frac{i}{C} \\ \frac{E - V_1}{L} \end{bmatrix}$$

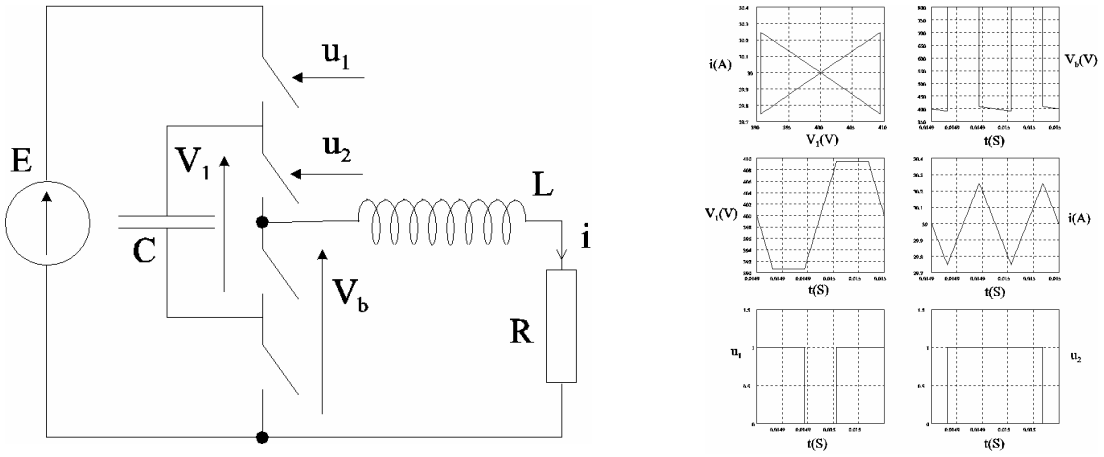

Fig 1- Structure of a two-cell chopper, and open loop signals of the converter for a duty cycle α=0,75

In our case, the values for the passive components are : E=800 V, C=40 µF, L=10 mH, R=20 Ω
Such a model of the converter can be controlled if the condition of accessibility is true on all the space state [8] :

$$\det([g_1(x), g_2(x)]) = -\frac{Ei}{LC} \neq 0 \text{ if } i \neq 0 \text{ A}$$

This means that when the output current is equal to zero, the voltage could not change, so can not be controlled. We will use this model for the synthesis of the two switching functions.

Synthesis of the control

The control has three goals :

- To control the voltage V_1 at $E/2$
- To control the current i at I_{ref}.
- To obtain optimal waveform for the output signals (V_b and i) *i.e.* a phase of π between u_1 and u_2.

We used the Lyapunov criterion for the synthesis of the control [5]. This method of synthesis has been defined in [6]. Let's consider a Lyapunov function as :

$$V(x) = x^T Q\, x > 0$$

With :

$$\Delta x = \begin{bmatrix} V_1 - \frac{E}{2} \\ i - I_{ref} \end{bmatrix}, \quad Q = \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix}$$

To have a stable system, the function $\dot{V}(x)$ should be negative [5]. Let's calculate this function :

$$\dot{V}(x) = \Delta x^T Q \Delta \dot{x}$$

$$\Leftrightarrow \dot{V}(x) = \Delta x^T Q(f(x) + g_1(x)u_1 + g_2(x)u_2)$$

We now consider that the equivalent controls U_{1eq} and U_{2eq} exist. Under the equivalent control, the converter is on open loop and so it is stable (this is a dissipative system). We can then write the control signal as :

$$u_i = U_{ieq} + \Delta u_i \text{ for } i \in \{1, 2\}$$

Because, the converter is stable on open loop we can say that :

$$\Delta x^T Q(f(x) + g_1(x)U_{1eq} + g_2(x)U_{2eq}) < 0$$

To have a stable system, we need to respect (if the equivalent control exists):

$$\Delta x^T Q g_i(x) \Delta u_i < 0$$

To respect this condition, we could take the switching surfaces respecting the following equation :

$$\begin{cases} s_1(x) = \dfrac{2I_{ref}}{E}(v_c - E) - (i - I_{ref}) \\ s_2(x) = -\dfrac{2I_{ref}}{E}(v_c - E) - (i - I_{ref}) \end{cases}$$

The control signal is given by :

$$u_i = \Phi(s_i(x)) \text{ with } \Phi(\alpha) = \begin{cases} 1 \text{ if } \alpha > \varepsilon \\ 0 \text{ if } \alpha < -\varepsilon \end{cases}$$

The function Φ is a simple hysteresis of a height of two ε.

We can note that the equivalent controls exist f the Lie derivatives of the switching function are different of zero [7]. It means that the voltage V_1 must be different of 0 and E when the current must be different of 0.

We make the following test :

- Initial condition are $V_1(0) = 0$ V, $i(0) = 0$ A, with $\varepsilon = 1$A
- Slope of input voltage of 8.10^5 V/s and a current reference of 30 A (duty cycle α=0,75)
- A step of the current I_{ref} = 15 A at 4 ms (α=0,375)
- A step of load of 100% at 6 ms (α=0,75)
- A step of reference I_{ref} = 10 A at 8 ms (α=0,5)
- A variation of the reference $I_{ref} = 10 + 5 \sin(2\pi f t + \varphi)$ A during 5 ms with f = 200 Hz and φ = 10.05 rad (α from 0,25 to 0,75)
- A variation of the input voltage of –25% with a current I_{ref} = 10 A at 15ms (α=0,66)

This test will be applied to at all the other controllers in the rest of this paper.

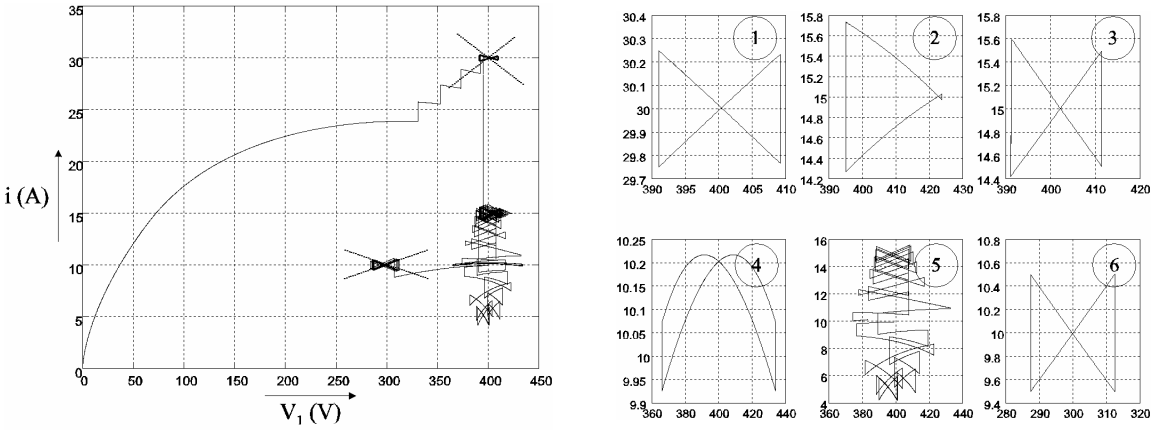

Fig 3 – The state trajectory in the plane V_1 – i

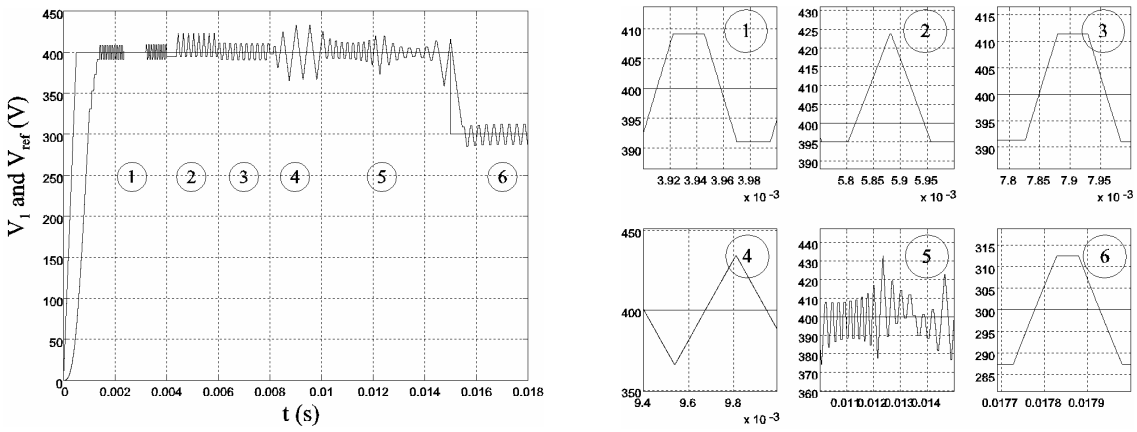

Fig 4 – The voltage V_1 and $E/2$

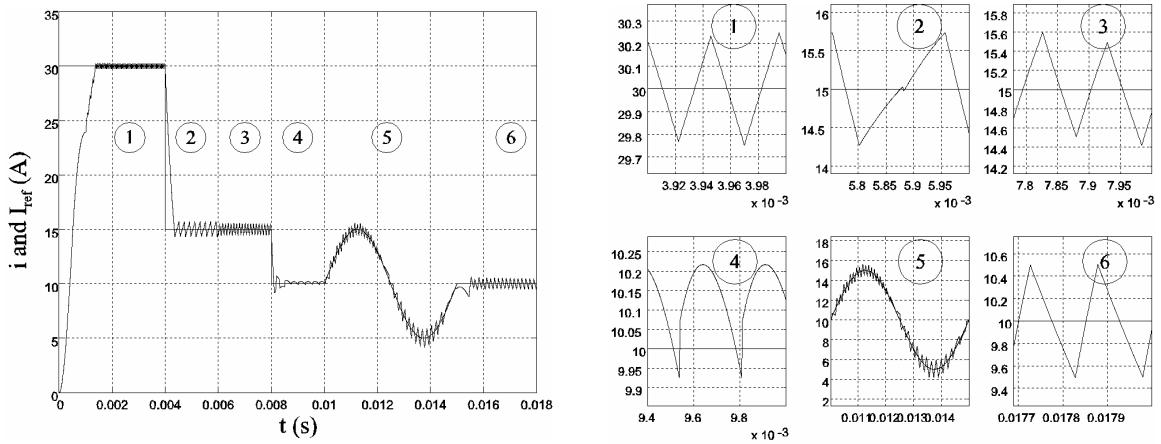

Fig 5 – Currents i and I_{ref} I

In steady state, the performances are not optimal. The errors on the voltage and on the current are *quasi* almost cancelled. But we can see on figure 3 (zoom 4) that the limit cycle is not symmetrical about $E/2$. It shows that the phase between the control signal is not of π. In this case, we cannot obtain optimal waves form for the output current i.

Study of the limit cycle :

To understand why the phase was not always of π, we have studied the installation of the limit cycle. When the control signal switch, the state variables verify :

$$\begin{cases} s_1(x) = \dfrac{2I_{ref}}{E}(v_c - E) - (i - I_{ref}) = \pm\varepsilon \\ \text{or} \\ s_2(x) = -\dfrac{2I_{ref}}{E}(v_c - E) - (i - I_{ref}) = \pm\varepsilon \end{cases}$$

A linear relation exists between the two state variables. The study could be reduced to a first order system [9]. So we can investigate the installation of the by studying the voltage recurrence for example (which the easier case).

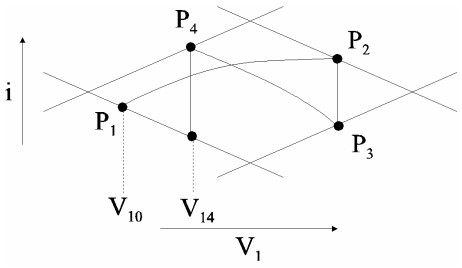

Fig 6 – Notation for the study of the limit cycle

Consider a limit cycle with four vertices as show on figure 6. To study the installation of the limit cycle we should find the function that links the voltage V_{10} to voltage V_{14} (*c.f.* figure 6) [4]. A numerical method allows us to find this function. Let us define function *F* who links V_{10} to V_{14}.

The slope of this function is close to 1, so that we present here, not the function *F*, but the difference of it with the identity function. In our case, when this difference is growing, it shows that the slope of *F* is greater than 1 and the opposite. As can be seen on figure 7, the absolute value of the slope of *F* is greater (respectively lower) than 1 if the duty cycle is lower (respectively grater) than 0,5. So the fixed point of this series will be stable only if the duty cycle is greater than 0,5. This is the reason why a symmetrical limit cycle only exists for reference current corresponding to a duty cycle greater than 0,5.

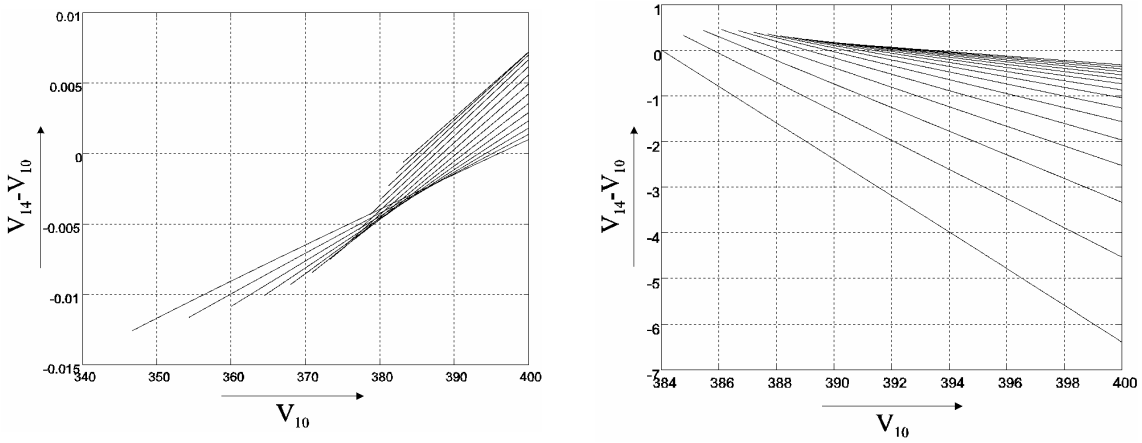

Fig 7 – $F(V_{10})$-V_{10} function of V_{10} for a duty cycle : lower and greater than 0,5.

Such a control has been implanted on an analog card using on analog multipliers (AD 734). It has been tested on an analog simulator of the studying two-cell chopper. The simulated converter is the one defined at he beginning.

The test done is

- Initial condition $V_1(0)$=0 V and i(0)=0
- Start up with an equivalent current reference of 34 A (duty cycle equal to 0,74), and a source of 800 V.
- At 2.8 ms a step of the current reference from 34 A to 4.8 A (duty cycle equal to 0,12).

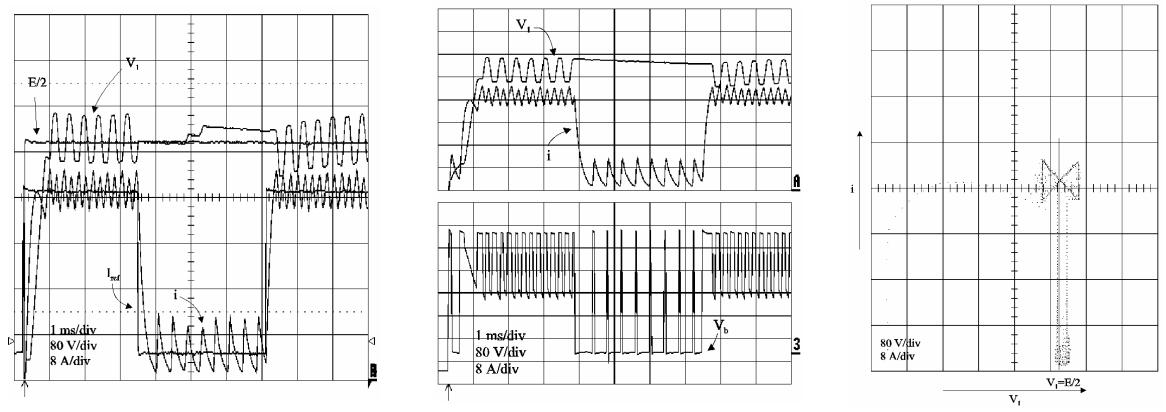

Fig 8 – The voltage V_1 and the current i with their reference for a 2 cell chopper and limit cycle of the chopper for two reference currents

We can observe the good performance of the law in transient (figure 8). As can be seen on voltage V_b on the figure 8 the voltage V_1 is correctly balanced from the start up to the current step. On the same voltage waveform, we can see that in steady state the frequency is twice that of V_1. It means that the phase of the control signals is π when the current reference is 34 A. When I_{ref} is equal to 4,8 A, voltage V_1 is not balanced. In this case, the control signals are to be in phase. In this case, the limit cycle is unsymmetrical (figure 8).

In conclusion, these experiences show that the previous study is correct

Some new controls

We found out a new control of a two-cell chopper that can give optimal output waveforms. The same switching function is used for the transient, but for the steady state, we decided to use the control presented on figure 10.

This control has been found by studying the stability of the limit cycle on the particular case of the triangle [4]. In steady state this control, we will used the lower triangle for a duty cycle greater than 0,5, and the lower triangle on the other case.

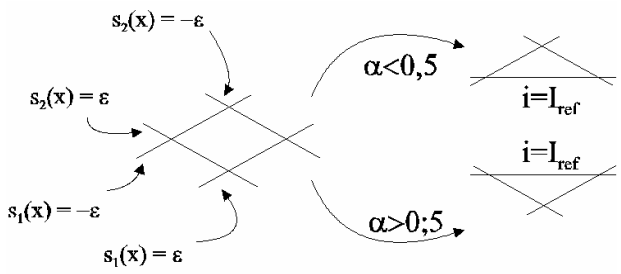

Fig 10 – Principle of steady state control

In this case, it can be shown that the limit cycle is always stable for any values of the duty cycle or of the hysteresis band control (as shown on figure 11) [4].

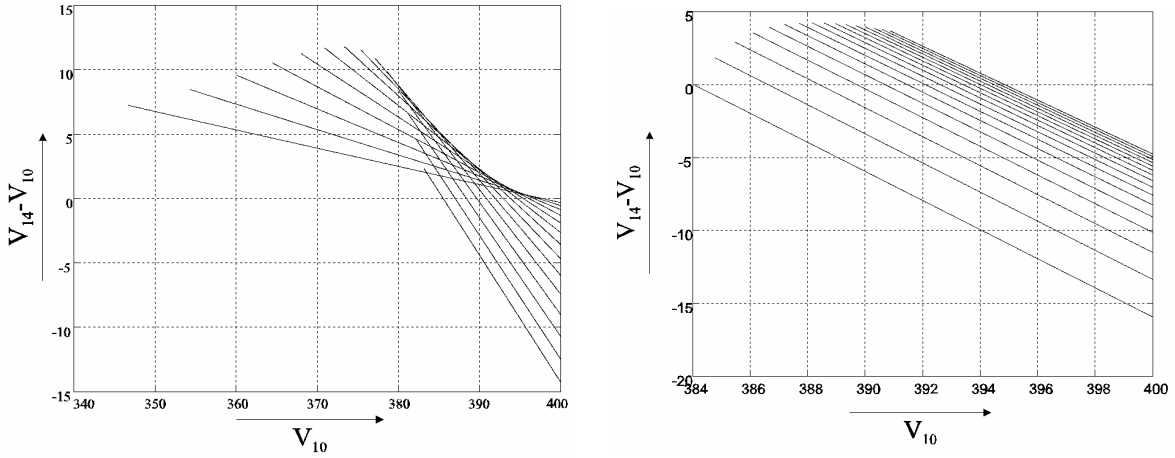

Fig 11 – V_{14}-V_{10} in function of V_{10} for duty cycle lower than 0,5 and greater than 0,5.

The same test sequence as in the previous section is carried with this control.

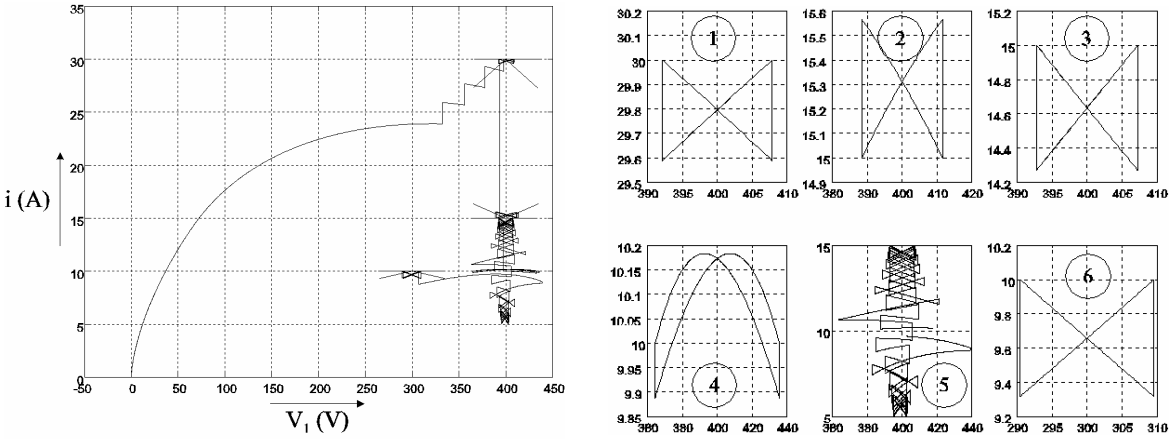

Fig 12 – State trajectory in the plane V_1 – i

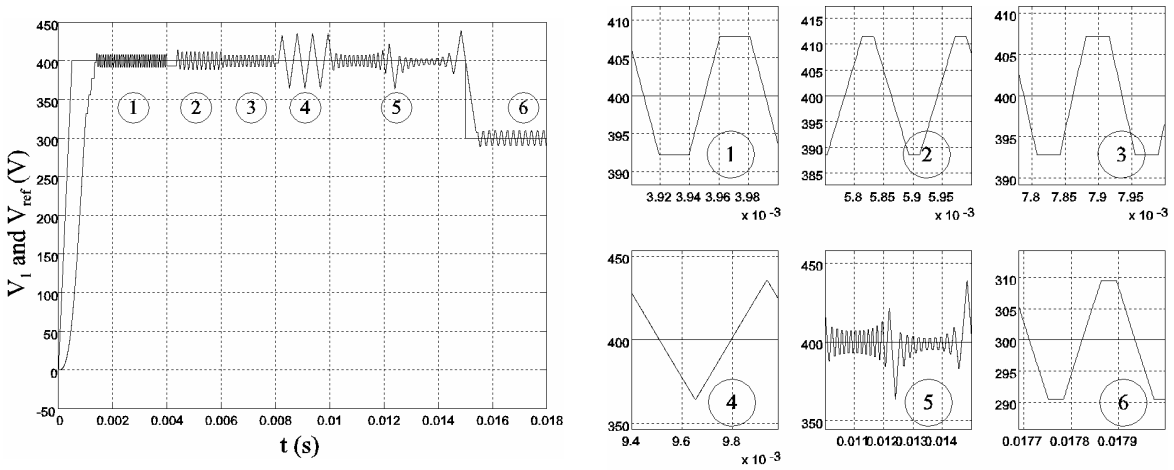

Fig 13 – Voltages V_1 and $E/2$

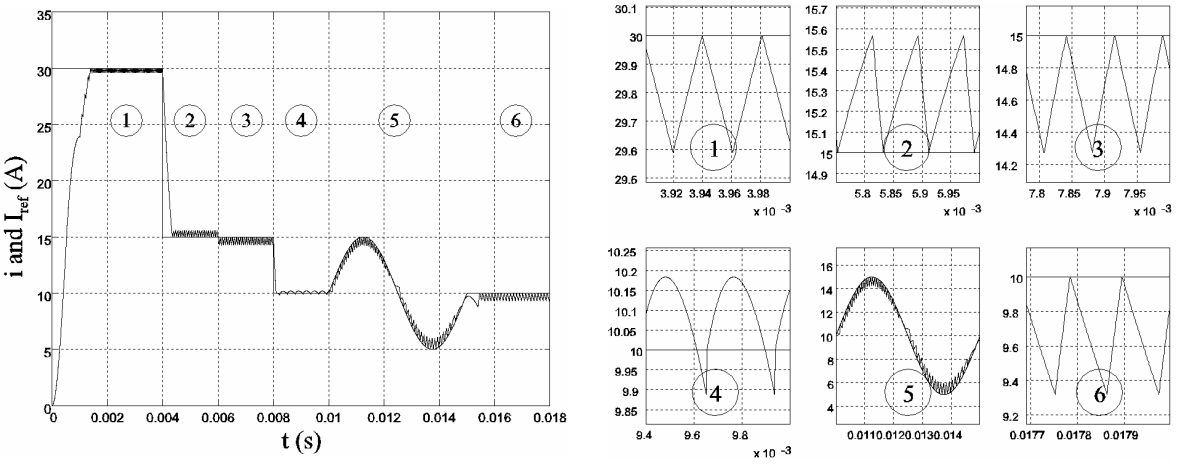

Fig 14 – Currents i and I_{ref}

Figures 12 to 14 present the state variables of the chopper. With this control, the transients are identical to the previous control. But now the performances in steady state are better. The symmetrical limit cycle on figure 12 (zoom 1 – 4) shows that the phase between the control signal is always π.
We can see that, in this case, the shape of the current is smaller than on figure 5. The frequency of the output signal is twice the frequency of the switching signal.
The error on the steady state current is due to the fact that we use a triangle that is not centred on I_{ref}. It could be significantly reduced by adding or deducting a constant to I_{ref} ($\varepsilon/2$ or $-\varepsilon/2$).
In conclusion, we always obtain very good performances in transient, and an optimal waveform for the current i for any value of the current reference (*i.e.* the duty cycle) or of the height of the hysteresis.

Fixed frequency control

For this control, we used the same switching function $S_1(x)$ and $S_2(x)$ (c.f. equation 4). Those function are now treated by two P.I. correctors delivering the duty cycle $\langle u_1 \rangle$ and $\langle u_2 \rangle$ (*c.f.* figure15).

To avoid over-switching, the correctors bandwidth is reduced by a low-pass filter (a Saley Nackey 4th order filter). This control is non-linear. So the dynamic in closed loop is non-linear too. It will only depend of the slope of the switching functions.

The gains are chosen for a large dynamical in front of the switching functions. The integrator's gains are lower in order to not disturb the sliding mode and to allow cancelling the static errors on the two state variables.

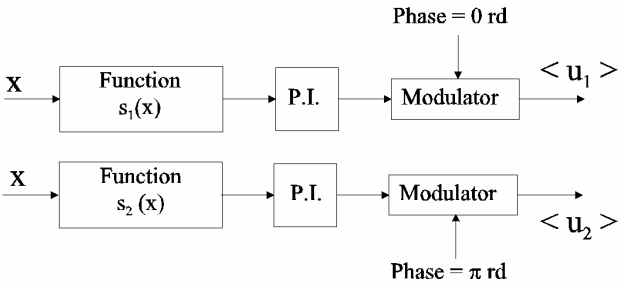

Fig 15 – Principle of the fixed frequency control in sliding mode

We make the same test with this control.

The results are presented on figures 16 to 18. We can see that in transient the results are not as good as with the other strategy due to over shoot (about 10% of the voltage). But, in steady state, the phase control gives optimal waveforms on the output current. The two state variables are stabilised for any value of their references (figures 17 and 18), of the input voltage (figures 17-18 zoom 6) and of the load (figure 17-18 zoom 3).

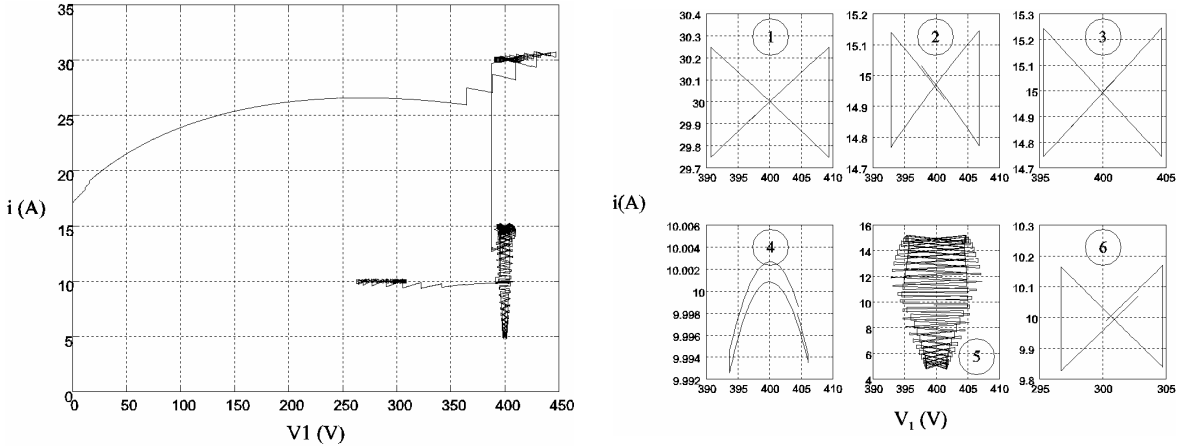

Fig 16 - State trajectory in the plane V_1 – i

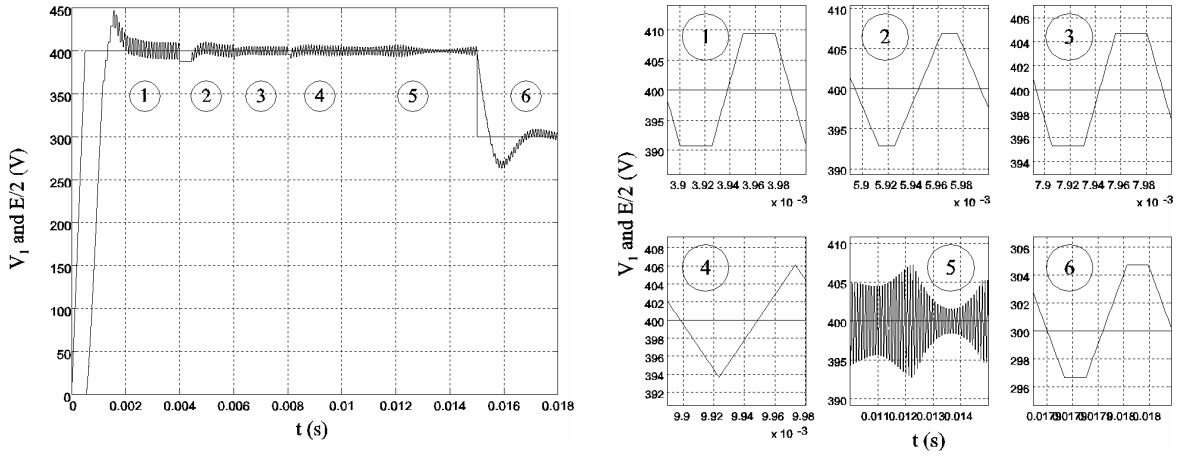

Fig 17 - Voltages V_1 and $E/2$

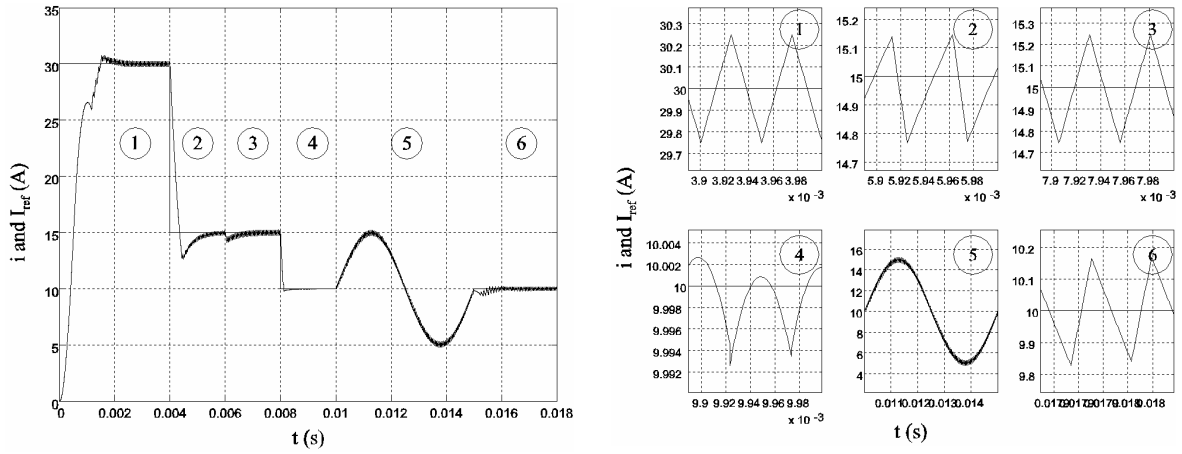

Fig 18 - Currents i and I_{ref}

This control has been realised with analog circuits. The figure 19 presents some results.
The test done is

- Initial condition $V_1(0)=0$ V and $i(0)=0$
- Start up with an equivalent current reference of 38 A (duty cycle equal to 0,95), and a source of 800 V.
- At 9.2ms a step of the current reference from 38 A to 12 A (duty cycle equal to 0.3).

The voltage V_1 is always balanced about E/2, so the frequency of the voltage V_b is twice the one of V_1 (c.f. figure 19). The shape of the output current is optimal.
There are three advantages to this control in front of the two above. The first : the frequency of the control signal is constant. The second : with this control we are sure to have an optimal waveform for the output current by controlling the phase between the two signals. The third, the dynamic is near from the one imposing by the Lyapunov design.

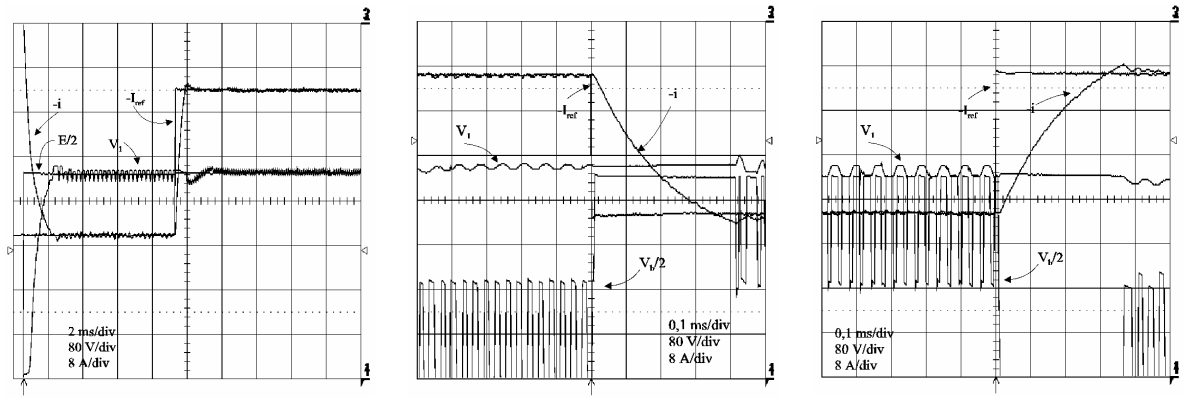

Fig 19 – Experimental results on an analog simulator of a two-cell chopper

Conclusion

We saw that the synthesis of the switching functions with Lyapunov criterion gives a control with very good performances in transient and not in steady state.
By studying of the limit cycle, we find a special control in steady state with a variable frequency that ensures optimal waveform of the output current. It creates a small error on the output current.
The control with a fixed frequency shows that with the sliding mode, we could obtain very good transient performances and an optimal waveform for the output current. The implantation is not difficult to do with analog circuit. The dynamic is near from the one imposing by the Lyapunov design.

Bibliography

1. T. MEYNARD, *Performances dynamiques des convertisseurs multiniveaux,* revue internationale de génie électrique, Volume 1 – N°1/1998, pages 37-52.
2. G. GATEAU, *Contribution à la commande des convertisseurs statiques multicellulaires séries - Commande non-linéaire et commande floue*, Thèse de doctorat, I.N.P. Toulouse, 1997.
3. O.Tachon , *Contribution à la commande découplante des convertisseurs multicellulaires*, Thèse de doctorat, I.N.P. Toulouse, 1997.
4. D. Pinon, M. Fadel, T. Meynard, *Commande par mode de glissement d'un hacheur à deux cellules : étude de l'installation des cycles limites*, revue internationale de génie électrique, Volume 1 – N°3/1998, pages 393-415.
5. J.J.E. SLOTINE and W.Li, *Applied non linear Control*, ed. Prentice Hall, 1990.
6. B. NICOLAS, *Contribution à la commande des convertisseurs statiques*, Thèse de doctorat, I.N.P. Toulouse, 1996.
7. V. I. UTKINS, *Sliding modes and their applications in Variable Structure System*, ed. M. I. R. Publisher, 1978.
8. H. NIJMEIJER, A. J. VAN DER SCHAFT, *Non linear Dynamical Control Systems*, ed. Springer Verlag, 1992.
9. C. MIRA, *Systèmes asservis non linéaires*, ed. Hermès, 1990.
10. H. SIRA-RAMIREZ, *Differential Geometric Methods in Variable-Structure Control*, International Journal of Control, p 1359-1390, 1988.

H. BUHLER, *Réglage par mode de glissement*, ed. Presses Polytechniques romandes, 1986.