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Enhanced Formulae for Determining Both Free Length and Rate of Cylindrical Compression Springs

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Paredes, Manuel
Institut Clément Ader (ICA), Université de Toulouse, INSA, CNRS FRE 3687, France.
135, avenue de Rangueil,
F-31077 Toulouse, France
manuel.paredes@insa-toulouse.fr

ABSTRACT

Cylindrical compression springs have been commonly exploited in mechanical systems for years and their behavior is considered as well identified. Nevertheless, it appears that, even though old research studies suggest correcting the rate formula, the main industrial software dedicated to spring design exploits the uncorrected one. In order to evaluate the accuracy of the analytical formulae for spring behavior, an experimental study was performed which tried to cover the common design space. This study was done using the 2 common coil ends: closed and ground ends, and closed and not ground ends. Moreover, the accuracy of the load-length relation was investigated whereas older studies focused only on the spring rate. It appears that the common uncorrected formulae give satisfactory results only when large numbers of coils are involved. We also highlight, for the first time, that it is interesting to correct not only the spring rate but also the free length of the spring.

Keywords

Compression springs, spring design, spring rate, active length.

1. INTRODUCTION

Mechanical compression springs are often used in mechanical devices for their ability to store and return energy. The range of applications is very wide. As recent examples, compression springs can be found in a suspended backpack for harvesting biomechanical energy [1], vibration absorbers [2], linear variable-

stiffness mechanisms [3] and Spring Operated Pressure Relief Valves [4]. Most industrial software available to help designers, such as Advanced Spring Design from SMI [5] and Spring Calculator from IST [6] or websites dedicated to spring design [7-8] exploit common analytical formulae from the reference book on spring design written several years ago by Wahl [9]. Since that time, many works have improved our knowledge of compression springs: recently, Rodriguez [10] and Paredes [11] determined analytical formulae for several types of conical springs, the dynamic aspect of compression springs has been studied by Pearson [12] and Yildirim [13], and Dym [14] proposes extended formulae to calculate the rate of cylindrical helical springs.

This study focuses on the most common spring: the cold formed cylindrical wire compression spring, which has a cylindrical shape, a constant axial pitch, a small pitch angle and no dead coils. To increase readability this will be referred to as a compression spring throughout this paper. Compression springs are usually manufactured with closed and ground ends or with closed ends that are not ground (see Fig. 1), to help the external load to be considered as a pure axial load. Thus the geometry of the spring is composed of a cylindrical helix of n_f free coils (with constant axial pitch and mean diameter D) plus one closed coil at each end that can be ground or not as illustrated in Fig. 1.

For a designer, a compression spring is expected to have linear load-length behavior, which is mainly defined by the free length L_0 and the spring rate k. For a given axial space, L_1, of the system where the spring is inserted, the associated load, P_1, given by the spring can be easily calculated:

$$P_1 = k\,(L_0 - L_1) \tag{1}$$

The accuracy of the calculated load depends directly on the accuracy of L_0 and k. Usually, the spring rate is calculated using the common formula:

$$k = \frac{G\,d^4}{8\,n_a\,D^3} \tag{2}$$

where $n_a = n_T - 2$ for closed ends, whether they are ground or not [8; 15].

In this case, n_a, which is exploited to calculate the spring rate, is equal to n_f, which corresponds to the number of coils with the constant axial pitch. This means that the calculation of the spring rate neglects the effects of the end coils on the global stiffness. In section 5-3 of his book [9], Wahl refers to three studies [16-18] to explain why the deflection of end coils should be considered. Vogt [16] studied two springs with closed and ground ends and showed that each end coil had an effect equivalent to 0.25 coils on the spring rate. Thus he proposed distinguishing between n_a and n_f and using:

$$n_a = n_f + 0.5 \tag{3}$$

Pletta et al. [17] tested seven sets of three springs each to show the variation of the number of active coils with the load. They tested springs with closed and ground ends, n_T between 2.5 and 9.85, and spring indexes between 3.09 and 7.46. They showed that the deflection of the spring was not strictly linear. Finally, Keysor [18] pointed out that a reversed deflection was found to occur in the closed end, at the back of the tip contact point, which increased the deflection of the spring as a whole and illustrated once again that it was not correct to consider the entire closed end as inactive.

Moreover, it is interesting to note that European standards related to compression springs [19] do not distinguish between n_a and n_f but recommend exploiting 2 active coils as a minimum.

Thus we see that the common formula tends to overestimate the spring rate and that the spring rate tends to be non-constant, so it could be interesting to evaluate the accuracy of both the common formula and the one proposed by Vogt. Moreover the spring rate tends to evolve during loading. Consequently, some spring manufacturers, such as the Lee Spring Company, suggest that the free length should be considered only as an approximate value, to be adjusted during manufacturing when necessary [15]. So, it could be interesting to investigate whether an additional correction of the free length could give more accurate results, as illustrated in Fig. 2.

As far as we are aware, no study has been performed to evaluate the accuracy of the common analytical load-length relation for compression springs and the correction of the spring rate [17] has been evaluated using very few springs that do not cover the range of the design parameter usually exploited by

designers. Therefore, in section 2, we propose an experimental study that tends to cover the design space and that deals with springs with closed and ground ends. The accuracy of the common formulae will thus be evaluated. Then, as proposed by Vogt, a spring rate correction will be defined in order to improve the accuracy. Finally, a combination of spring rate correction and free length correction will be proposed for even better accuracy. The same study, but concerning compression springs with closed ends that are not ground, is presented in section 3, and conclusions are drawn in section 4.

2. STUDY ON COMPRESSION SPRINGS WITH CLOSED AND GROUND ENDS

2.1 EXPERIMENTAL STUDY

The experimental study was intended to cover a large part of the design space. Thus springs with 2, 5, 10 and 13 free coils were evaluated (2 is considered as a minimum by European standards and 13 appears to be a maximum to avoid buckling). For each given number of coils, three spring indexes were tested (low: c = 5, medium: c = 8, and high: c = 16). As in Pletta [17], three specimens were tested for each configuration.

To avoid the influence of variations in the mechanical properties of the wire material, all the springs were made from the same wire by a spring manufacturer that works with the aeronautical and aerospace industries, in order to reach the highest standards.

Finally, 4x3x3=36 springs made from the same wire (d=1.8 mm in stainless steel 1.4310) were tested. Table 1 gives the detailed geometry of the springs tested.

Each spring was tested on a Spring Test 1 test bench from Andilog [20] (see Fig. 3). The force gauge had a 500 N capacity with 0.1% accuracy and 0.04 N resolution. The handle enabled a stroke of 2 mm per revolution. The displacement transducer with digital display had a resolution of 0.01 mm. The axial stiffness of the test bench was taken into account to correct the lengths and reach a precision of 0.02 mm.

2.2 EVALUATION OF THE ACCURACY OF COMMON FORMULAE

In order to evaluate the accuracy of the common analytical formulae for the spring rate and the free length, the following strategy was followed. The idea was to have a designer-oriented study. So, for each spring tested, the individual error, e, between analytical formulae and experimental results was calculated as the sum of the relative errors on loads at two points covering the usual operating range of the spring, one at 25% of the potential travel and the other at 75% of the potential travel as illustrated in Fig. 4.

$$L_1 = L_0 - 0.25(L_0 - L_c) \quad (4)$$

$$L_2 = L_0 - 0.75(L_0 - L_c) \quad (5)$$

$$e = \left|\frac{P_{1e} - P_1}{P_{1e}}\right| + \left|\frac{P_{2e} - P_2}{P_{2e}}\right| \quad (6)$$

The results presented in Fig. 5 show the average individual errors for the 3 springs tested for each configuration.

The results clearly show that the accuracy of the common formulae can be considered as quite good when 5 free coils are considered (e is less than 0.17) and very good with more free coils (e is less than 0.08). It is a first result, showing that the common formulae give satisfactory results with most of the usual designs and explains why those formulae are exploited in industrial software. On the other hand, the accuracy is quite poor when large indexes and small numbers of free coils are considered. The example of reference CG9, which has the worst accuracy is shown in Fig. 4. The geometry of spring CG9#1 gives L_1 = 18.26 mm and L_2 = 10.29 mm. The common formulae predict F_1 = 7.7 N and F_2 = 23.0 N but the experimental results give F_1 = 4.8 N and F_2 = 19.2 N, leading to an individual error e = 0.58. The same study for spring CG9#2 gives e = 0.40 and spring CG9#3 gives e = 0.43.

It is thus interesting to investigate further in order to find more accurate analytical formulae.

2.3 EVALUATION OF THE ACCURACY OF FORMULAE WITH CORRECTED RATE

In this section, we try to exploit the approach proposed by Vogt by correcting the number of active coils in the calculation of the spring rate. An optimal value has been calculated so as to minimize the sum of the individual errors on each spring. We thus obtain the following correction:

$$n_a = n_f + 0.35 \quad (7)$$

The results presented on Fig. 6 show that the accuracy for springs with few free coils and large indexes has been significantly increased and can be considered as quite good (e remains lower than 0.20). We can also observe that the accuracy for numerous free coils and small indexes has been slightly degraded as e is about 0.08 for configuration CG4 with correction whereas it was only 0.04 without correction. This phenomenon is accentuated when using the correction proposed by Vogt (which leads to e = 0.10) and may explain why this correction has not been integrated in industrial software and standards.

2.4 EVALUATION OF THE ACCURACY OF FORMULAE WITH CORRECTED RATE AND LENGTH

As illustrated in Fig. 2, we investigated the effects of a correction on both the rate and the free length. The load-length relation was still considered as linear to match the common assumption of designers related to compression spring behavior. At this stage, many correction rules for L_{0a} could be evaluated. To make the correction on the free length easily exploitable by designers, we chose it to be proportional to the wire diameter and linearly dependent on the spring index. Once again, the optimal parameter values were calculated so as to minimize the sum of the individual errors on each spring. We thus obtained the following correction:

$$n_a = n_f + 0.12 \quad (8)$$

$$L_{0a} = L_0 - n_e\, d \quad (9)$$

$$\text{with } n_e = -0.22 + 0.044\, c \quad (10)$$

This correction appeared to give satisfactory results for each spring tested as e remained lower than 0.14 (see Fig. 7). It can be seen that introducing a length correction reduced the associated rate correction

(compared to the rate correction only). Investigating further, we found that a single length correction with $n_e = -0.24 + 0.052\ c$ (and $n_a = n_f$ as usually considered by designers) enabled almost the same accuracy to be obtained.

3. STUDY ON COMPRESSION SPRINGS WITH CLOSED ENDS NOT GROUND

3.1 EXPERIMENTAL STUDY

The same experimental study was performed for compression springs with closed ends that were not ground. The springs were all guided by an inner rod of external diameter D_r. Compared to the springs with closed and ground ends, all the lengths were increased to maintain the same axial pitch for each configuration. Once again, 36 springs were tested (see Table 2).

3.2 EVALUATION OF THE ACCURACY OF COMMON FORMULAE

The individual errors on loads were calculated as for compression springs with closed and ground ends and the results are presented in Fig. 8. They show that accurate results were obtained for large values of both n_f and c but that the quality was very poor for springs with small values of n_f and c. The worst individual error was 1.35, for reference CNG1, which can be considered as totally unsatisfactory for a designer.

3.3 EVALUATION OF THE ACCURACY OF FORMULAE WITH CORRECTED RATE

Correcting the number of active coils ($n_a = n_f + 0.58$) to calculate the spring rate improved the results as shown in Fig. 9. In this case, the worst individual error decreased to 0.82 but was still unsatisfactory. Moreover, the accuracy was reduced for large values of n_f and c. As an example e = 0.03 without correction for reference CNG12 but e = 0.12 after correction. It was thus necessary to investigate further and find a more accurate correction.

3.4 EVALUATION OF THE ACCURACY OF FORMULAE WITH CORRECTED RATE AND LENGTH

Using the same approach, we obtained the following optimal correction parameters:

$$n_a = n_f + 0.44 \tag{11}$$

$L_{0a} = L_0 - n_e\, d$ (12)

with $n_e = 0.33 - 0.036\, c$ (13)

The results presented in Fig. 10 show a significant increase in accuracy. Nevertheless, accuracy must still be considered as only of average quality as the worst individual error is equal to 0.35.

We therefore analyzed the experimental load-length curves of reference CNG1 to identify possibilities for improvement. Fig. 11 shows the common analytical curve without correction and the 3 experimental curves obtained for CNG1. The experimental curves show a significant evolution of the spring rate during compression. The load length curve should probably be considered as bilinear rather than linear as illustrated by the discontinuous lines in Fig. 11.

For that reason, we developed a bilinear correction defined by a first linear part starting at L_0 (uncorrected) and having a corrected rate, and a second linear part exploiting the usual rate (uncorrected) and a corrected length L_{0a}. The axial load, P, related to a given length, L, can be calculated as follows:

$$\mathrm{P(L)} = \max\left[\frac{\mathrm{G\,d^4}}{8\,\mathrm{n_a\ D^3}}\left(L_0 - L\right); \frac{\mathrm{G\,d^4}}{8\,\mathrm{n_f\ D^3}}\left(L_{0a} - L\right)\right] \tag{14}$$

Finding accurate formulae for compression springs with closed and not ground ends appears to be more difficult than for compression springs with closed and ground ends. For that reason we tried to optimize the correction parameters in order to minimize the sum of the average errors on springs with $n_f = 2$ and $n_f = 5$. We thus obtained:

$n_a = n_f + 2.3$ (15)

$L_{0a} = L_0 - n_e\, d$ (16)

with $n_e = 0.54 - 0.022\, c$ (17)

The bilinear correction appears to significantly improve the results for springs with less than five free coils as the individual error e remains lower than 0.16 (see Fig. 12). For larger numbers of free coils, the corrections of rate and length are more accurate.

4. CONCLUSIONS

Most software dedicated to compression spring design considers the number of active coils n_a to be equal to the number of free coils n_f when calculating the spring rate, whereas several authors [9,16-18] suggest that n_a should be corrected. This fact was the starting point of our study. We developed experiments to test the accuracy of the common formulae that define spring behavior. Cylindrical compression springs with a constant pitch were tested. To be sure that the tests were reliable, we sought to avoid any influence of variations in wire material properties. For that reason, all the tested springs were manufactured with the same wire. Seventy-two springs were tested, which had various numbers of coils, various spring indexes and the 2 main end coils, in order to cover the usual design space. The accuracy of the whole load-length curve of each spring was investigated instead of focusing only on the spring rate as was done in the older studies.

We showed that the common formulae gave quite accurate results when at least 5 free coils were considered for springs with closed and ground ends but results could be of poor accuracy for large indexes and few free coils. A correction on the spring rate proved to be efficient for such springs, as shown by the older studies. We also highlighted, for the first time, that a correction on the external length gave more accurate results.

The experimental study on springs with closed and not ground ends showed that the accuracy of the common formulae was good only for springs with more than 10 free coils for a medium spring index, and more than 5 free coils for a large spring index. Results could be of poor quality when small numbers of coils and small spring index values were considered. For this kind of spring, simultaneous correction on both rate and free length appeared to be very efficient. Such springs showed a bilinear load-length curve. For that reason a bilinear correction was evaluated and gave even more accurate results for springs with few coils.

Of course, it is important to keep in mind that this experimental study was performed using a single wire dimension. It can thus be considered as a first, but important, step in assessing the quality of the analytical formulae concerning compression springs. Several simulations using Abaqus were performed on springs having other geometry and indicated the same conclusions but further investigations will be necessary to fully cover the design space and adjust the correction parameters or define more complex correction strategies.

NOMENCLATURE

c	spring index = D/d
d	wire diameter
D	mean diameter of the spring
D_r	external diameter of the inner rod
e	individual error on loads
G	torsional modulus
k	spring rate
L_{0a}	active free external length
L_0	free external length
L_S	length at solid
L_1	length related to P_1
L_2	length related to P_2
n_e	number related to the effect of end coils on L_{0a}

n_a	number of active coils
n_f	number of free coils
n_T	total number of coils
P	axial load
P_1	axial load related to L_1
P_2	axial load related to L_2

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Table 2 Details of the compression springs with closed and not ground ends

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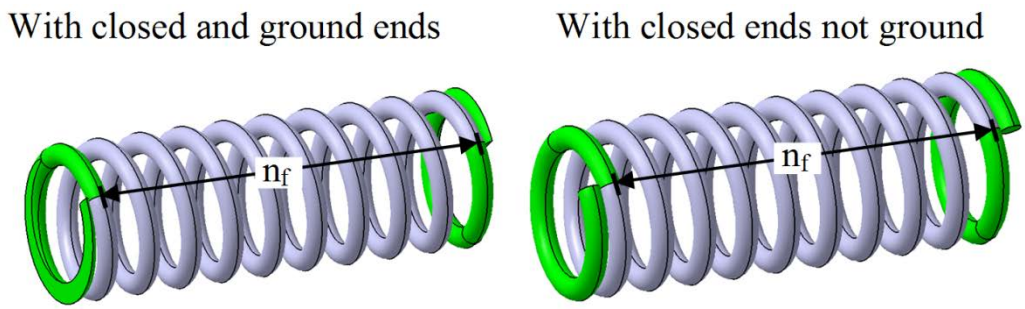

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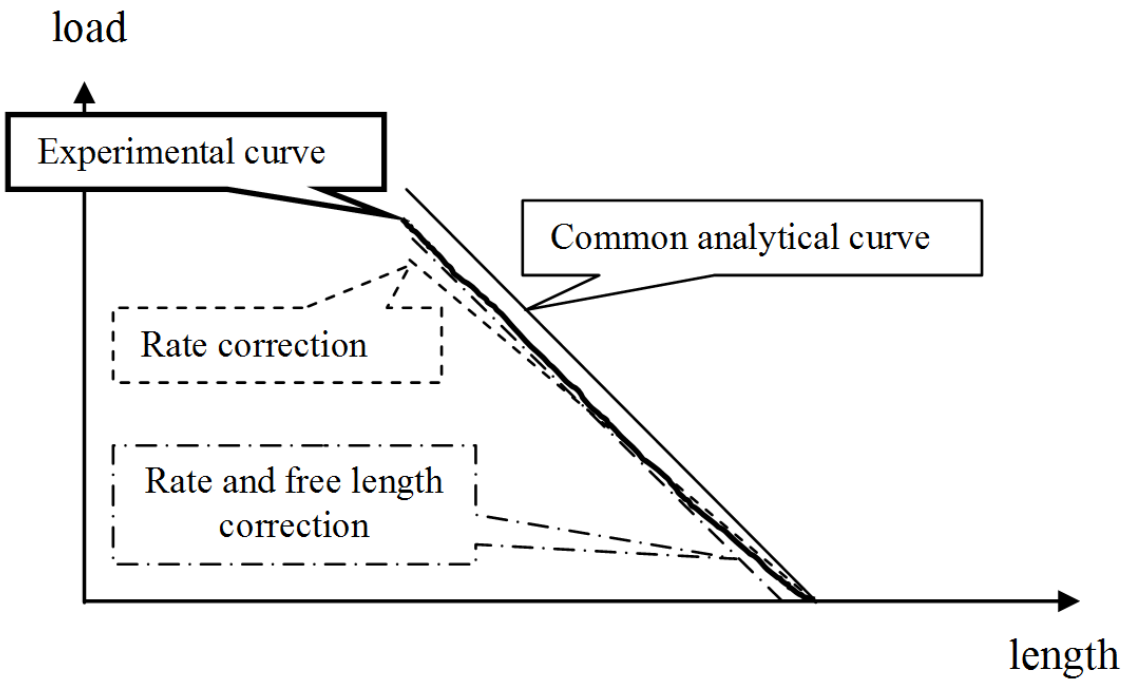

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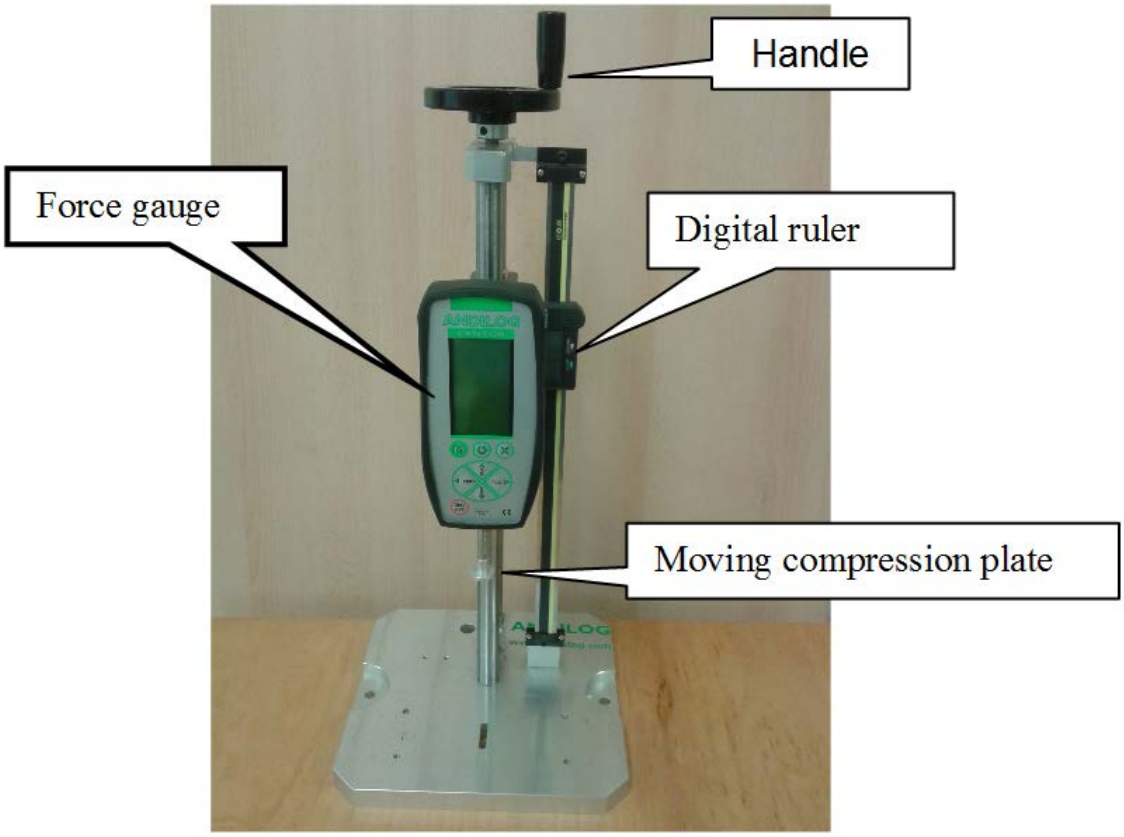

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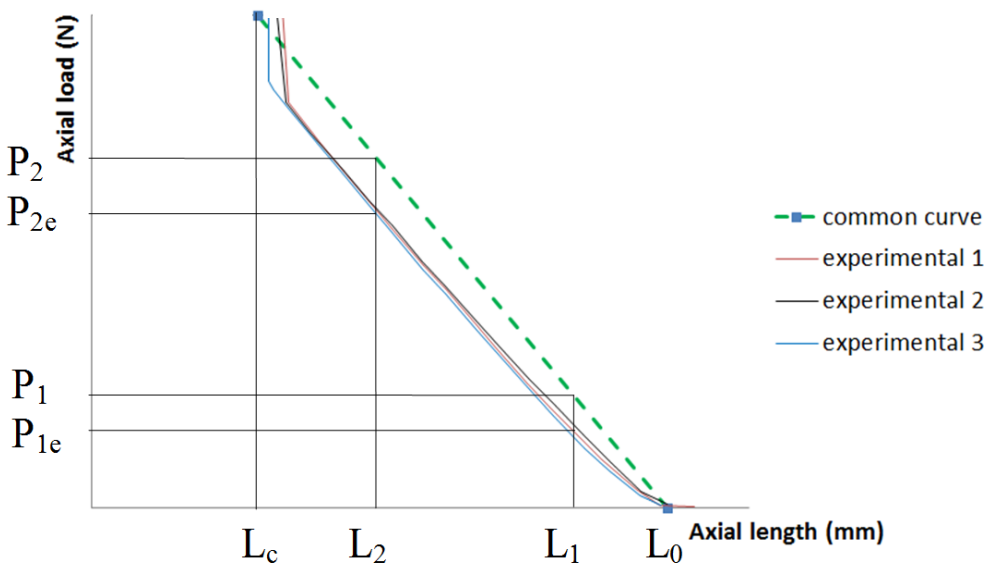

Fig. 4 Details on the lengths and loads used to evaluate the individual error, e, related to configuration CG9

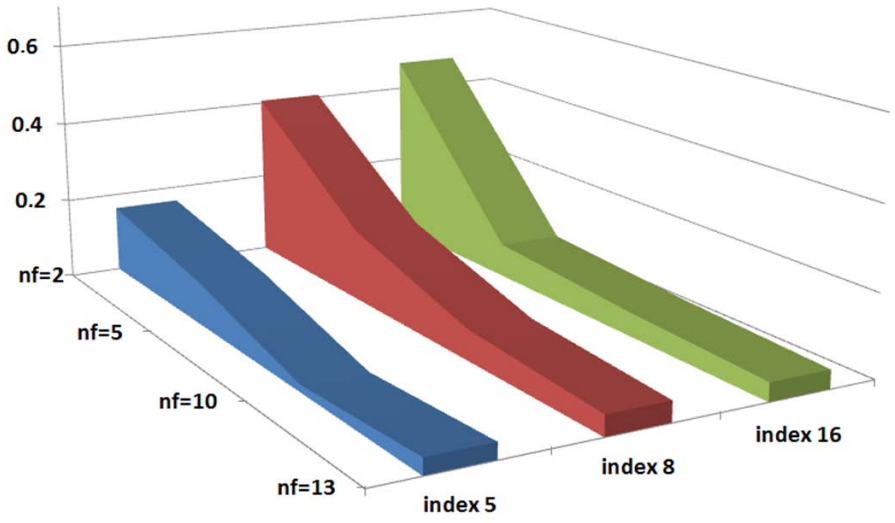

Fig. 5 Individual errors, e, obtained with the common formulae

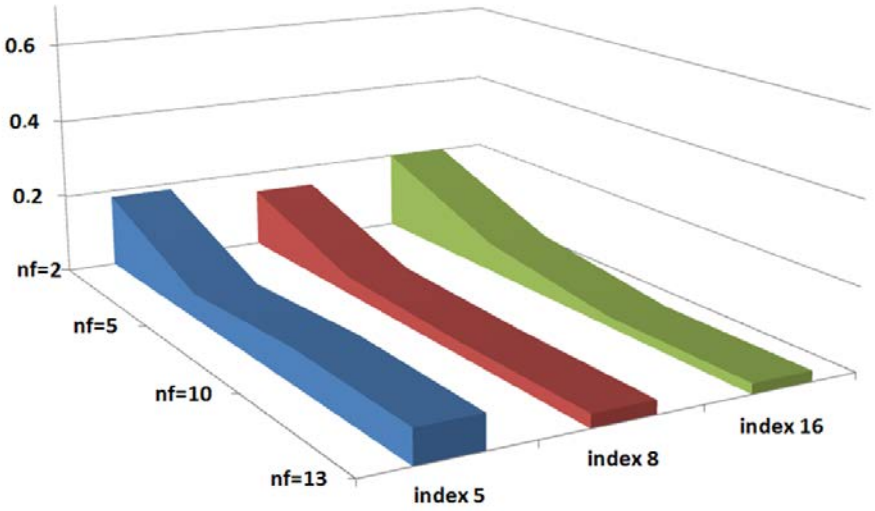

Fig. 6 Individual errors, e, obtained with rate correction

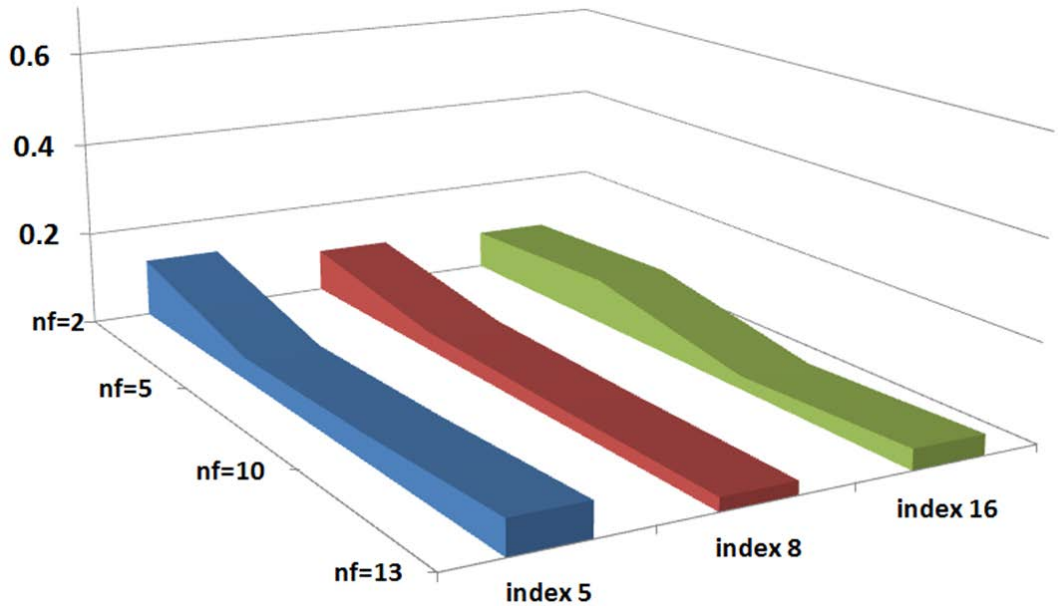

Fig. 7 Individual errors, e, obtained with rate and length correction

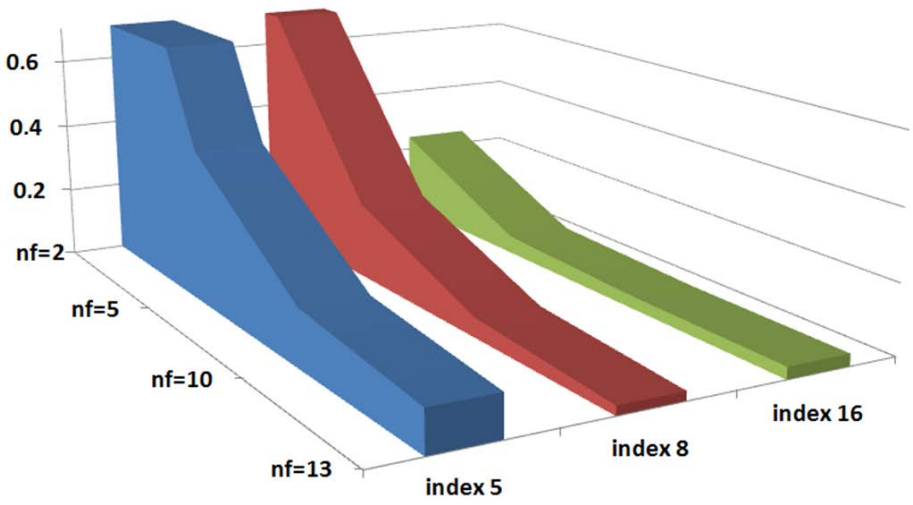

Fig. 8 Individual errors, e, obtained with common formulae

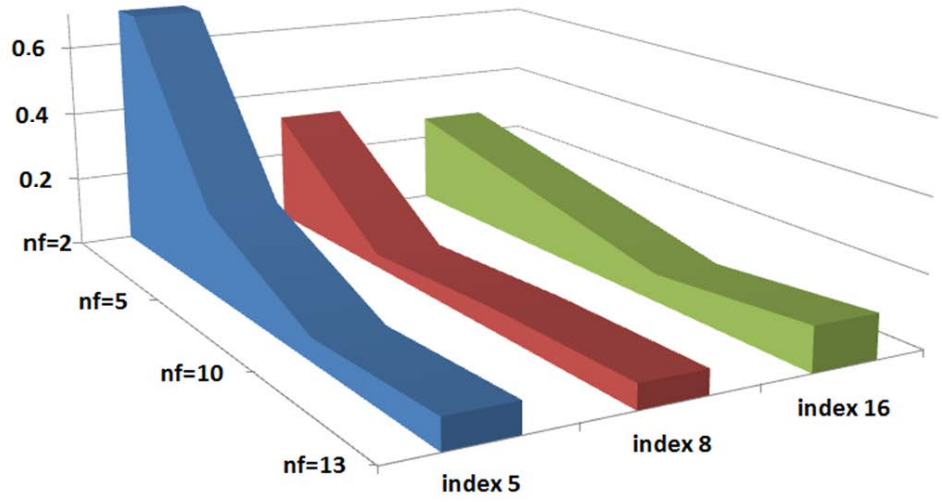

Fig.9 Individual errors, e, obtained with rate correction

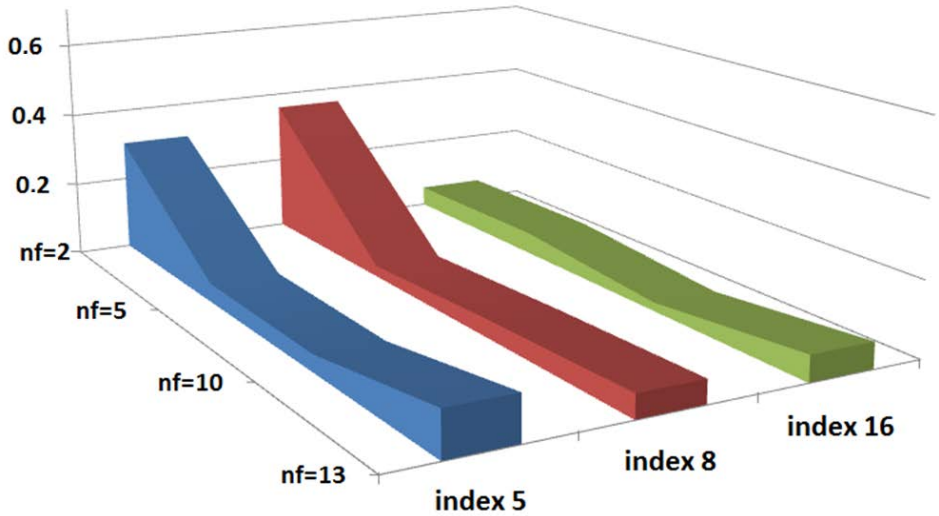

Fig. 10 Individual errors, e, obtained with rate and length correction

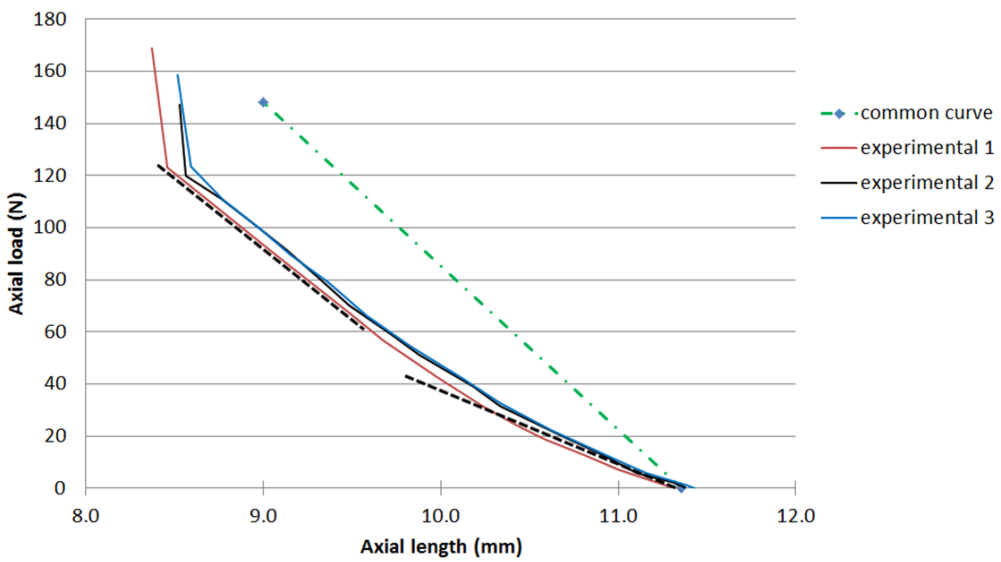

Fig. 11 Experimental load-length curves for reference CNG1

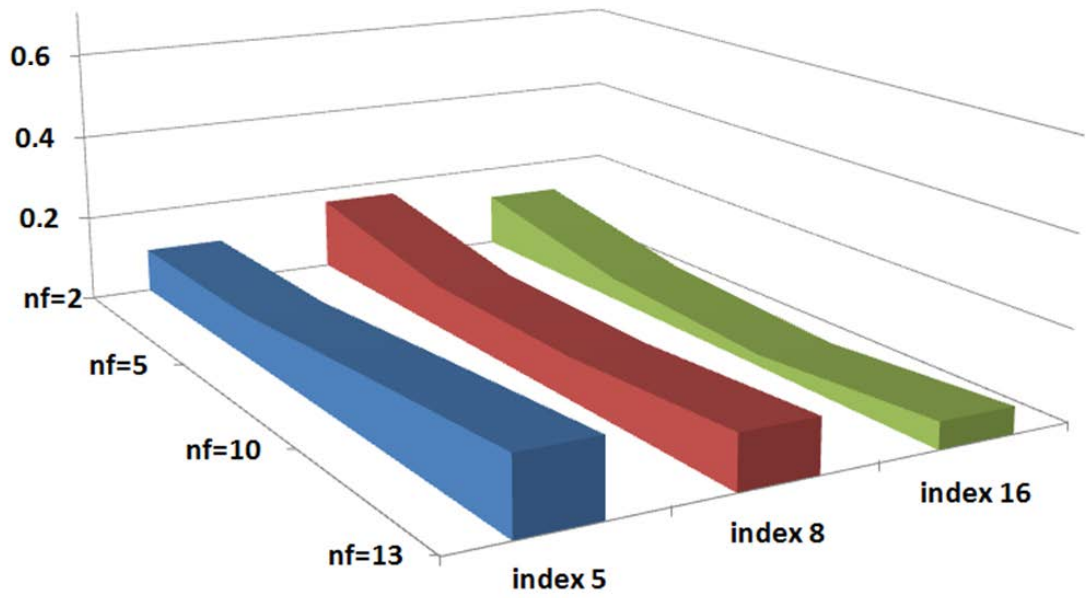

Fig. 12 Individual errors, e, obtained with bilinear correction

Table 1: Details of the compression springs with closed and ground ends

ref	d (mm)	D (mm)	n_f	L_0 (mm)
CG1	1.8	9	2	8.7
CG2	1.8	9	5	17.7
CG3	1.8	9	10	32.7
CG4	1.8	9	13	32.7
CG5	1.8	14.4	2	12.3
CG6	1.8	14.4	5	26.7
CG7	1.8	14.4	10	50.7
CG8	1.8	14.4	13	50.7
CG9	1.8	28.8	2	21.9
CG10	1.8	28.8	5	50.7
CG11	1.8	28.8	10	98.7
CG12	1.8	28.8	13	98.7

Table 2: Details of the compression springs with closed and not ground ends

ref	d (mm)	D (mm)	n_f	L_0 (mm)	D_r (mm)
CNG1	1.8	9	2	11.4	7.00
CNG2	1.8	9	5	20.4	7.00
CNG3	1.8	9	10	35.4	7.00
CNG4	1.8	9	13	38.1	7.00
CNG5	1.8	14.4	2	15.0	12.37
CNG6	1.8	14.4	5	29.4	12.37
CNG7	1.8	14.4	10	53.4	12.37
CNG8	1.8	14.4	13	56.1	12.37
CNG9	1.8	28.8	2	24.6	26.60
CNG10	1.8	28.8	5	53.4	26.60
CNG11	1.8	28.8	10	101.4	26.60
CNG12	1.8	28.8	13	104.1	26.60